Multiplicity Fluctuations and Resonances in Heavy-Ion Collisions

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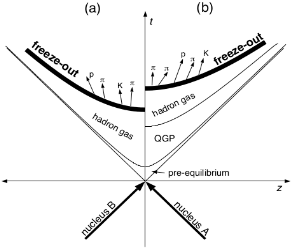
18. 1. 2019

Outline

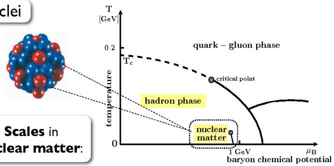
Quark-Gluon Plasma and Heavy Ion Collisions within the Statistical Model

The Quark-Gluon Plasma (QGP) is a state of matter where partons are deconfined, i. e. not confined in hadrons. Deconfinement is phenomenologically (i. e. within the QCD framework) defined as a phase transition.

- the position of the phase transition fully described (at sufficiently high collision energies) by a set of two parameters - T and μ_B
- Hadron Resonance Gas model - reproduction of the equilibrium lQCD results for the lowest order susceptibilities and their ratios reasonably well reproduced
- A+A collisions - Grandcanonical formalism (GCE), $pp, p\bar{p}, e^+e^-$ - Canonical and Microcanonical formalism (CE and MCE)
- moments of net-particle multiplicity distributions from the experiment related to susceptibilities of conserved charges



nuclei



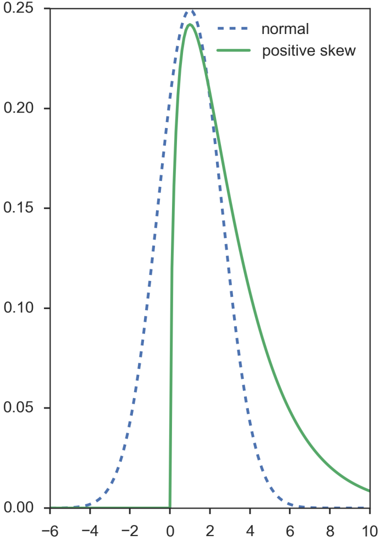
Scales in **nuclear matter**:

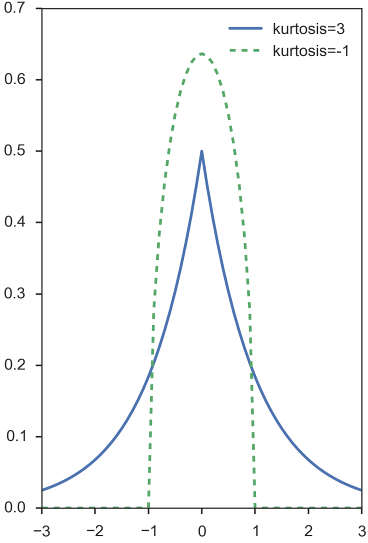
- momentum scale: **Fermi momentum** $p_F \simeq 1.4\ \text{fm}^{-1} \sim 2m_\pi$
- NN distance: $d_{NN} \simeq 1.8\ \text{fm} \simeq 1.3\ m_\pi^{-1}$
- energy per nucleon: $E/A \simeq -16\ \text{MeV}$
- compression modulus: $K = (260 \pm 30)\ \text{MeV} \sim 2m_\pi$

Statistical moments

- m-th statistical moment $\varphi_m(X)^{'}$: $\varphi_m(X)^{'} = E(X^m)$
- m-th central moment $\varphi_m(X)$: $\varphi_m(X) = E(X - EX)^m$
- first four central moments are of great significance
- **mean:** $M = \varphi_1$, **variance:** $\sigma^2 = \varphi_2$
- **skewness:** $S = \varphi_3/\varphi_2^{3/2}$ - measure of the assymetry of the probability distribution
- **kurtosis:** $\kappa = \varphi_4/\varphi_2^2$ - measure of the "tailedness" of the probability distribution

Skewness (left) and kurtosis (right).





Calculation of the multiplicity fluctuations within the statistical model

- grandcanonical and canonical ensemble assumed, event-by-event distributions of conserved quantities - characterized by the moments (M, σ, S, κ)
- introduction of the following products: $S\sigma = \varphi_3/\varphi_2$, $\kappa\sigma^2 = \varphi_4/\varphi_2, M/\sigma^2 = \varphi_1/\varphi_2$, $S\sigma^3/M = \varphi_3/\varphi_1$ -the volume term in the distribution gets obviously cancelled; direct comparison of experimental measurement and theoretical calculation possible
- large volume limit (V$\rightarrow \infty$) - all statistical ensembles (MCE, CE, GCE) equivalent

Partition functions in statistical ensembles - GC formalism

- HRG model - all relevant degrees of freedom contained in the partition function
- confined, strongly interacting matter - interactions that result in resonance formation included
- **GC partition function:** $Z_{GC}(\lambda_j) = \prod_j \exp\left[\sum_{n_j=1}^{+\infty} \frac{z_j(n_j)\lambda_j^{n_j}}{n_j}\right]$ where $z_j(n_j) = (\mp 1)^{n_j+1} \frac{d_j V}{2\pi^2 n_j} T m_j^2 K_2\left(\frac{n_j m_j}{T}\right)$ is the single particle partition function
- $K_2 \ldots$ modified Bessel function, $V \ldots$ volume of the hadron gas
- $\lambda_j = \exp(\frac{\mu_j}{T}) \ldots$ fugacity for each particle species j, $m_j \ldots$ hadron mass
- $\mu_j \ldots$ chemical potential of a particle species j, $d_j = 2J_j + 1 \ldots$ spin degeneracy
- $\mp \ldots$ upper sign for fermions, lower sign for bosons

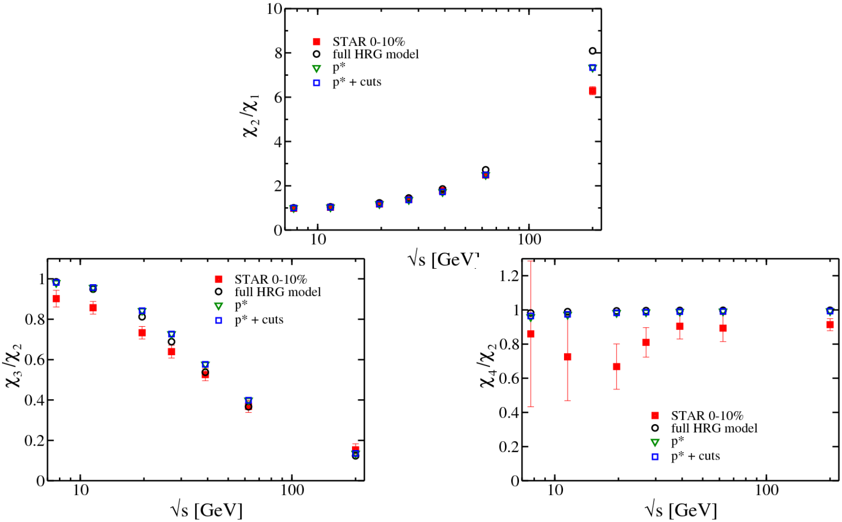
Fluctuations in a hadron resonance gas model with chemical equilibrium

Susceptibilities and cumulants:

- $\chi_l^{(i)} = \frac{\partial^l (P/T)^4}{\partial(\mu_i/T)^l} |_T$
- $\chi_1^{(i)} = \frac{1}{VT^3} \langle N_i \rangle_c = \frac{1}{VT^3} \langle N_i \rangle$
- $\chi_2^{(i)} = \frac{1}{VT^3} \left\langle (\Delta N_i)^2 \right\rangle_c = \frac{1}{VT^3} \left\langle (\Delta N_i)^2 \right\rangle$
- $\chi_3^{(i)} = \frac{1}{VT^3} \left\langle (\Delta N_i)^3 \right\rangle_c = \frac{1}{VT^3} \left\langle (\Delta N_i)^3 \right\rangle$
- $\chi_4^{(i)} = \frac{1}{VT^3} \left\langle (\Delta N_i)^4 \right\rangle_c = \frac{1}{VT^3} \left(\left\langle (\Delta N_i)^4 \right\rangle - 3 \left\langle (\Delta N_i)^2 \right\rangle^2 \right)$

Equilibrium pressure:

- $P/T^4 = \frac{1}{VT^3} \sum_i \ln Z_{m_i}^{M/B}(V, T, \mu_B, \mu_Q, \mu_S)$
- $\ln Z_{m_i}^{M/B} = \mp \frac{V d_i}{(2\pi)^3} \int d^3k \ln(1 \mp z_i \exp(-\epsilon_i/T))$
- $\epsilon_i = \sqrt{k^2 + m_i^2}$
- $z_i = \exp\left(\frac{\mu_i}{T}\right),\ \mu_i = B_i \mu_B + S_i \mu_S + Q_i \mu_Q.$


STAR 0-10%
full HRG model
p*
p* + cuts
χ_2/χ_1
$\sqrt{s}$ [GeV]
χ_3/χ_2
$\sqrt{s}$ [GeV]
χ_4/χ_2
$\sqrt{s}$ [GeV]

Inclusion of resonances

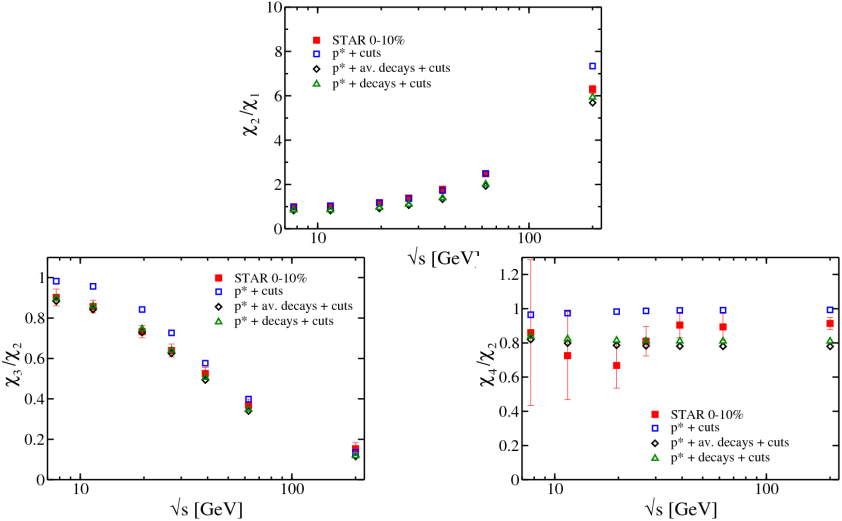
$$VT^3 \frac{\partial(P/T^4)}{\partial(\mu_h/T)}|_T = \langle N_h \rangle + \sum_R \langle N_R \rangle \langle n_h \rangle_R$$

where $\langle N_h \rangle$ and $\langle N_R \rangle$ are the means of the primordial numbers of hadrons and resonances, respectively. The sum runs over all the resonances in the model.

- $\langle n_h \rangle_R \equiv \sum_r b_r^R n_{h,r}^R$
- b_r^R - the branching ratio of the decay-channel and $n_{h,r}^R = 0, 1, \ldots$ - number of hadrons h formed in that specific decay-channel.
- The related susceptibilities are then given by

$$\hat{\chi}_l{}^{(h)} = \chi_l{}^{(h)} + \sum_R \chi_l^{(R)} \langle n_h \rangle_R^l$$

$\rightarrow$ formulae for statistical quantities (and their respective ratios) retain their form, only hats are added.


STAR 0-10%
p* + cuts
p* + av. decays + cuts
p* + decays + cuts
χ2/χ1
χ3/χ2
χ4/χ2
√s [GeV]

Fluctuations in a hadron resonance gas model with chemical non-equilibrium I.

Non-equilibrium pressure:

- $P/T^4 = \frac{1}{VT^3} \sum_i \ln Z_{m_i}^{M/B}(V, T, \mu_i)$
- $\ln Z_{m_i}^{M/B} = \mp \frac{Vd_i}{(2\pi)^3} \int d^3k \ln(1 \mp z_i \exp(-\epsilon_i/T))$
- $\epsilon_i = \sqrt{k^2 + m_i^2}$
- $z_i = \exp\left(\frac{\mu_i}{T}\right)$, $\mu_i = \sum_j N_{ji}\mu_j$.
- N_{ji} - average number of stable particles emerging in the decay of the level i
- μ_j - chemical potential of the j-th stable particle
- 26 particle species we consider stable: $\pi^0, \pi^+, \pi^-, K^+, K^-, K^0, \bar{K}_0, \eta$ and $p, n, \lambda^0, \sigma^+, \sigma^0, \sigma^-, \Xi^0, \Xi^-, \Omega^-$ and their respective anti-baryons
- assumption: chemical potential of the mother equal to the sum of the chemical potentials of the daughters

Fluctuations in a hadron resonance gas model with chemical non-equilibrium II.

The thermodynamic susceptibility χ_l of particle species a is given by

$$\chi_l^{(a)} = \frac{\partial^l (P/T^4)}{\partial(\mu_a/T)^l} = T^l \frac{\partial^l (P/T^4)}{\partial \mu_a^l}.$$

For partial pressure P/T^4, we obtain

$$\frac{P}{T^4} = \frac{1}{2\pi^2 T^2} \sum_i \sum_{k=1}^{+\infty} d_i m_i^2 \frac{(-1)^{k+1}}{k^2} \exp\left(\frac{k}{T} \sum_{j \in A} N_{ji} \mu_j\right) K_2\left(\frac{k m_i}{T}\right),$$

then the corresponding thermodynamic susceptibility reads

$$\chi_l^{(a)} = \frac{1}{2\pi^2 T^2} \sum_i \sum_{k=1}^{+\infty} d_i m_i^2 (-1)^{k+1} k^{l-2} N_{ai}^l \exp\left(\frac{k}{T} \sum_{j \in A} N_{ji} \mu_j\right) K_2\left(\frac{k m_i}{T}\right).$$

Fluctuations in a hadron resonance gas model with chemical non-equilibrium III.

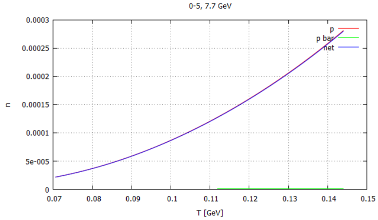
Obviously, the ratio of any two thermodynamic susceptibilities of the same particle species a, denoted $\chi_l^{(a)}$ and $\chi_n^{(a)}$, $l \neq n$, can be written as

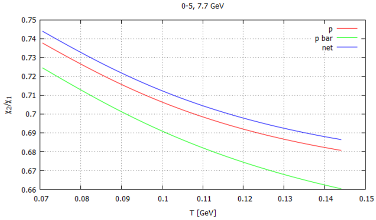
$$\frac{\chi_l^{(a)}}{\chi_n^{(a)}} = \frac{\sum_i \sum_{k=1}^{+\infty} d_i m_i^2 (-1)^{k+1} k^{l-2} N_{ai}^l \exp\left(\frac{k}{T}\sum_{j\in A} N_{ji}\mu_j\right) K_2\left(\frac{km_i}{T}\right)}{\sum_i \sum_{k=1}^{+\infty} d_i m_i^2 (-1)^{k+1} k^{n-2} N_{ai}^n \exp\left(\frac{k}{T}\sum_{j\in A} N_{ji}\mu_j\right) K_2\left(\frac{km_i}{T}\right)}.$$

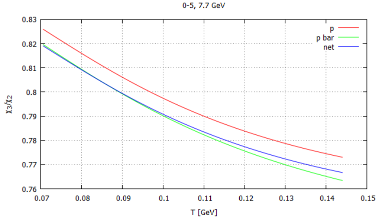
Results

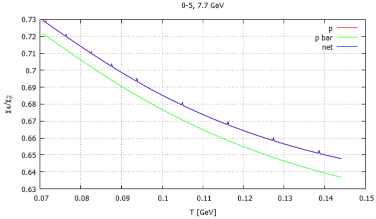
- implementation of the derived formulae using data from DRAGON with the newest PDG update
- calculations performed for the most central $Au + Au$ collisions (centrality 0-5 and 5-10) and for seven collision energies $\sqrt{s_{NN}} = 7.7, 11.5, 19.6, 27.0, 39.0, 62.4, 200$ GeV
- RHIC Beam Energy Scan program - ratio fits (GCER) have been used - corresponding chemical freeze-out parameters for grand canonical ensemble
- temperature dependencies of the (net-)proton number densities and the ratios of thermodynamic susceptibilities $\omega = \frac{\chi_2}{\chi_1}$, $S\sigma = \frac{\chi_3}{\chi_2}$ and $\kappa\sigma^2 = \frac{\chi_4}{\chi_2}$ for each of the collision energies and each centrality

Results for centrality 0-5 and $\sqrt{s_{NN}} = 7.7$ GeV

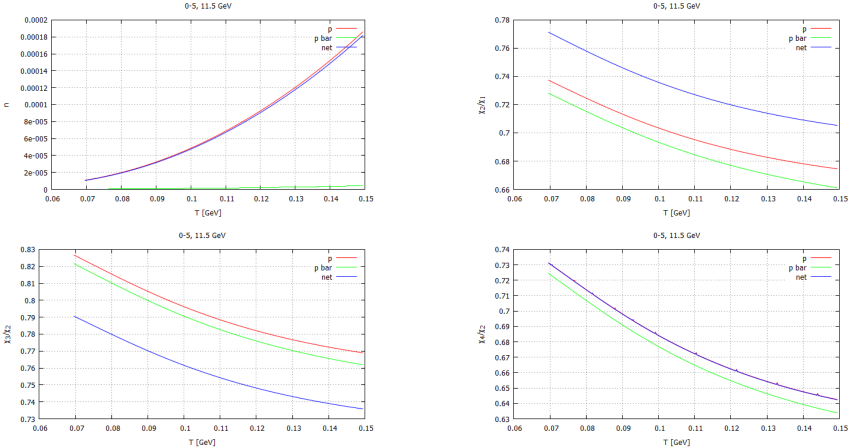




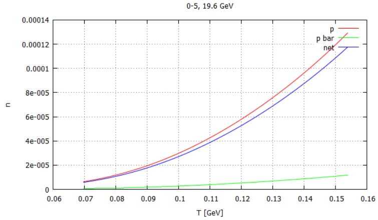


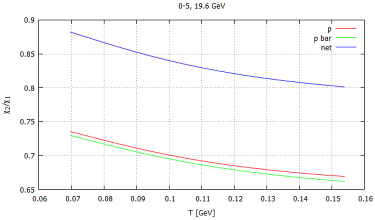


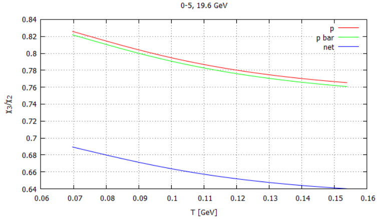
Results for centrality 0-5 and $\sqrt{s_{NN}} = 11.5$ GeV

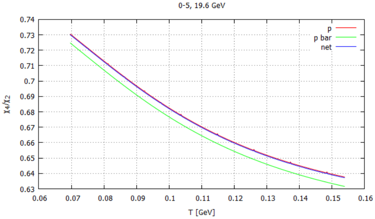


Results for centrality 0-5 and $\sqrt{s_{NN}} = 19.6$ GeV

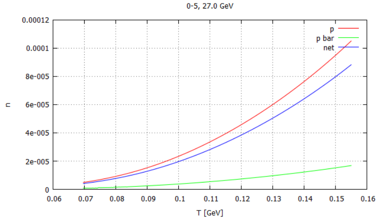


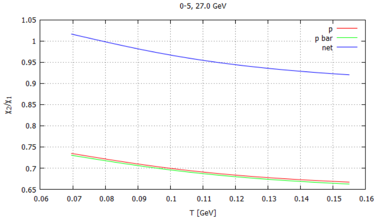


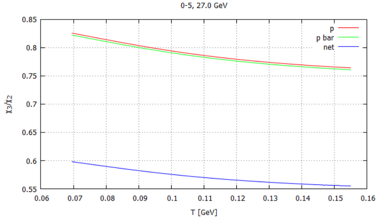


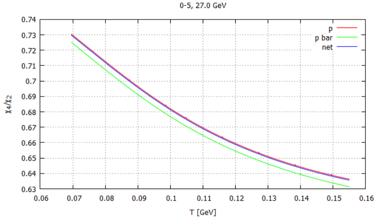


Results for centrality 0-5 and $\sqrt{s_{NN}} = 27.0$ GeV

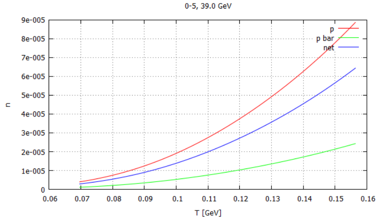


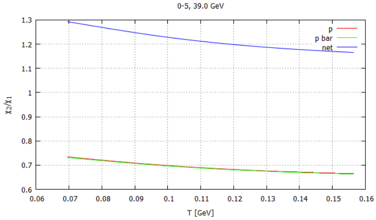


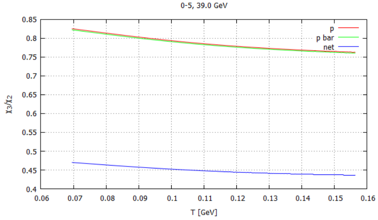


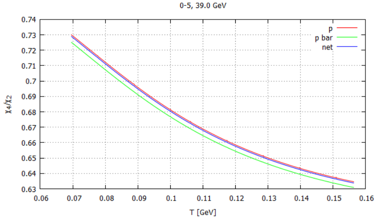


Results for centrality 0-5 and $\sqrt{s_{NN}} = 39.0$ GeV

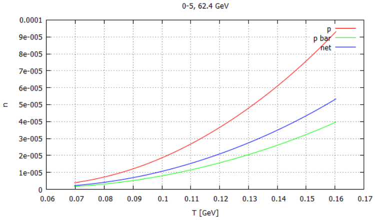


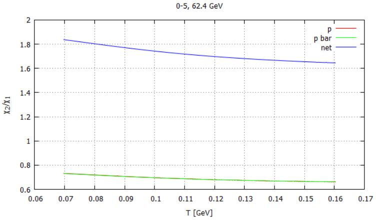


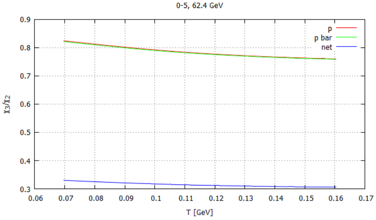


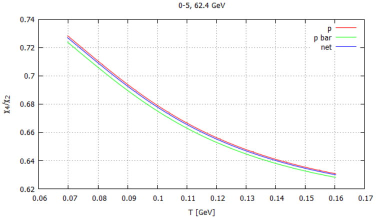


Results for centrality 0-5 and $\sqrt{s_{NN}} = 62.4$ GeV

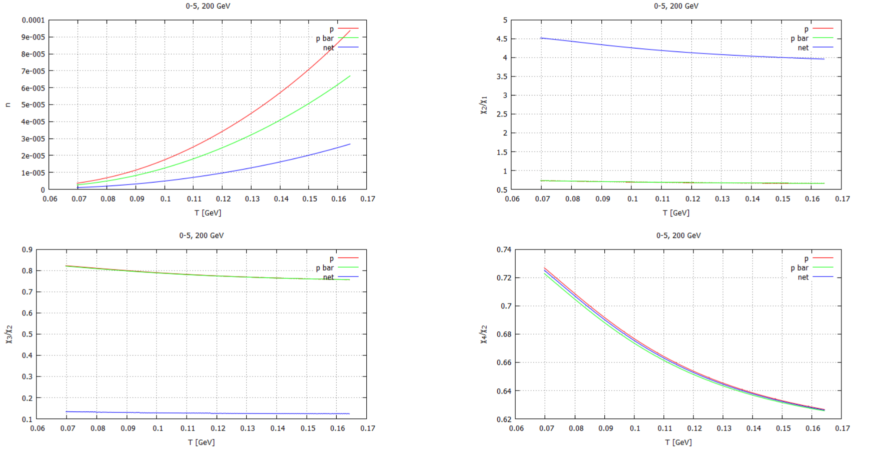




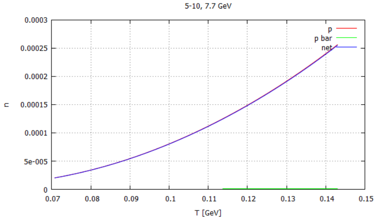


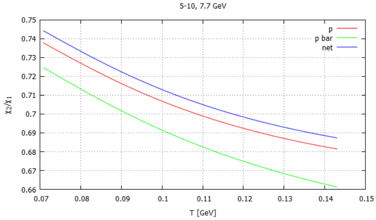


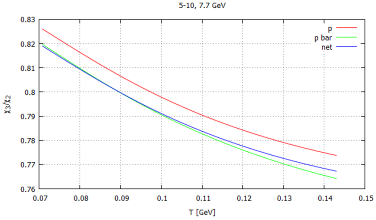
Results for centrality 0-5 and $\sqrt{s_{NN}} = 200$ GeV

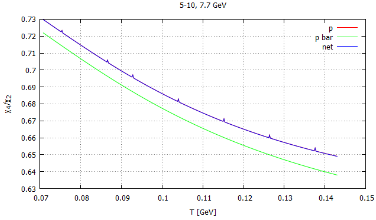


Results for centrality 5-10 and $\sqrt{s_{NN}} = 7.7$ GeV

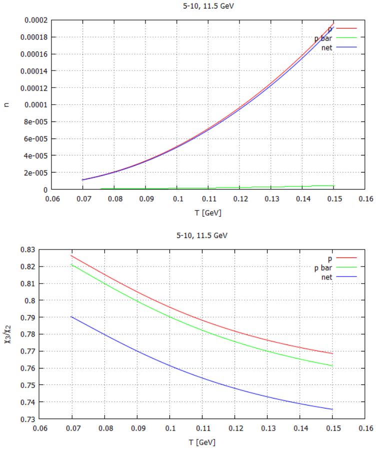




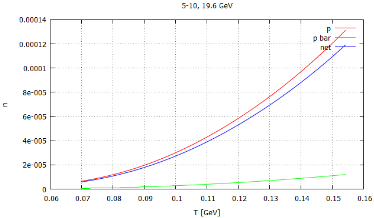


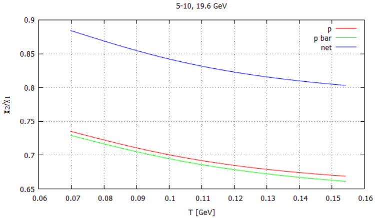


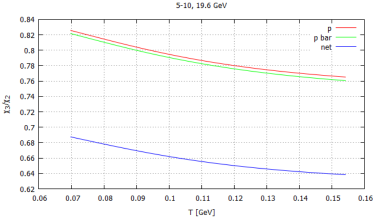
Results for centrality 5-10 and $\sqrt{s_{NN}} = 11.5$ GeV

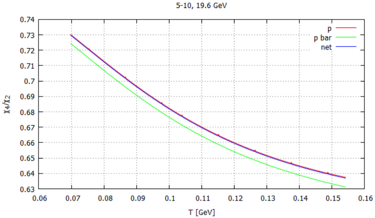


Results for centrality 5-10 and $\sqrt{s_{NN}} = 19.6$ GeV

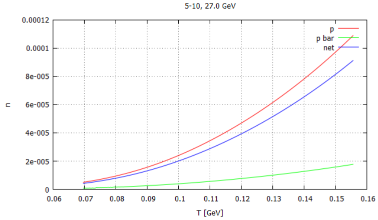


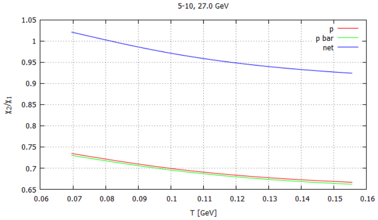


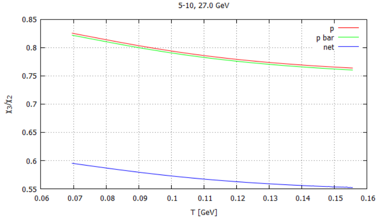


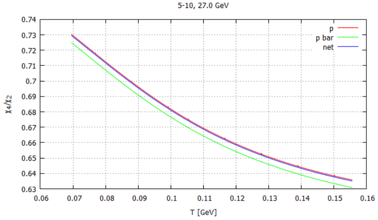


Results for centrality 5-10 and $\sqrt{s_{NN}} = 27.0$ GeV

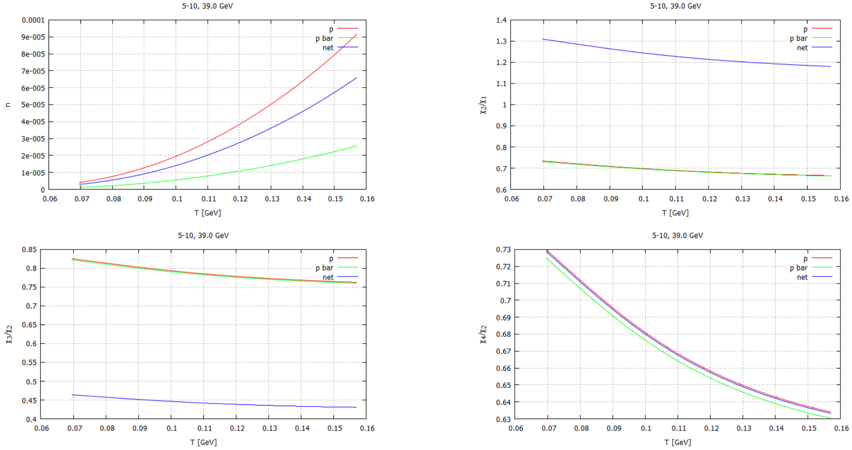




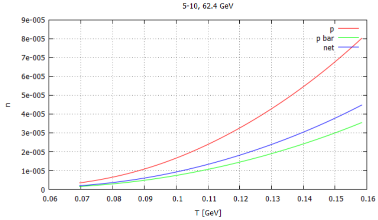


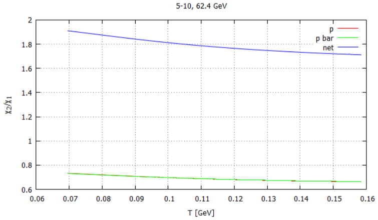


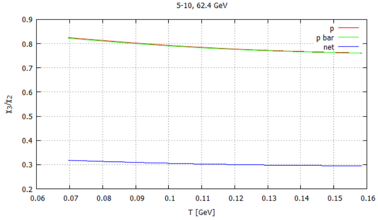
Results for centrality 5-10 and $\sqrt{s_{NN}} = 39.0$ GeV

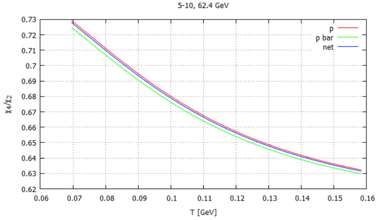


Results for centrality 5-10 and $\sqrt{s_{NN}} = 62.4$ GeV

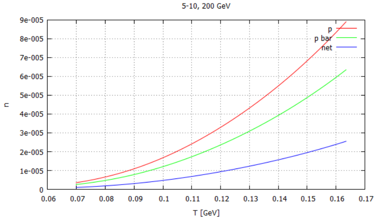


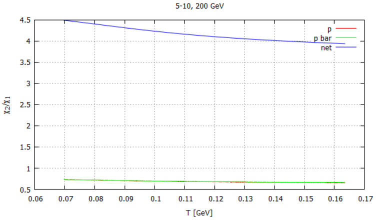


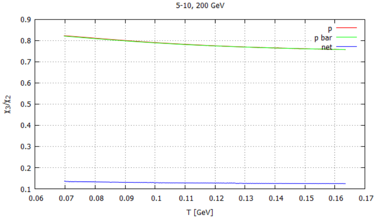


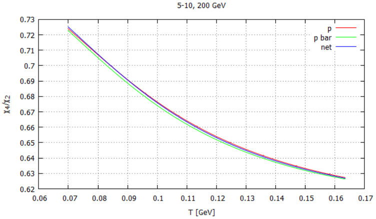


Results for centrality 5-10 and $\sqrt{s_{NN}} = 200$ GeV

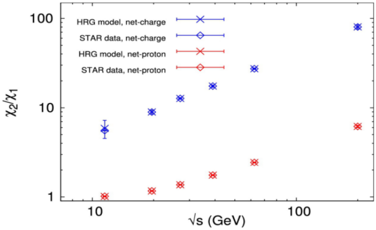


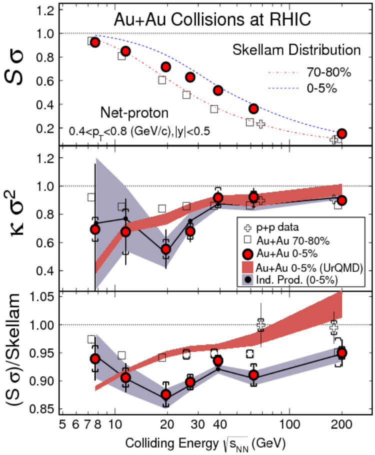






Experimental data





Conclusion

- derivation of the first four moments corresponding to the state of chemical non-equilibrium using the fact that chemical potentials appear for **each** stable type of hadrons $\rightarrow$ calculation of the moments of baryon number distribution and proton multiplicity depending on the temperature of the system
- temperature dependence of specific ratios of thermodynamic susceptibilities for protons, antiprotons and net protons explored using the RHIC BES program
- comparison with relevant experimental data performed