Coupling of Λ With One-Phonon Excitation of Nuclear Core

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Outline

- Hypernuclear Physics – Theory and Experiment
- Models of NN and ΛN Interactions
- Mean-Field Model of Hypernuclei
- Tamm-Dancoff Approximation
  - $N\Lambda$ TDA
- Equation of Motion Phonon Method
  - Coupling of Λ to Phonon Excitations
  - Coupling of $N\Lambda$ TDA to Phonon Excitations
- Results
- Conclusions
- Future Plans

Motivation

- Hyperons (except Σ^0) – lifetime $\approx 10^{-10}$s – enough time to bind with nucleons
- Hypernuclei – discovered in 1952 by Pniewski and Danysz [1]
  - study of hyperon-nucleon (YN) and hyperon-hyperon (YY) interaction
    - 38 experimental data points for low-energy hyperon-proton scattering
    - hyperon-hyperon scattering is impossible to measure
  - strange particles in the lowest bound states – probes of nuclear interior
  - hypothesis: neutron stars – condensed neutron matter – hyperonic degrees of freedom [2]
    - hyperon puzzle – too soft EoS
- observed about 30 hypernuclei with one Λ from ${}^{3}_{\Lambda}$H to ${}^{208}_{\Lambda}$Bi and ${}^{208}_{\Lambda}$Pb
- 3 hypernuclei with two Λ – ${}^{6}_{\Lambda\Lambda}$He, ${}^{10}_{\Lambda\Lambda}$Be, and ${}^{13}_{\Lambda\Lambda}$B
- one hypernuclei with Σ – ${}^{4}_{\Sigma}$He
- BNL – antihypernucleus ${}^{3}_{\Lambda}\bar{\mathrm{H}}$ (2010)

[1] M. Danysz, J. Pniewski, Phil. Mag. **44**, 348 (1953)
[2] J. Schaffner-Bielich, Nucl. Phys. A **804**, 309-321 (2008)

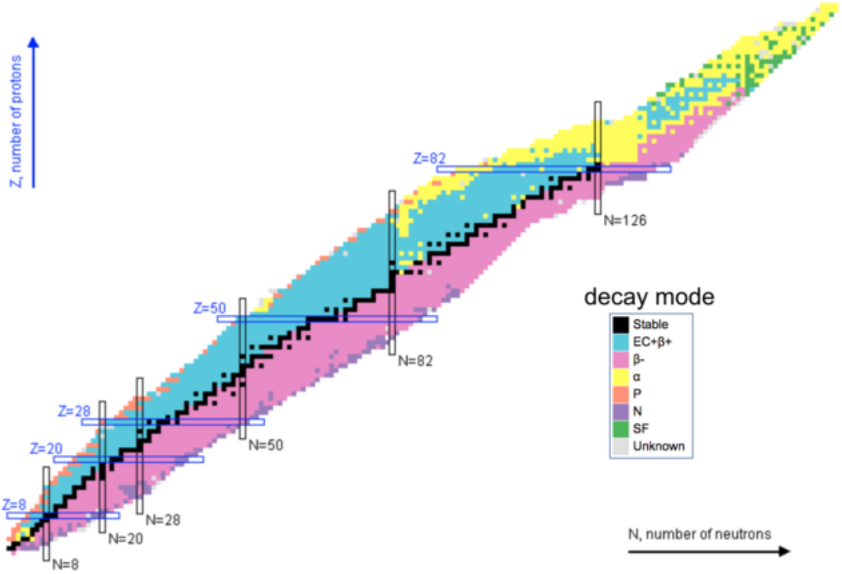

Figure: Two-dimensional nuclear chart.

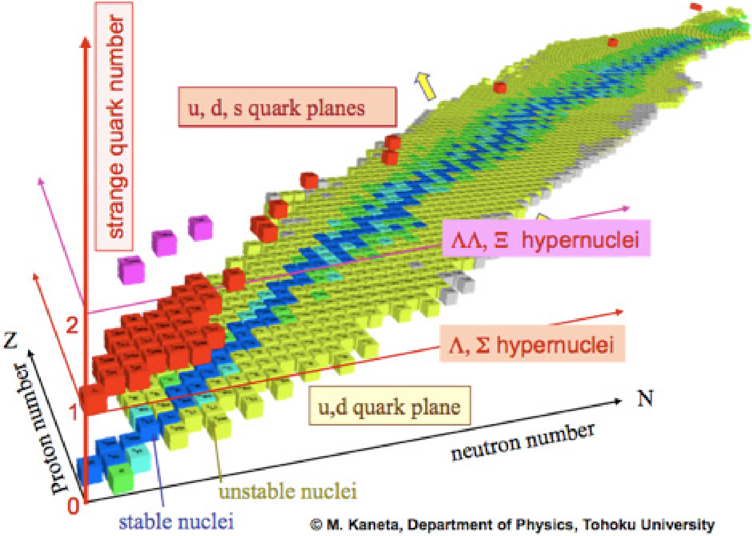

Figure: Three-dimensional nuclear chart.

Experimental Background

- production mechanisms (one nucleon changes to Λ)
  - (a) strangeness exchange $K^- + n \rightarrow \Lambda + \pi^-$ [1]
  - (b) associated production $\pi^+ + n \rightarrow \Lambda + K^+$ [2]
  - (c) electroproduction $e^- + p \rightarrow e^{-'} + \Lambda + K^+$ [3]
    - inelastic scattering of e^- on a nuclear target
- heavy ion collisions (CERN,BNL) [4,5,6]

[1] Heidelberg-Saclay Collaboration, Phys. Lett. B, **158**, 1, 19-22 (1985)
[2] P. H. Pile et al., Phys. Rev. Lett., **66**, 2585 (1991)
[3] Jefferson Lab Hall A Collaboration, Phys. Rev. Lett **990**, 052501 (2007)
[4] CERN-Lyons-Warsaw Collaboration, Phys. Lett. B **83**, 252-256 (1979)
[5] Armstrong et al., Phys. Rev. C **70** 024902 (2004)
[6] L. Xue et al., Phys. Rev. C **92**, 059901 (2015)

Models of NN and ΛN Interactions

- phenomenological (Skyrme [1], Gogny [2]) – construction of energy-density functional (EDF) **or** NN potential
- realistic – based on meson exchanges – CD-Bonn, Argonne V18 [3,4]
  - mean-field calculations require renormalization procedures – UCOM, G-Matrix, SRG [5,6,7]
- effective field theory (EFT) – field theory with broken chiral SU(2) symmetry [8]
  - nucleons and pions as degrees of freedom
  - coupling constants fitted to experimental data of nucleon-nucleon scattering, phase shifts, properties of light nuclei

[1] T. Skyrme, Nucl. Phys. **9**, 615 (1958)
[2] D. Gogny et al., Phys. Rev. C **21**, 1568 (1980)
[3] R. Machleidt et al., Phys. Rev. C **53**, R1483 (1996)
[4] R. B. Wiringa et al., Phys. Rev. C **51**, 38 (1995)
[5] H. Feldmeier et al., Nucl. Phys. A **632**, 61 (1998)
[6] M. Hjorth-Jensen et al., Phys. Rep. **261**, 125 (1995)
[7] S. Bogner et al., Prog. Part. Nucl. Phys. **65**, 94 (2010)
[8] E. Epelbaum et al., Rev. Mod. Phys. **81**, 1773 (2009)

Models of NN and ΛN Interactions

- effective ΛN potentials – Nijmegen, Jülich groups [1]
  - soft-core potentials ESC08 [2,3]
- chiral potential at LO [4] or NLO [5]
  - strong cut-off dependence

[1] J. Haidenbauer et al., Phys. Rev. C **72**, 044005 (2005),
[2] M. Isaka et al., Phys Rev. C **89**, 024310 (2014),
[3] Y. Yamamoto et al., Prog. Theor. Phys. Supp. **185**, 72 (2010),
[4] H. Polinder et al., Nucl. Phys. A **779**, 244 (2006),
[5] J. Haidenbauer et al., Nucl. Phys. A **915**, 24 (2013)

Mean-Field Model of Hypernuclei

- hypernucleus $\equiv \underbrace{\text{A-1 nucleons}}_{\text{nuclear core}} + \underbrace{1\,\Lambda}_{\text{hyperon}} \equiv {}^{A}_{\Lambda}\mathrm{X}$
- interactions among baryons in the hypernuclear system
  - nuclear core: $NN + 3N$ potential $\mathrm{N^2LO_{sat}}$ [1]
  - hyperon-nucleons: $\Lambda N - \Lambda N$ channel of LO YN potential [2]
- construction of Hartree-Fock (HF) single-particle (s.p.) basis
  - unitary transformation between harmonic oscillator (HO) and HF basis
  - creation and annihilation operators $a_i^\dagger, a_i$ (p), $b_i^\dagger, b_i$ (n), $c_i^\dagger, c_i$ (Λ)

$$a_i^\dagger|0\rangle = |i\rangle = |n_i, l_i, j_i, m_i\rangle \quad \& \quad a_i|i\rangle = |0\rangle \tag{1}$$

  - ground-state wave function:

$$|\mathrm{HF}\rangle = \prod_{i=1}^{Z} a_i'^\dagger|0\rangle \otimes \prod_{i=1}^{N} b_i'^\dagger|0\rangle \otimes c_1'^\dagger|0\rangle \tag{2}$$

[1] A. Ekström et al., Phys. Rev. C **91**, 051301 (2015)
[2] H. Polinder et al., Phys. Rev. C **72**, 044005 (2005)

Hartree-Fock (HF) Method

- wave functions of fermionic systems – totally antisymmetric – Slater determinats
- second quantization – operators as matrix elements

$$\widehat{T}^{\mathrm{p}} = \sum_{ij} t^{\mathrm{p}}_{ij} a^{\dagger}_{i} a_{j}, \quad t^{\mathrm{p}}_{ij} = \langle i | \widehat{T}^{\mathrm{p}} | j \rangle \tag{1}$$

$$\widehat{V}^{\mathrm{pp}} = \frac{1}{2} \sum_{ijkl} V^{\mathrm{pp}}_{ijkl} a^{\dagger}_{i} a^{\dagger}_{j} a_{l} a_{k}, \quad V^{\mathrm{pp}}_{ijkl} = \langle ij | \widehat{V}^{\mathrm{pp}} | kl - lk \rangle \tag{2}$$

- Hamiltonian of the hypernuclear system

$$\widehat{H} = \widehat{T}^{N} + \widehat{T}^{\Lambda} + \widehat{V}^{NN} + \widehat{V}^{NNN} + \widehat{V}^{\Lambda N} - \widehat{T}^{CM} \tag{3}$$

Hartree-Fock Equation

$$\frac{\delta}{\delta U} \left(\frac{\langle \mathrm{HF} | \widehat{H} | \mathrm{HF} \rangle}{\langle \mathrm{HF} | \mathrm{HF} \rangle} \right) = 0 \tag{4}$$

Tamm-Dancoff Approximation (TDA)

- Wick's theorem – Hamiltonian normal ordered in HF basis

$$\widehat{H} = \widehat{H}^{(0)} + \widehat{H}^{(1)} + \widehat{H}^{(2)} + \widehat{H}^{(3)} \tag{1}$$

- ground state – all states below Fermi level are occupied (p,n,Λ separately)
- particle states $a^\dagger_p, a_p$, hole states $a^\dagger_h, a_h$
- *phonon* creation operator

$$Q^\dagger_\mu = \sum_{ph} c^{\mu,\mathrm{p}}_{ph} a^\dagger_p a_h + c^{\mu,\mathrm{n}}_{ph} b^\dagger_p b_h \tag{2}$$

- one-phonon excitation

$$|\mu\rangle = Q^\dagger_\mu |\mathrm{HF}\rangle \tag{3}$$

TDA Equation of Motion

$$\langle \mathrm{HF} | Q_{\nu'} [\widehat{H}, Q^\dagger_\nu] | \mathrm{HF} \rangle = (E_\nu - E_{\mathrm{HF}}) \delta_{\nu\nu'} \tag{4}$$

NΛ TDA

- starting point – doubly magic nucleus (e.g. ^{16}O) – construction of mean field and HF basis
- $N\Lambda$ TDA – annihilation of one neutron (proton) and creation of one Λ

$$R^{\dagger}_{\mu,\mathrm{n}\Lambda} = \sum_{ph} r^{\mu,\mathrm{n}\Lambda}_{ph} c^{\dagger}_{p} b_{h} \tag{1}$$

$N\Lambda$ TDA Equation of Motion

$$\langle \mathrm{HF} | R_{\nu',\mathrm{n}\Lambda} [\widehat{H}, R^{\dagger}_{\nu,\mathrm{n}\Lambda}] | \mathrm{HF} \rangle = (E^{\mathrm{n}\Lambda}_{\nu} - E_{\mathrm{HF}}) \delta_{\nu\nu'} \tag{2}$$

Equation of Motion Phonon Method (EMPM)

- starting point is TDA (1 phonon)
- construction of n phonon basis from $n-1$ phonon basis
- many-particle correlations
- wave function of correlated ground state

Equation of Motion

$$\langle n+1;\beta_{n+1}|[\widehat{H},Q_{\mu}^{\dagger}]|n;\alpha_n\rangle = (E_{\beta_{n+1}} - E_{\alpha_n})\langle n+1;\beta_{n+1}|Q_{\mu}^{\dagger}|n;\alpha_n\rangle \quad (1)$$

- successfully implemented in nuclear calculations [1,2,3]

[1] D. Bianco et al., Phys. Rev. C **85**, 014313 (2012)
[2] G. De Gregorio et al., Phys. Rev. C **94**, 061301 (2016)
[3] G. De Gregorio et al., Phys. Rev. C **93**, 044314 (2016)

Coupling of Λ with TDA Phonons

- analogy to [1,2,3] – EMPM of doubly magic nuclei with one valence proton or neutron
- EMPM of doubly magic nuclei with one valence Λ
- used for hypernuclei with even-even core (${}^{17}_{\Lambda}$O, ${}^{5}_{\Lambda}$He)
- corrections to mean-field calculations
- Hilbert space $\mathcal{H}$ is a direct sum

$$\begin{aligned}\mathcal{H} &= \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \ldots \oplus \mathcal{H}_n,\\ \mathcal{H}_0 &= c_p^\dagger |\mathrm{HF}\rangle,\\ \mathcal{H}_1 &= c_p^\dagger Q_{\mu_1}^\dagger |\mathrm{HF}\rangle,\\ \mathcal{H}_2 &= c_p^\dagger Q_{\mu_1}^\dagger Q_{\mu_2}^\dagger |\mathrm{HF}\rangle \end{aligned} \tag{1}$$

[1] G. De Gregorio et al., Phys. Rev. C **94**, 061301 (2016)
[2] G. De Gregorio et al., Phys. Rev. C **95**, 034327 (2017)
[3] G. De Gregorio et al., Phys. Rev. C **97**, 034311 (2018)

Coupling of $N\Lambda$ TDA Phonon with TDA Phonons

- used for hypernuclei with even-odd core ($^{16}_{\Lambda}$O, $^{40}_{\Lambda}$Ca, $^{40}_{\Lambda}$K, ...)
  - actual hypernuclei produced in experiments (or planned to be produced)
- Hilbert space $\mathcal{H}$ is a direct sum

$$
\begin{aligned}
\mathcal{H} &= \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \ldots \oplus \mathcal{H}_n, \\
\mathcal{H}_0 &= R^{\dagger}_{\nu,\mathrm{p(n)}\Lambda} |\mathrm{HF}\rangle, \\
\mathcal{H}_1 &= R^{\dagger}_{\nu,\mathrm{p(n)}\Lambda} Q^{\dagger}_{\mu_1} |\mathrm{HF}\rangle, \\
\mathcal{H}_2 &= R^{\dagger}_{\nu,\mathrm{p(n)}\Lambda} Q^{\dagger}_{\mu_1} Q^{\dagger}_{\mu_2} |\mathrm{HF}\rangle
\end{aligned}
\tag{1}
$$

Results

- mean field calculations
  - nucleon spectra
  - charged radii, densities
  - ground-state energies
  - Λ spectra
- $N\Lambda$ TDA
  - energy spectra
- $\Lambda - 1ph$ coupling
  - correction to mean-field calculations of Λ s.p. spectra

Neutron Single-Particle Spectra (MF)

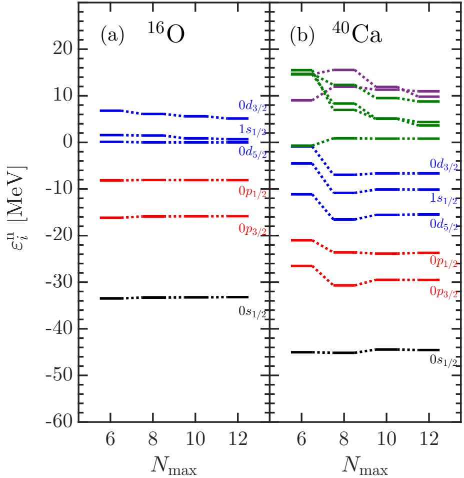

Figure: Convergence of neutron s.p. energies of ^{16}O (a) and ^{40}Ca (b).

Neutron Single-Particle Spectra (MF)

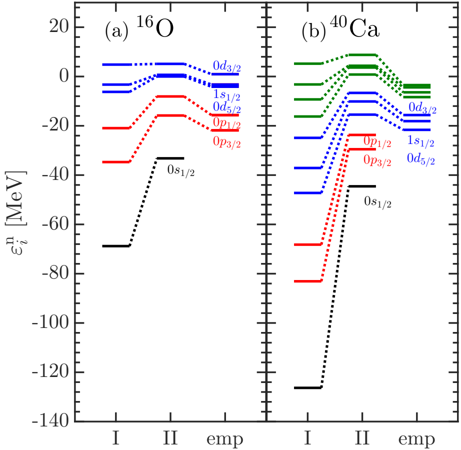

Figure: Neutron s.p. energies of ^{16}O (a) and ^{40}Ca (b) calculated with NN (I) or $NN + 3N$ (II) parts of potential $\mathrm{N^2LO_{sat}}$.

Radial Densities (MF)

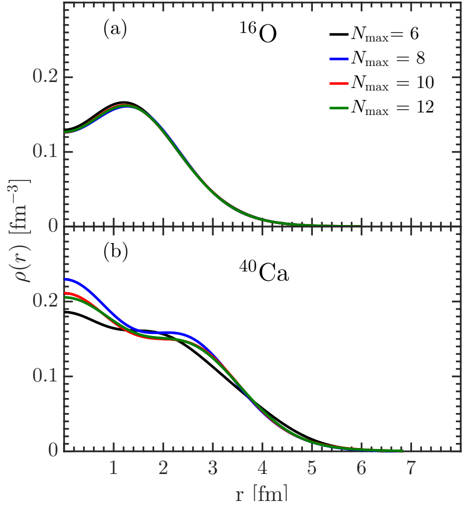

Figure: Convergence of radial densities of ^{16}O (a) and ^{40}Ca (b).

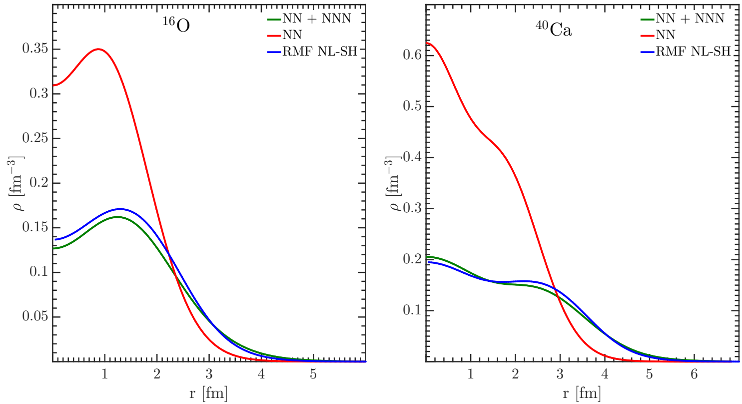

Figure: Radial densities of ^{16}O and ^{40}Ca calculated with NN or $NN + 3N$ parts of the potential N^2LO_{sat} compared to calculation of RMF model with parametrization NL-SH.

Charged Radii and Binding Energies (MF)

r_{ch} [fm]			
^{A}X	NN	$NN+3N$	exp
^{16}O	2.24	2.96	2.70
^{40}Ca	2.62	3.68	3.48

BE/A [MeV]			
^{A}X	NN	$NN+3N$	exp
^{16}O	7.36	2.66	7.98
^{40}Ca	11.65	2.31	8.55

Λ Single-Particle Spectra (MF)

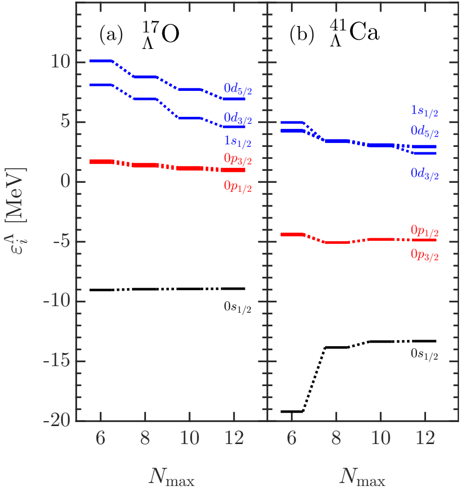

Figure: Convergence of Λ s.p. energies of $^{17}_{\Lambda}$O (a) and $^{41}_{\Lambda}$Ca (b).

Λ S.P. Spectrum of $^{17}_{\Lambda}$O

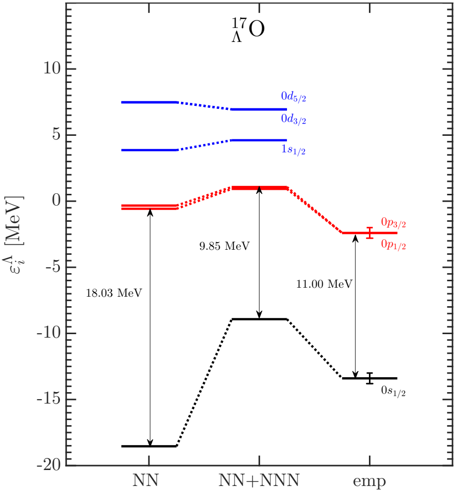

Figure: Λ s.p. energies calculated with NN and $NN + 3N$ parts of the $\mathrm{N^2LO_{sat}}$.

Energy Spectra of $^{16}_{\Lambda}$O ($N\Lambda$ TDA)

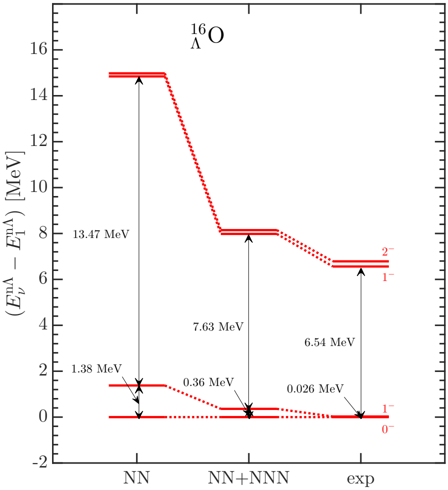

Figure: Energy spectra of $^{16}_{\Lambda}$O calculated with $N\Lambda$ TDA compared to experiment.

Energy Spectra of $^{40}_{\Lambda}$K ($N\Lambda$ TDA)

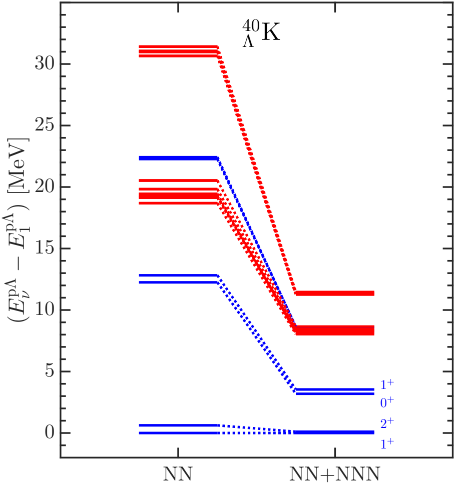

Figure: Energy spectra of $^{40}_{\Lambda}$K calculated with $N\Lambda$ TDA.

$\Lambda - 1ph$ Coupling

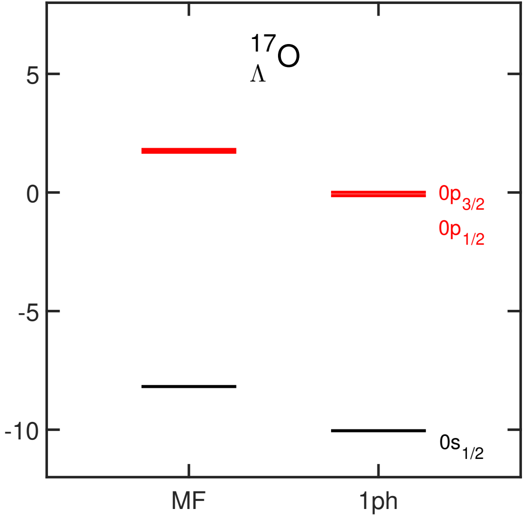

Figure: Energies calculated in mean-field and $\Lambda - 1ph$ coupling.

Conclusions

- 1st method – mean field
  - HF method in p-n-Λ formalism
- 2nd method – TDA
  - *N*Λ TDA
- 3rd method – EMPM
  - Λ coupled to phonons
  - *N*Λ TDA coupled to phonons
- spectrum of $^{17}_{\Lambda}$O drops in $\Lambda - 1ph$
- binding energy too low at the mean field level – we expect it do increase with $\Lambda - 1ph$
- *many-body correlations*

Future Plans

- continue with the analysis of the $\Lambda - 1ph$ and $N\Lambda$ TDA $- 1ph$ coupling
- new methods – natural orbitals
- new inputs
  - Σ degrees of freedom $\rightarrow \Lambda - \Sigma$ mixing
  - implementation of different ΛN potentials

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