# CP Violation in $B_{s}^{0} \rightarrow J / \psi \phi$ <br> - Theoretical Background 

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## Agenda

1 CP Violation

2 CKM Mechanism

3 Mixing of neutral $B$ mesons

4 Current experimental status of $\phi_{s}$

## Parity and Charge Conjugation

## Parity:

## Parity and Charge Conjugation

## Parity:

- The parity converts a right handed coordinate system to left handed ( $x, y, z \rightarrow-x,-y,-z$ ).

$$
\mathcal{P} \psi(\mathbf{r})=\psi(-\mathbf{r})
$$

- Eigenvalues are $\pm 1$
- Any physical process will happen identically when viewed in a mirror image
- does not affect time, charge and angular momentum


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## Charge conjugation

- changes the sign of the all quantum charges

$$
\mathcal{C} \psi(\mathbf{r})=\bar{\psi}(-\mathbf{r})
$$

- does not affect the mass, linear momentum and spin


## Violation of Parity and Charge Conjugation

## Parity Violation

- Chien-Shiung Wu 1956, ${ }^{60}$ Co decay

$$
{ }_{27}^{60} \mathrm{Co} \rightarrow{ }_{28}^{60} \mathrm{Ni}+e^{-}+\bar{\nu}_{e}+2 \gamma
$$

- cobalt nuclei were placed in the magnetic field
- electrons would have no preferred direction of decay relative to the nuclear spin
- Most of the electrons favoured a very specific direction of decay, opposite to that of the nuclear spin


## CP Violation

- Parity violated - is combination of $\mathcal{P}$ and $\mathcal{C}$ violated?
- Strong and EM interactions: $\mathcal{C P}$ conserved
- Weak interactions: $\mathcal{C P}$ violated:
- Christenson, Cronin, Fitch and Turlay 1964
- study of two neutral $K$ mesons in the kaon decays, $K_{S}^{0}$ and $K_{L}^{0}$
- if $\mathcal{C P}$ conserved:

$$
K_{S}^{0} \rightarrow 2 \pi \quad K_{L}^{0} \rightarrow 3 \pi
$$

- $K_{L}^{0} \rightarrow 2 \pi$ observed!!
- $K^{0} \bar{K}^{0}$ oscilation, $\mathcal{C P}$ violated
- Three types of $\mathcal{C P}$ violation:
- in decay
- in mixing
- in interference of mixing and decay


## $\mathcal{C P}$ Violation in Mixing

- probability of oscillation from meson to anti-meson is different from the probability of oscillation from anti-meson to meson

$$
\operatorname{Prob}\left(P^{0} \rightarrow \bar{P}^{0}\right) \neq \operatorname{Prob}\left(\bar{P}^{0} \rightarrow P^{0}\right)
$$

- Mass eigenstates are not CP eigenstates
- Charged-current semileptonic neutral meson decays $M, \bar{M} \rightarrow I^{ \pm} X$



## $\mathcal{C P}$ Violation in Decay

- decay amplitude of particle into the final state is different from the decay amplitude of its antiparticle into its final anti-state

$$
\Gamma(M \rightarrow f) \neq \Gamma(\bar{M} \rightarrow \bar{f})
$$

- In charged meson (and all baryon) decays, where mixing effects are absent, this is the only possible source of $\mathcal{C P}$ asymmetries



## $\mathcal{C P}$ Violation in Interference of Mixing and

## Decay

- occurs in case both meson and antimeson decay into the same final state

$$
M \rightarrow f \quad M \rightarrow \bar{M} \rightarrow f
$$


$\bar{f}$

$\bar{f}$

## $\mathcal{C P}$ Violation in Interference of Mixing and

## Decay

- occurs in case both meson and antimeson decay into the same final state

$$
M \rightarrow f \quad M \rightarrow \bar{M} \rightarrow f
$$



## CP Violation in Standard Model

- Charged current part of Lagrangian for weak interactions of quarks

$$
\mathcal{L}_{Y}^{q}=-\frac{g}{\sqrt{2}} \bar{u}_{L_{i}}^{\prime} \gamma^{\mu} \delta_{i j} \bar{d}_{L_{j}}^{\prime} W_{\mu}^{+}+\text {h.c. }
$$

- interaction eigenvectors in term of mass eigenvectors $d_{L}^{\prime}=V_{d L}^{\dagger} d_{L}$ and $\bar{u}_{L}^{\prime}=V_{u L} u_{L}$

$$
\mathcal{L}_{Y}^{q}=-\frac{g}{\sqrt{2}} \bar{u}_{L i} \gamma^{\mu} \bar{V}_{i j} \bar{d}_{L j} W_{\mu}^{+}+\text {h.c. }
$$

- CKM matrix $V_{i j}=V_{u L}^{\dagger} V_{d L}:$

$$
V_{C K M}=\left(\begin{array}{lll}
V_{u d} & V_{u s} & V_{u b} \\
V_{c d} & V_{c s} & V_{c b} \\
V_{t d} & V_{t s} & V_{t b}
\end{array}\right)
$$

## CKM Matrix

- Complex $3 \times 3$ unitary matrix
- only four parameters are independent - 3 Euler mixing angles and one CP-violating CKM phase

$$
\begin{align*}
V_{C K M} & =\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{array}\right)\left(\begin{array}{ccc}
c_{13} & 0 & s_{13} e^{-\imath \delta} \\
0 & 1 & 0 \\
-s_{13} e^{\imath \delta} & 0 & c_{13}
\end{array}\right)\left(\begin{array}{ccc}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{array}\right)= \\
& =\left(\begin{array}{ccc}
c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-\imath \delta} \\
-s_{12} c_{23}-c_{12} s_{23} s_{13} e^{\imath \delta} & c_{12} c_{23}-s_{12} s_{23} s_{13}{ }^{\imath \delta} & s_{23} c_{13} \\
s_{12} s_{23}-c_{12} c_{23} s_{13} e^{\imath \delta} & -c_{12} s_{23}-s_{12} c_{23} s_{13} e^{\imath \delta} & c_{23} c_{13}
\end{array}\right) \tag{1}
\end{align*}
$$

- Angle $\theta_{12}$ identified as Cabibbo angle


## CKM Matrix

- Wolfenstein parametrization

$$
\begin{align*}
& V_{C K M}=\left(\begin{array}{ccc}
1-\frac{1}{2} \lambda^{2}-\frac{1}{8} \lambda^{4} & \lambda & A \lambda^{3}(\rho-\imath \eta) \\
-\lambda & 1-\frac{1}{2} \lambda^{2}-\frac{1}{8} \lambda^{4}\left(1+4 A^{2}\right) & A \lambda^{2} \\
A \lambda^{3}(1-\rho-\imath \eta) & -A \lambda^{2}+\frac{1}{2} A \lambda^{4}(1-2(\rho+\imath \eta)) & 1-\frac{1}{2} A^{4} \lambda^{4}
\end{array}\right) \\
&+\mathcal{O}\left(\lambda^{5}\right)  \tag{2}\\
&(2) \\
& ■ \lambda=0.22506 \pm 0.00050 \\
& ■ A=0.811 \pm 0.026 \\
& ■ \bar{\rho}=\rho\left(1-\frac{\lambda}{2}\right)=0.124_{-0.018}^{+0.019} \\
& \square \bar{\eta}=\eta\left(1-\frac{\lambda}{2}\right)=0.356 \pm 0.011
\end{align*}
$$

## CKM Matrix




## Unitarity Triangles

- unitarity $1=V_{C K M} V_{C K M}^{\dagger}$ :
- orthonormality of columns or rows in $V_{C K M}$ expressed as

$$
\sum_{\alpha=u, c, t} V_{\alpha i} V_{\alpha j}^{*}=\delta_{i j}, \sum_{i=d, s, b} V_{\alpha i} V_{\beta i}^{*}=\delta_{\alpha \beta}
$$

- One of triangles:

$$
\begin{aligned}
& V_{u d} V_{u b}^{*}+V_{c d} V_{c b}^{*}+V_{t d} V_{t b}^{*}=0 \\
& V_{u s} V_{u b}^{*}+V_{c s} V_{c b}^{*}+V_{t s} V_{t b}^{*}=0
\end{aligned}
$$

## Unitarity Triangles - Experimental Results



## Unitarity Triangles - Experimental Results



## Mixing of Neutral $B$ Mesons

- flavour eigenstates of the $B^{0}$ meson and the $B_{s}^{0}$ meson:

$$
\begin{aligned}
\left|B^{0}\right\rangle & =|\bar{b} d\rangle & & \left|B_{s}^{0}\right\rangle
\end{aligned}=|\bar{b} s\rangle,
$$

- Decay amplitudes of neutral meson (and anti-mesons) decays into final states:

$$
\begin{array}{ll}
A_{f}=\langle f| \mathcal{H}\left|B_{q}^{0}\right\rangle & A_{\bar{f}}=\langle\bar{f}| \mathcal{H}\left|B_{q}^{0}\right\rangle \\
\bar{A}_{f}=\langle f| \mathcal{H}\left|\bar{B}_{q}^{0}\right\rangle & \bar{A}_{\bar{f}}=\langle\bar{f}| \mathcal{H}\left|\bar{B}_{q}^{0}\right\rangle
\end{array}
$$



## Time developement of the mass eigenstates

of $B$ mesons

$$
\imath \hbar \frac{\partial}{\partial t}\binom{\left|B^{0}(t)\right\rangle}{\left.\bar{B}^{0}(t)\right\rangle}=\boldsymbol{H}\binom{\left|B^{0}(t)\right\rangle}{\left.\bar{B}^{0}(t)\right\rangle}=\left(\boldsymbol{M}-\frac{\imath}{2} \boldsymbol{\Gamma}\right)\left(\begin{array}{l}
\left.\left\lvert\, \begin{array}{l}
\left.B^{0}(t)\right\rangle \\
\left.\bar{B}^{0}(t)\right\rangle
\end{array}\right.\right)
\end{array}\right)
$$

- $\left|B^{0}(t)\right\rangle$ denotes the flavour state at $t=0$
- $\boldsymbol{M}, \boldsymbol{\Gamma}$ hermitian matrices

$$
\boldsymbol{M}=\left(\begin{array}{ll}
M_{11} & M_{12} \\
M_{12}^{*} & M_{22}
\end{array}\right), \boldsymbol{\Gamma}=\left(\begin{array}{ll}
\Gamma_{11} & \Gamma_{12} \\
\Gamma_{12}^{*} & \Gamma_{22}
\end{array}\right)
$$

- Mass terms: M, Decay matrix $\boldsymbol{\Gamma}$
- $\mathcal{C P} \mathcal{T}$ conserved:

$$
M_{11}=M_{22}=M, \Gamma_{11}=\Gamma_{22}=\Gamma
$$

- non-diagonal elements correspond to the $B_{s}^{0}-\bar{B}_{s}^{0}$ mixing


## Time developement of the mass eigenstates

of $B$ mesons

$$
\begin{gathered}
\Gamma=\frac{1}{2}\left(\Gamma_{L}+\Gamma_{H}\right), \quad M=\frac{1}{2}\left(M_{L}+M_{H}\right) \\
\Delta \Gamma=\Gamma_{L}-\Gamma_{H}=2\left|\Gamma_{12}\right| \cos \phi_{S}, \quad \Delta m=M_{H}-M_{L}=2\left|M_{12}\right|
\end{gathered}
$$

$\square \Delta m>0, \Delta \Gamma$ can have either signs ( $\Delta \Gamma>0$ sign convention in SM)

- Solving eigenvalues problem:

$$
\begin{gathered}
(\Delta m)^{2}-\frac{1}{4}(\Delta \Gamma)^{2}=4\left|M_{12}\right|^{2}-\left|\Gamma_{12}\right|^{2} \\
\Delta m \Delta \Gamma=4 \mathfrak{R e}\left(M_{12} \Gamma_{12}\right)
\end{gathered}
$$

## Time developement of the mass eigenstates

## of $B$ mesons

- Diagonalization of Hamiltonian leads to mass eigenstates:

$$
\begin{align*}
& \left|B_{L}\right\rangle=p\left|B^{0}\right\rangle+q\left|\bar{B}^{0}\right\rangle \\
& \left|B_{H}\right\rangle=p\left|B^{0}\right\rangle-q\left|\bar{B}^{0}\right\rangle \tag{3}
\end{align*}
$$

- Solution of the Schrödinger equation:

$$
\begin{align*}
\left|B_{L}(t)\right\rangle & =e^{-i M_{L} t} e^{\frac{-\Gamma_{L}}{2} t}\left|B_{L}(0)\right\rangle,  \tag{4}\\
\left|B_{H}(t)\right\rangle & =e^{-i M_{H} t} e^{\frac{-\Gamma_{H}}{2} t}\left|B_{H}(0)\right\rangle,
\end{align*}
$$

## Time developement of the flavor eigenstates

of $B$ mesons

- Time development of $B^{0}$ and $\bar{B}^{0}$ :

$$
\begin{align*}
& \left|B^{0}(t)\right\rangle=\frac{1}{2 p}\left(\left|B_{L}(t)\right\rangle+\left|B_{H}(t)\right\rangle\right) \\
& \left|\bar{B}^{0}(t)\right\rangle=\frac{1}{2 q}\left(\left|B_{L}(t)\right\rangle-\left|B_{H}(t)\right\rangle\right) \tag{5}
\end{align*}
$$

- Substituting the solution of Schrödinger equation:

$$
\begin{align*}
\left|B^{0}(t)\right\rangle & =\frac{1}{2 p}\left\{e^{-i M_{L} t} e^{\frac{-\Gamma_{L}}{2} t}\left|B_{L}(0)\right\rangle+e^{-i M_{H} t} e^{\frac{-\Gamma_{H}}{2} t}\left|B_{H}(0)\right\rangle\right\} \\
\left|\bar{B}^{0}(t)\right\rangle & =\frac{1}{2 q}\left\{e^{-i M_{L} t} e^{\frac{-\Gamma_{L}}{2} t}\left|B_{L}(0)\right\rangle-e^{-i M_{H} t} e^{\frac{-\Gamma_{H}}{2} t}\left|B_{H}(0)\right\rangle\right\} \tag{6}
\end{align*}
$$

## Time development of the flavor eigenstates

of $B$ mesons

$$
\begin{align*}
\left|B^{0}(t)\right\rangle & =\frac{1}{2 p}\left\{p e^{-i M_{L} t} e^{\frac{-\Gamma_{L}}{2} t}|B(0)\rangle+q e^{-i M_{L} t} e^{\frac{-\Gamma_{L}}{2} t}|\bar{B}(0)\rangle\right\}  \tag{7}\\
& +\frac{1}{2 p}\left\{p e^{-i M_{H} t} e^{\frac{-\Gamma_{H}}{2} t}|B(0)\rangle-q e^{-i M_{H} t} e^{\frac{-\Gamma_{H}}{2} t}|\bar{B}(0)\rangle\right\}
\end{align*}
$$

- using $g_{ \pm}(t)=\frac{1}{2}\left(e^{-i M_{L} t} e^{\frac{-\Gamma_{L}}{2} t} \pm e^{-i M_{H} t} e^{\frac{-\Gamma_{H}}{2} t}\right)$ :

$$
\begin{aligned}
& \left|B^{0}(t)\right\rangle=g_{+}(t)\left|B^{0}\right\rangle+\frac{q}{p} g_{-}(t)\left|\bar{B}^{0}\right\rangle \\
& \left|\bar{B}^{0}(t)\right\rangle=g_{+}(t)\left|\bar{B}^{0}\right\rangle-\frac{p}{q} g_{-}(t)\left|B^{0}\right\rangle
\end{aligned}
$$

## Time development of the flavor eigenstates

of $B$ mesons

- $g_{ \pm}(t)=\frac{1}{2}\left(e^{-i M_{L} t} e^{\frac{-\Gamma_{L}}{2} t} \pm e^{-i M_{H} t} e^{\frac{-\Gamma_{H}}{2} t}\right)$ squared:

$$
\begin{align*}
\left|g_{ \pm}(t)\right|^{2} & =\frac{1}{2} e^{-\Gamma t}\left(\cosh \frac{\Delta \Gamma}{2} t \pm \cos \Delta m t\right) \\
g_{+}(t) g_{-}^{\star}(t) & =\frac{1}{2} e^{-\Gamma t}\left(-\sinh \frac{\Delta \Gamma}{2} t-i \sin \Delta m t\right)  \tag{8}\\
g_{+}^{\star}(t) g_{-}(t) & =\frac{1}{2} e^{-\Gamma t}\left(-\sinh \frac{\Delta \Gamma}{2} t+i \sin \Delta m t\right)
\end{align*}
$$

- Helpful parameter

$$
\lambda_{f}=\frac{q \bar{A}_{f}}{p A_{f}}=\eta_{f} e^{-i \phi_{s}}
$$

## Branching ratio (decay rate)

$$
\begin{align*}
\Gamma\left(B_{s}^{0}(t) \rightarrow f\right) & = \\
& \left|A_{f}\right|^{2} \mathrm{e}^{-\Gamma t}\left[\left(1+\left|\lambda_{f}\right|\right) \cosh \frac{\Delta \Gamma_{s} t}{2}+\left(1-\left|\lambda_{f}\right|\right) \cos \Delta M t\right. \\
& \left.+2 \operatorname{Re}\left(\lambda_{f}\right) \sinh \frac{\Delta \Gamma_{s} t}{2}-2 \operatorname{Im}\left(\lambda_{f}\right) \sin \Delta M t\right]  \tag{9}\\
\Gamma\left(\bar{B}^{0}(t) \rightarrow f\right) & = \\
& \left|A_{f}\right|^{2} \mathrm{e}^{-\Gamma t}\left[\left(1+\left|\lambda_{f}\right|\right) \cosh \frac{\Delta \Gamma_{s} t}{2}-\left(1-\left|\lambda_{f}\right|\right) \cos \Delta M t\right. \\
& \left.+2 \operatorname{Re}\left(\lambda_{f}\right) \sinh \frac{\Delta \Gamma_{s} t}{2}+2 \operatorname{Inm}_{m}\left(\lambda_{f}\right) \sin \Delta M t\right] \tag{10}
\end{align*}
$$

## $\mathcal{C} \mathcal{P}$-violating phase $\phi_{s}$ in decay $B_{s}^{0} \rightarrow J / \psi \phi$



## $\mathcal{C}$ P-violating phase $\phi_{s}$ in decay $B_{s}^{0} \rightarrow J / \psi \phi$

- $\mathcal{C P}$ eigenvalues of the $J / \psi \phi$ final state are $\eta_{f}= \pm 1$, depending on the total angular momentum $L$ :

$$
\mathcal{C P}|J / \psi \phi\rangle=\eta_{f}|J / \psi \phi\rangle=(-1)^{L}|J / \psi \phi\rangle
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$$

- $B_{s}^{0}$ is a spinless particle, total angular momentum: $\mathcal{J}=0$
- Adding the spin of the the $J / \psi$ and $\phi$ results into total spin

$$
S \in\{0,1,2\}
$$

- conservation of angular momentum: $L+S=0$


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- conservation of angular momentum: $L+S=0$
- $\mathcal{C P}$ even states: $\eta_{f}=1, L=0,2$
- parallel $(\|)$ state: $J / \psi$ and $\phi$ parallel with respect to the direction of their momentum
- zero state: polarisation of $J / \psi$ perpendicular to $\phi$ perpendicular with respect to the direction of their momentum
$\square \mathcal{C P}$ odd states: $\eta_{f}=-1, L=1$


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- zero state: polarisation of $J / \psi$ perpendicular to $\phi$ perpendicular with respect to the direction of their momentum
- $\mathcal{C P}$ odd states: $\eta_{f}=-1, L=1$
- perpendicular $(\perp)$ state: polarisation of $J / \psi$ perpendicular to $\phi$ and parallel with respect to the direction of their momentum


## $B_{s}^{0}$ differential decay rate - time part

$$
\begin{aligned}
& \frac{\mathrm{d} \Gamma\left(B_{\mathrm{s}}^{0} \rightarrow J / \psi \phi\right)}{\mathrm{d} t \mathrm{~d} \Omega}=\sum_{i=1}^{6} A_{i}(t) f_{i}(\cos \theta, \varphi, \cos \psi) \\
&\left|A_{0}(\mathrm{t})\right|^{2}=\left|A_{0}(0)\right|^{2} e^{-\Gamma_{\mathrm{s}} t}\left[\cosh \left(\frac{\Delta \Gamma_{\mathrm{s}}}{2} t\right)-\cos \phi_{s} \sinh \left(\frac{\Delta \Gamma_{\mathrm{s}}}{2} t\right)+\sin \phi_{s} \sin \left(\Delta m_{\mathrm{s}} t\right)\right] \\
&\left|A_{\|}(\mathrm{t})\right|^{2}=\left|A_{\|}(0)\right|^{2} e^{-\Gamma_{\mathrm{s}} t}\left[\cosh \left(\frac{\Delta \Gamma_{\mathrm{s}}}{2} t\right)-\cos \phi_{s} \sinh \left(\frac{\Delta \Gamma_{\mathrm{s}}}{2} t\right)+\sin \phi_{s} \sin \left(\Delta m_{\mathrm{s}} t\right)\right] \\
&\left|A_{\perp}(\mathrm{t})\right|^{2}=\left|A_{\perp}(0)\right|^{2} e^{-\Gamma_{s} t}\left[\cosh \left(\frac{\Delta \Gamma_{\mathrm{s}}}{2} t\right)+\cos \phi_{s} \sinh \left(\frac{\Delta \Gamma_{\mathrm{s}}}{2} t\right)-\sin \phi_{s} \sin \left(\Delta m_{\mathrm{s}} t\right)\right] \\
& \Im\left(A_{\|}^{*}(\mathrm{t}) A_{\perp}(\mathrm{t})\right)=\left|A_{\|}(0)\right|\left|A_{\perp}(0)\right| e^{-\Gamma_{\mathrm{s}} t}\left[-\cos \left(\delta_{\perp}-\delta_{\|}\right) \sin \phi_{s} \sinh \left(\frac{\Delta \Gamma_{\mathrm{s}}}{2} t\right)\right. \\
&\left.+\sin \left(\delta_{\perp}-\delta_{\|}\right) \cos \left(\Delta m_{\mathrm{s}} t\right)-\cos \left(\delta_{\perp}-\delta_{\|}\right) \cos \phi_{s} \sin \left(\Delta m_{\mathrm{s}} t\right)\right] \\
& \Re\left(A_{0}^{*}(\mathrm{t}) A_{\|}(\mathrm{t})\right)=\left|A_{0}(0)\right|\left|A_{\|}(0)\right| e^{-\Gamma_{s} t} \cos \delta_{\|}\left[\cosh \left(\frac{\Delta \Gamma_{\mathrm{s}}}{2} t\right)-\cos \phi_{s} \sinh \left(\frac{\Delta \Gamma_{\mathrm{s}}}{2} t\right)\right. \\
&\left.+\sin \phi_{s} \sin \left(\Delta m_{\mathrm{s}} t\right)\right] \\
& \Im\left(A_{0}^{*}(\mathrm{t}) A_{\perp}(\mathrm{t})\right)=\left|A_{0}(0)\right|\left|A_{\perp}(0)\right| e^{-\Gamma_{s} t}\left[-\cos \delta_{\perp} \sin \phi_{s} \sinh \left(\frac{\Delta \Gamma_{\mathrm{s}}}{2} t\right)\right. \\
&\left.+\sin \delta_{\perp} \cos \left(\Delta m_{\mathrm{s}} t\right)-\cos \delta_{\perp} \cos \phi_{s} \sin \left(\Delta m_{\mathrm{s}} t\right)\right]
\end{aligned}
$$

## $B_{s}^{0}$ differential decay rate - angular part



## $B_{s}^{0}$ differential decay rate - angular part

| i | $A_{i}(\mathrm{t})$ | $\bar{A}_{i}(\mathrm{t})$ | $f_{i}(\cos \theta, \varphi, \cos \psi)$ |
| :---: | :---: | :---: | :---: |
| 1 | $\left\|A_{0}(\mathrm{t})\right\|^{2}$ | $\left\|\bar{A}_{0}(\mathrm{t})\right\|^{2}$ | $\frac{9}{32 \pi} 2 \cos ^{2} \psi\left(1-\sin ^{2} \theta \cos ^{2} \varphi\right)$ |
| 2 | $\left\|A_{\\|}(\mathrm{t})\right\|^{2}$ | $\left\|\bar{A}_{\\| \mid}(\mathrm{t})\right\|^{2}$ | $\frac{9}{32 \pi} \sin ^{2} \psi\left(1-\sin ^{2} \theta \sin ^{2} \varphi\right)$ |
| 3 | $\left\|A_{\perp}(\mathrm{t})\right\|^{2}$ | $\left\|\bar{A}_{\perp}(\mathrm{t})\right\|^{2}$ | $\frac{9}{32 \pi} \sin ^{2} \psi \sin ^{2} \theta$ |
| 4 | $\Im\left(A_{\\|}^{*}(\mathrm{t}) A_{\perp}(\mathrm{t})\right)$ | $\Im\left(\bar{A}_{\\|}^{*}(\mathrm{t}) \bar{A}_{\perp}(\mathrm{t})\right)$ | $-\frac{9}{32 \pi} \sin ^{2} \psi \sin 2 \theta \sin \varphi$ |
| 5 | $\Re\left(A_{0}^{*}(\mathrm{t}) A_{\\|}(\mathrm{t})\right)$ | $\Re\left(\bar{A}_{0}^{*}(\mathrm{t}) \bar{A}_{\\|}(\mathrm{t})\right)$ | $\frac{9}{32 \pi \sqrt{2}} \sin 2 \psi \sin ^{2} \theta \sin 2 \varphi$ |
| 6 | $\Im\left(A_{0}^{*}(\mathrm{t}) A_{\perp}(\mathrm{t})\right)$ | $\Im\left(\bar{A}_{0}^{*}(\mathrm{t}) \bar{A}_{\perp}(\mathrm{t})\right)$ | $\frac{9}{32 \pi \sqrt{2}} \sin 2 \psi \sin 2 \theta \cos \varphi$ |

## S-wave contribution

- $\phi$ is a vector meson - the $K^{+} K^{-}$system is in a P-wave configuration
- Detected $K^{+} K^{-}$results from a non-resonant contribution or decay of the $f_{0}(980)$ which is a scalar meson
- In both cases: S-wave configuration
- Introduce the S -wave amplitude $A_{s}(t)$ with phase $\delta_{s}$
- S-wave amplitude can also interfere with the P -wave
- The S-wave contributions have also its angular distributions

$$
\frac{\mathrm{d} \Gamma\left(B_{s}^{0} \rightarrow J / \psi \phi\right)}{\mathrm{d} t \mathrm{~d} \Omega}=\sum_{i=1}^{10} A_{i}(t) f_{i}(\cos \theta, \varphi, \cos \psi)
$$

## S-wave contribution



## S-wave contribution

| $k$ | $\mathcal{O}^{(k)}(t) \quad \pm \rightarrow B_{s} / \bar{B}_{s}$ | $g^{(k)}\left(\theta_{T}, \psi_{T}, \phi_{T}\right)$ |
| :---: | :---: | :---: |
| 1 | $\frac{1}{2}\left\|A_{0}(0)\right\|^{2}\left[\left(1+\cos \phi_{s}\right) e^{-\Gamma_{\mathrm{L}}^{(s)} t}+\left(1-\cos \phi_{s}\right) e^{-\Gamma_{\mathrm{H}}^{(s)} t} \pm 2 e^{-\Gamma_{s} t} \sin \left(\Delta m_{s} t\right) \sin \phi_{s}\right]$ | $2 \cos ^{2} \psi_{T}\left(1-\sin ^{2} \theta_{T} \cos ^{2} \phi_{T}\right)$ |
| 2 | $\left.\frac{1}{2}\left\|A_{\\|}(0)\right\|^{2}\left[\left(1+\cos \phi_{s}\right) e^{-\Gamma_{\mathrm{L}}^{(s)} t}+\left(1-\cos \phi_{s}\right) e^{-\Gamma_{\mathrm{H}}^{(s)} t} \pm 2 e^{-\Gamma_{s} t} \sin \left(\Delta m_{s} t\right) \sin \phi_{s}\right]\right]$ | $\sin ^{2} \psi_{T}\left(1-\sin ^{2} \theta_{T} \sin ^{2} \phi_{T}\right)$ |
| 3 | $\left.\frac{1}{2}\left\|A_{\perp}(0)\right\|^{2}\left[\left(1-\cos \phi_{s}\right) e^{-\Gamma_{\mathrm{L}}^{(s)} t}+\left(1+\cos \phi_{s}\right) e^{-\Gamma_{\mathrm{H}}^{(s)} t} \mp 2 e^{-\Gamma_{s} t} \sin \left(\Delta m_{s} t\right) \sin \phi_{s}\right]\right]$ | $\sin ^{2} \psi_{T} \sin ^{2} \theta_{T}$ |
| 4 | $\begin{aligned} & \frac{1}{2}\left\|A_{0}(0) \\| A_{\\|}(0)\right\| \cos \delta_{\\|} \\ & \quad\left[\left(1+\cos \phi_{s}\right) e^{-\Gamma_{\mathrm{L}}^{(s)} t}+\left(1-\cos \phi_{s}\right) e^{-\Gamma_{\mathrm{H}}^{(s)} t} \pm 2 e^{-\Gamma_{s} t} \sin \left(\Delta m_{s} t\right) \sin \phi_{s}\right] \end{aligned}$ | $-\frac{1}{\sqrt{2}} \sin 2 \psi_{T} \sin ^{2} \theta_{T} \sin 2 \phi_{T}$ |
| 5 | $\begin{aligned} & \left\|A_{\\|}(0) \\| A_{\perp}(0)\right\|\left[\frac{1}{2}\left(e^{-\Gamma_{\mathrm{L}}^{(s)} t}-e^{-\Gamma_{\mathrm{H}}^{(s)} t}\right) \cos \left(\delta_{\perp}-\delta_{\\|}\right) \sin \phi_{s}\right. \\ & \left.\quad \pm e^{-\Gamma_{s} t}\left(\sin \left(\delta_{\perp}-\delta_{\\|}\right) \cos \left(\Delta m_{s} t\right)-\cos \left(\delta_{\perp}-\delta_{\\|}\right) \cos \phi_{s} \sin \left(\Delta m_{s} t\right)\right)\right] \end{aligned}$ | $\sin ^{2} \psi_{T} \sin 2 \theta_{T} \sin \phi_{T}$ |
| 6 | $\begin{aligned} \left\|A_{0}(0)\right\|\left\|A_{\perp}(0)\right\|\left[\frac { 1 } { 2 } \left(e^{-\Gamma_{\mathrm{L}}^{(s)} t}-\right.\right. & \left.e^{-\Gamma_{\mathrm{H}}^{(s)} t}\right) \cos \delta_{\perp} \sin \phi_{s} \\ & \left. \pm e^{-\Gamma_{s} t}\left(\sin \delta_{\perp} \cos \left(\Delta m_{s} t\right)-\cos \delta_{\perp} \cos \phi_{s} \sin \left(\Delta m_{s} t\right)\right)\right] \end{aligned}$ | $\frac{1}{\sqrt{2}} \sin 2 \psi_{T} \sin 2 \theta_{T} \cos \phi_{T}$ |
| 7 | $\left.\frac{1}{2}\left\|A_{S}(0)\right\|^{2}\left[\left(1-\cos \phi_{s}\right) e^{-\Gamma_{\mathrm{L}}^{(s)} t}+\left(1+\cos \phi_{s}\right) e^{-\Gamma_{\mathrm{H}}^{(s)} t} \mp 2 e^{-\Gamma_{s} t} \sin \left(\Delta m_{s} t\right) \sin \phi_{s}\right]\right]$ | $\frac{2}{3}\left(1-\sin ^{2} \theta_{T} \cos ^{2} \phi_{T}\right)$ |
| 8 | $\begin{aligned} & \left\|A_{S}(0) \\| A_{\\|}(0)\right\|\left[\frac{1}{2}\left(e^{-\Gamma_{\mathrm{L}}^{(s)} t}-e^{-\Gamma_{\mathrm{H}}^{(s)} t}\right) \sin \left(\delta_{\\|}-\delta_{S}\right) \sin \phi_{s}\right. \\ & \left.\quad \pm e^{-\Gamma_{s} t}\left(\cos \left(\delta_{\\|}-\delta_{S}\right) \cos \left(\Delta m_{s} t\right)-\sin \left(\delta_{\\|}-\delta_{S}\right) \cos \phi_{s} \sin \left(\Delta m_{s} t\right)\right)\right] \end{aligned}$ | $\frac{1}{3} \sqrt{6} \sin \psi_{T} \sin ^{2} \theta_{T} \sin 2 \phi_{T}$ |
| 9 | $\begin{aligned} & \frac{1}{2}\left\|A_{S}(0)\right\|\left\|A_{\perp}(0)\right\| \sin \left(\delta_{\perp}-\delta_{S}\right) \\ & \quad\left[\left(1-\cos \phi_{s}\right) e^{-\Gamma_{\mathrm{L}}^{(s)} t}+\left(1+\cos \phi_{s}\right) e^{-\Gamma_{\mathrm{H}}^{(s)} t} \mp 2 e^{-\Gamma_{s} t} \sin \left(\Delta m_{s} t\right) \sin \phi_{s}\right] \end{aligned}$ | $\frac{1}{3} \sqrt{6} \sin \psi_{T} \sin 2 \theta_{T} \cos \phi_{T}$ |
| 10 | $\begin{aligned} \left\|A_{0}(0)\right\|\left\|A_{S}(0)\right\|\left[\frac { 1 } { 2 } \left(e^{-\Gamma_{\mathrm{H}}^{(s)} t}-\right.\right. & \left.e^{-\Gamma_{\mathrm{L}}^{(s)} t}\right) \sin \delta_{S} \sin \phi_{s} \\ & \left. \pm e^{-\Gamma_{s} t}\left(\cos \delta_{S} \cos \left(\Delta m_{s} t\right)+\sin \delta_{S} \cos \phi_{s} \sin \left(\Delta m_{s} t\right)\right)\right] \mid \end{aligned}$ | $\frac{4}{3} \sqrt{3} \cos \psi_{T}\left(1-\sin ^{2} \theta_{T} \cos ^{2} \phi_{T}\right)$ |

## S-wave contribution



## S-wave contribution


-Theory

## Theory Measurements

- Our analysis: 5 dimensional fit: mass, lifetime, 3 angles



## Theory Measurements



## Theory Measurements



## Theory Measurements



## Theory Measurements



## Results

$$
\begin{aligned}
\phi_{s} & =-0.055 \pm 0.053 \text { (stat.) } \pm 0.0069 \text { (syst.) } \mathrm{rad} \\
\Delta \Gamma_{s} & =0.080 \pm 0.007 \text { (stat.) } \pm 0.0024 \text { (syst.) } \mathrm{ps}^{-1} \\
\Gamma_{s} & =0.667 \pm 0.002 \text { (stat.) } \pm 0.0010 \text { (syst.) } \mathrm{ps}^{-1} \\
\left|A_{\|}(0)\right|^{2} & =0.219 \pm 0.002 \text { (stat.) } \pm 0.0012 \text { (syst.) } \\
\left|A_{0}(0)\right|^{2} & =0.518 \pm 0.002 \text { (stat.) } \pm 0.0028 \text { (syst.) } \\
\left|A_{S}(0)\right|^{2} & =0.053 \pm 0.005 \text { (stat.) } \pm 0.0072 \text { (syst.) } \\
\delta_{\perp} & =2.767 \pm 0.161 \text { (stat.) } \pm 0.0390 \text { (syst.) rad } \\
\delta_{\|} & =3.123 \pm 0.140 \text { (stat.) } \pm 0.0054 \text { (syst.) rad } \\
\delta_{\perp}-\delta_{S} & =-0.113 \pm 0.025 \text { (stat.) } \pm 0.0079 \text { (syst.) rad. }
\end{aligned}
$$

## Results



