BALITSKY-KOVCHEGOV EQUATION

Marek Matas
UPC group meeting at Decin 26.9.-27.9.2019
Workshop supported by grant SVK30/19/F4

DEEP INELASTIC SCATTERING

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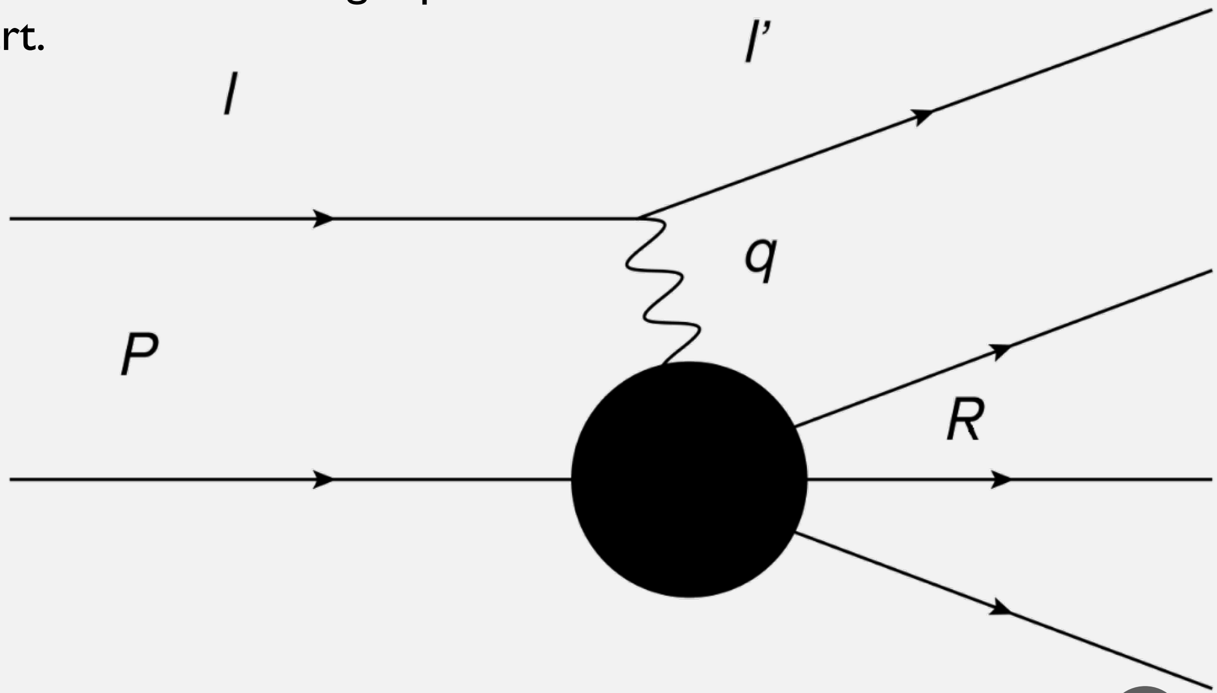
The electron-proton collisions are considered to happen as:

1. The incoming electron emits a virtual photon.
2. The virtual photon interacts with the target proton
3. The proton breaks apart.

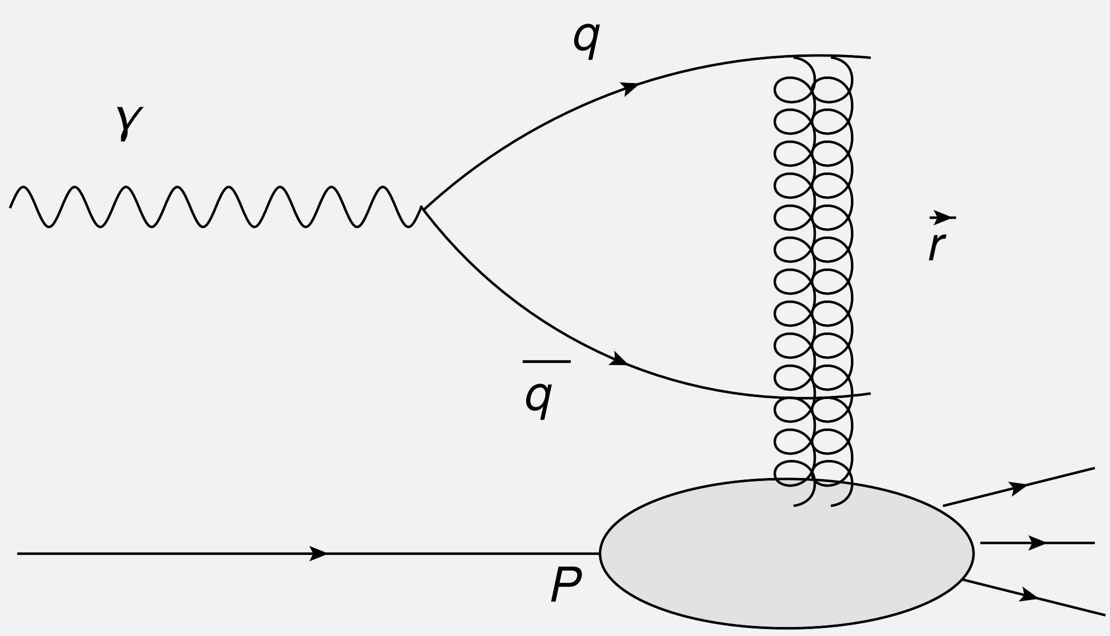


HOW DOES A PHOTON INTERACT WITH A PROTON?

DIPOLE MODEL

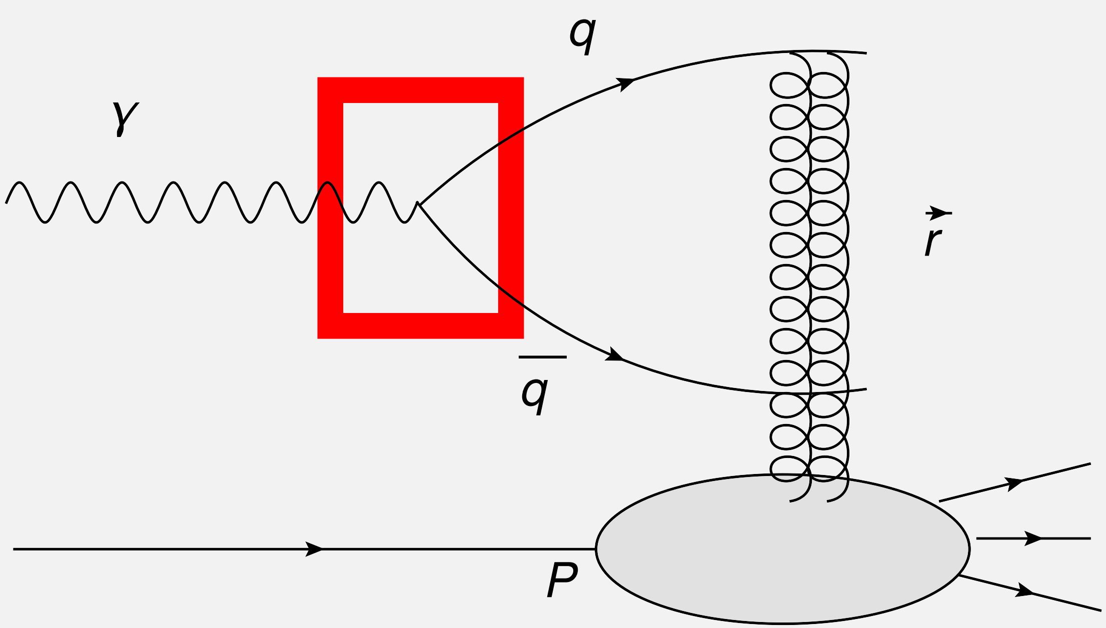
The photon must interact strongly with the target proton, how is that possible?

1. The virtual photon first fluctuates into a quark-antiquark pair
2. Then it exchanges an object with vacuum quantum numbers with the proton



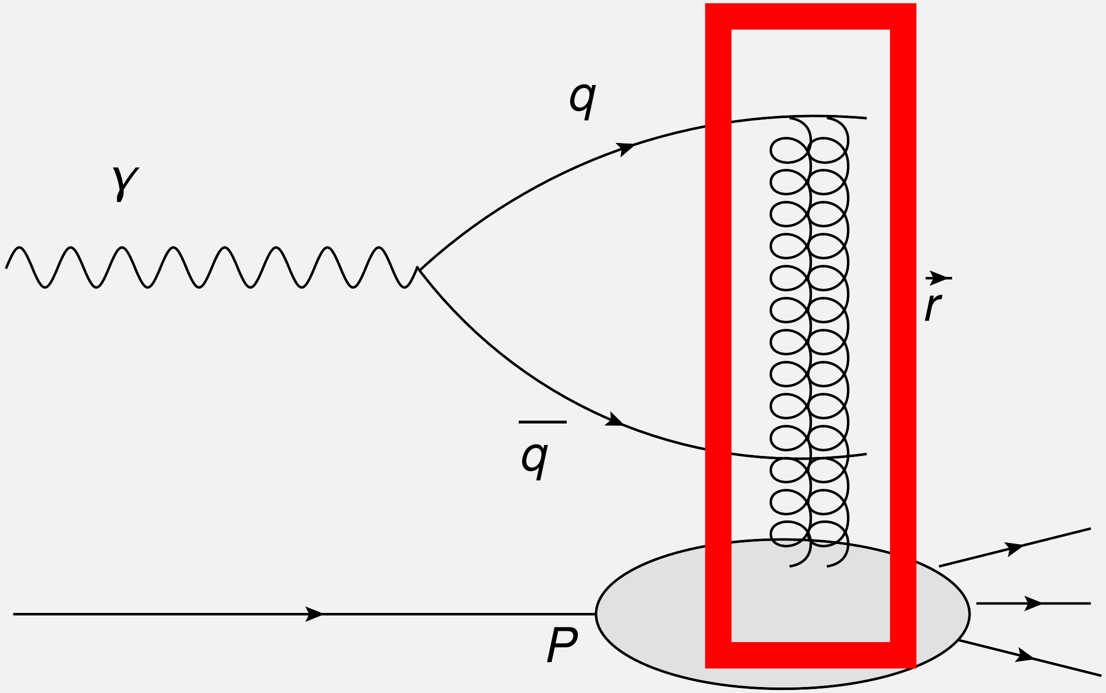
DIPOLE MODEL

The probability of a photon splitting to a quark-antiquark pair is computed from QFT.



DIPOLE MODEL

To compute the cross section of the interaction, we are missing the $\sigma_{\text{dipole-proton}}$



HOW DO WE OBTAIN THE DIPOLE-PROTON CROSS SECTION?

BK EQUATION

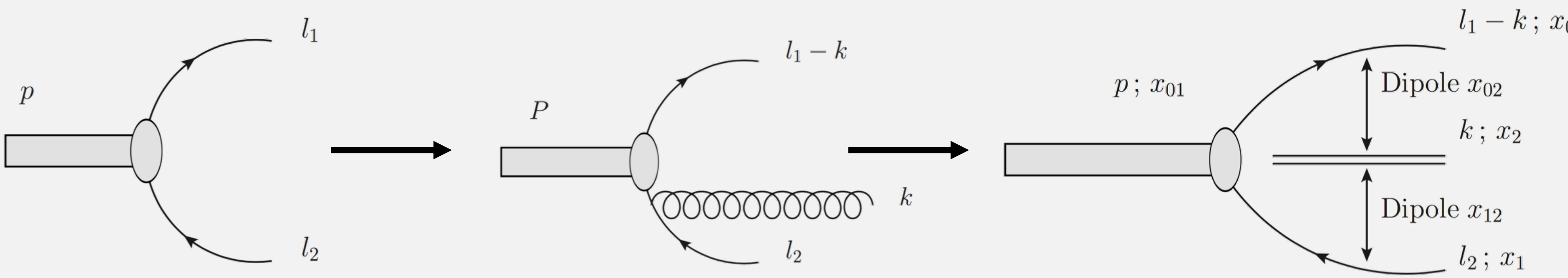
The BK equation governs the $\sigma_{\text{dipole-proton}}$, also called the scattering amplitude.
(Thanks to the optical theorem)

The main idea of the computation of the scattering amplitude is as follows:

1. Boost into a frame, where the dipole is at rest. Here the dipole is bare.
2. Then boost a bit, so that we add a bit energy into the system.
3. One of the quarks emits a gluon.
4. In the limit of high number of colors, this gluon fluctuates into another qq pair.
5. Two daughter dipoles are created. These contribute independently to the scattering amplitude

BK EQUATION

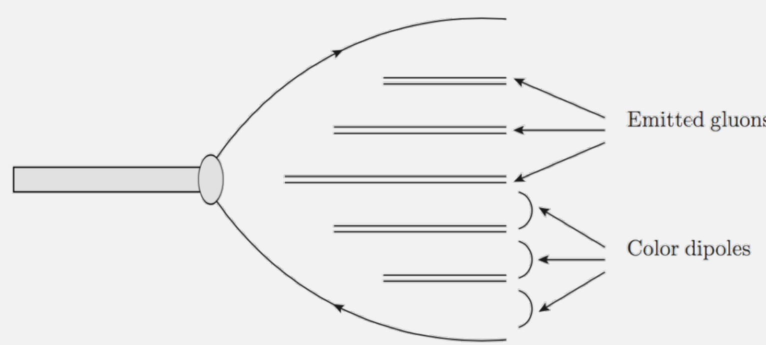
Schematically this means:



Add a bit of energy

High N_c limit

After some time, the initial dipole becomes dressed.



BK EQUATION

Mathematically, this realates to:

$$\frac{\partial N(r,Y)}{\partial \ln Y} = \int d\vec{r}_1 K(\vec{r},\vec{r}_1,\vec{r}_2)(N(\vec{r}_1,Y) + N(\vec{r}_2,Y) - N(\vec{r},Y) - N(\vec{r}_1,Y)N(\vec{r}_2,Y))$$

$$K(\vec{r},\vec{r}_1,\vec{r}_2) = \frac{\alpha_s(r^2)N_c}{2\pi}\left(\frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_1^2}\left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1\right) + \frac{1}{r_2^2}\left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1\right)\right)$$

BK EQUATION

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$$\boxed{\frac{\partial N(r,Y)}{\partial lnY} = \int} d\vec{r}_1 K(\vec{r},\vec{r}_1,\vec{r}_2)(N(\vec{r}_1,Y) + N(\vec{r}_2,Y) - N(\vec{r},Y) - N(\vec{r}_1,Y)N(\vec{r}_2,Y))$$

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This is the change of the scattering amplitude, when we add a bit of energy into the system.

BK EQUATION

Mathematically, this realates to:

$$\frac{\partial N(r,Y)}{\partial \ln Y} = \int d^2\vec{r}_1 \boxed{K(\vec{r},\vec{r}_1,\vec{r}_2)}\left(N(\vec{r}_1,Y) + N(\vec{r}_2,Y) - N(\vec{r},Y) - N(\vec{r}_1,Y)N(\vec{r}_2,Y)\right)$$

$$K(\vec{r},\vec{r}_1,\vec{r}_2) = \frac{\alpha_s(r^2)N_c}{2\pi}\left(\frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_1^2}\left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1\right) + \frac{1}{r_2^2}\left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1\right)\right)$$

Kernel is computed from QCD to reflect the probability of the gluon emission.

BK EQUATION

Mathematically, this realates to:

$$\frac{\partial N(r,Y)}{\partial lnY} = \int d\vec{r}_1 K(\vec{r},\vec{r}_1,\vec{r}_2) \boxed{(N(\vec{r}_1,Y) + N(\vec{r}_2,Y) - N(\vec{r},Y) - N(\vec{r}_1,Y)N(\vec{r}_2,Y))}$$

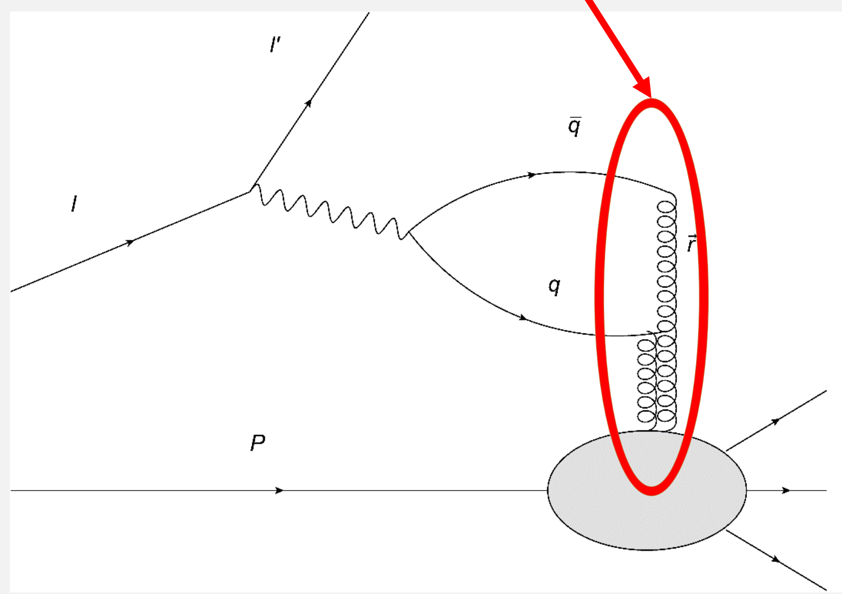
$$K(\vec{r},\vec{r}_1,\vec{r}_2) = \frac{\alpha_s(r^2)N_c}{2\pi}\left(\frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_1^2}\left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1\right) + \frac{1}{r_2^2}\left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1\right)\right)$$

Dipole-proton scattering amplitudes.

BK EQUATION

Okay, we know how to evolve to higher energy, but how about the initial condition?

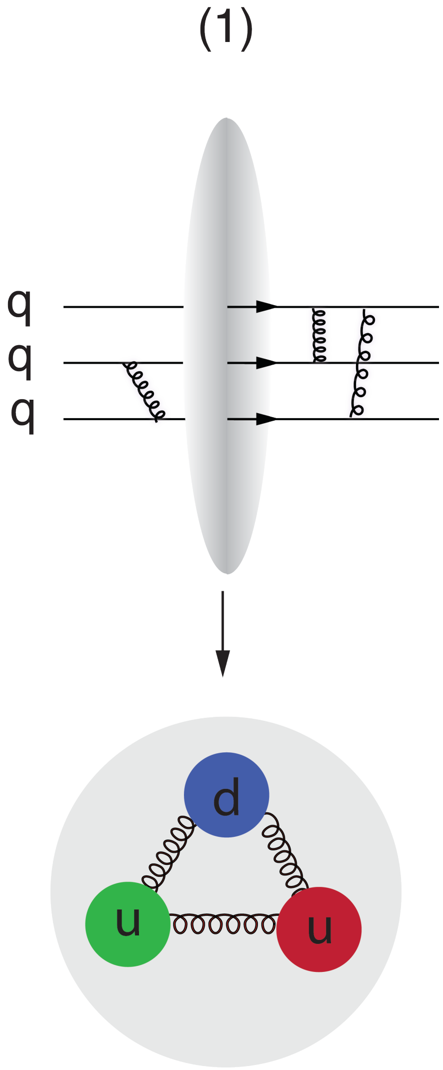
- How do we determine this variable?
  - We use the McLerran Venugopalan model (MV).
  - This model tries to address the interaction of the dilute dipole with the dense gluonic field.



$$N^{MV}(r) = 1 - \exp\left(\frac{-(r^2 Q_{s0}^2)^{\gamma}}{4} \ln\left(\frac{1}{r^2 \Lambda_{QCD}^2} + e\right)\right)$$

Where Λ_{QCD}, γ and Q_{s0}^2 are constants.

WHAT DOES THE BK TELL US ABOUT THE PROTON?



At large values of x (carried momentum fraction), the proton is made of valence quarks

(1)

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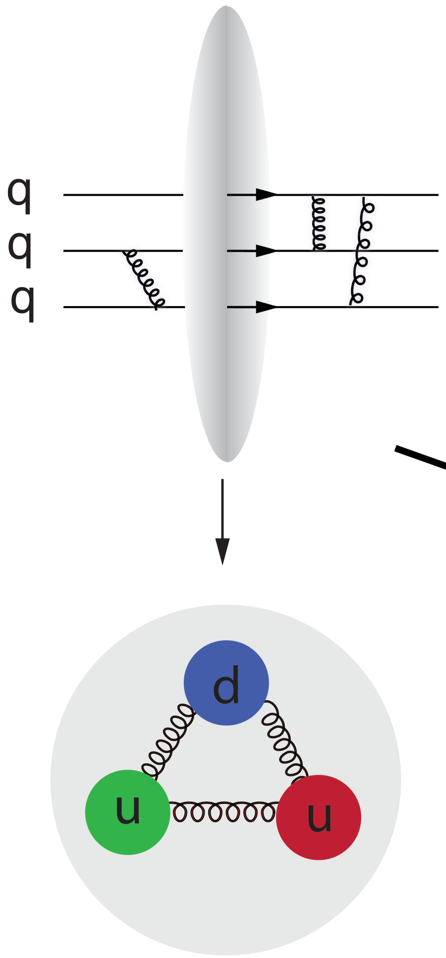
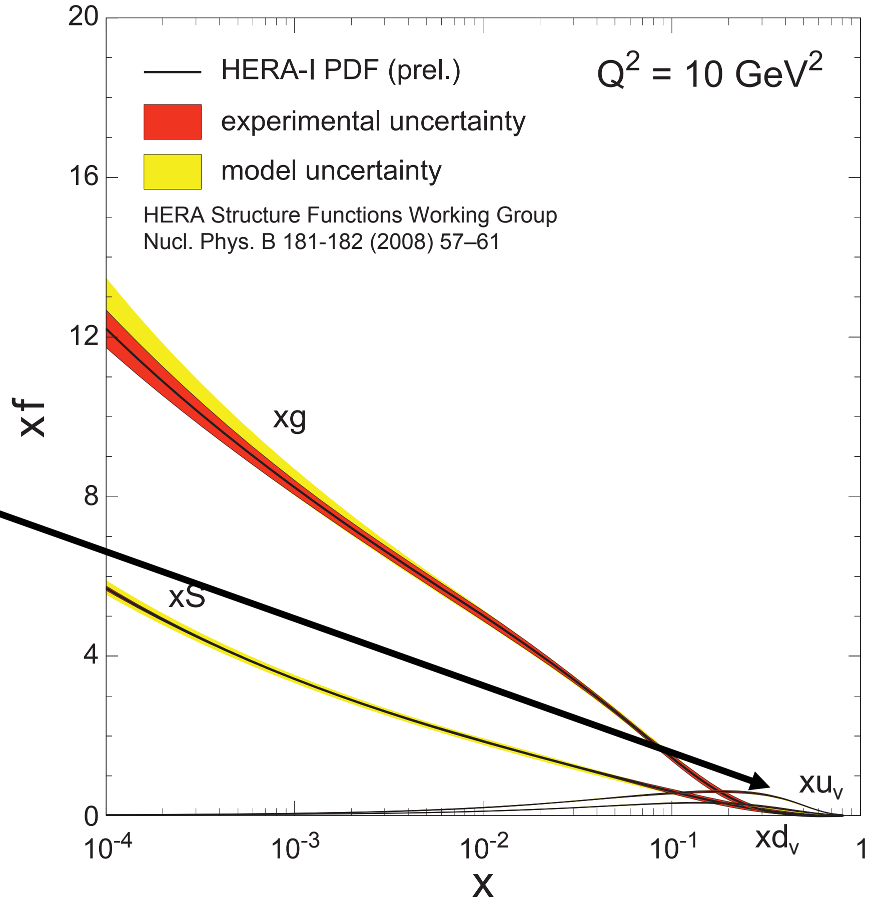
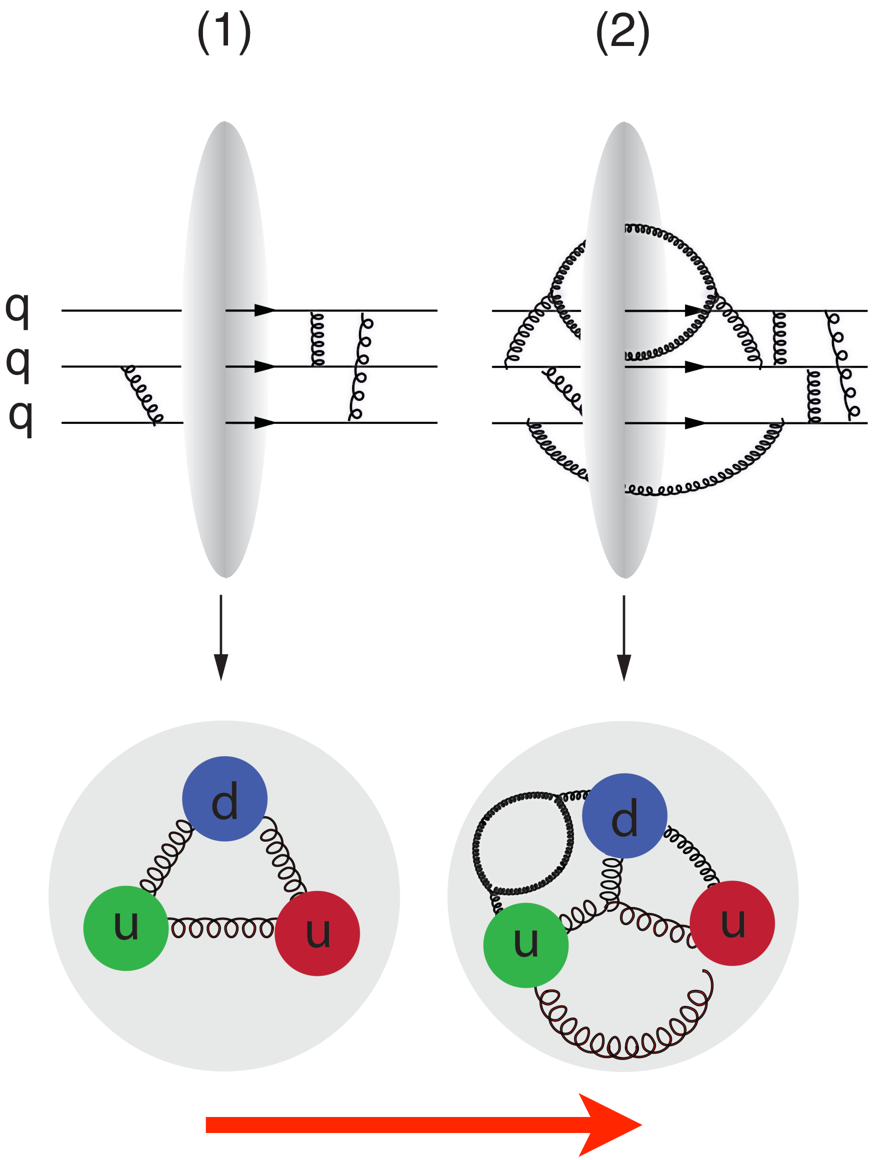


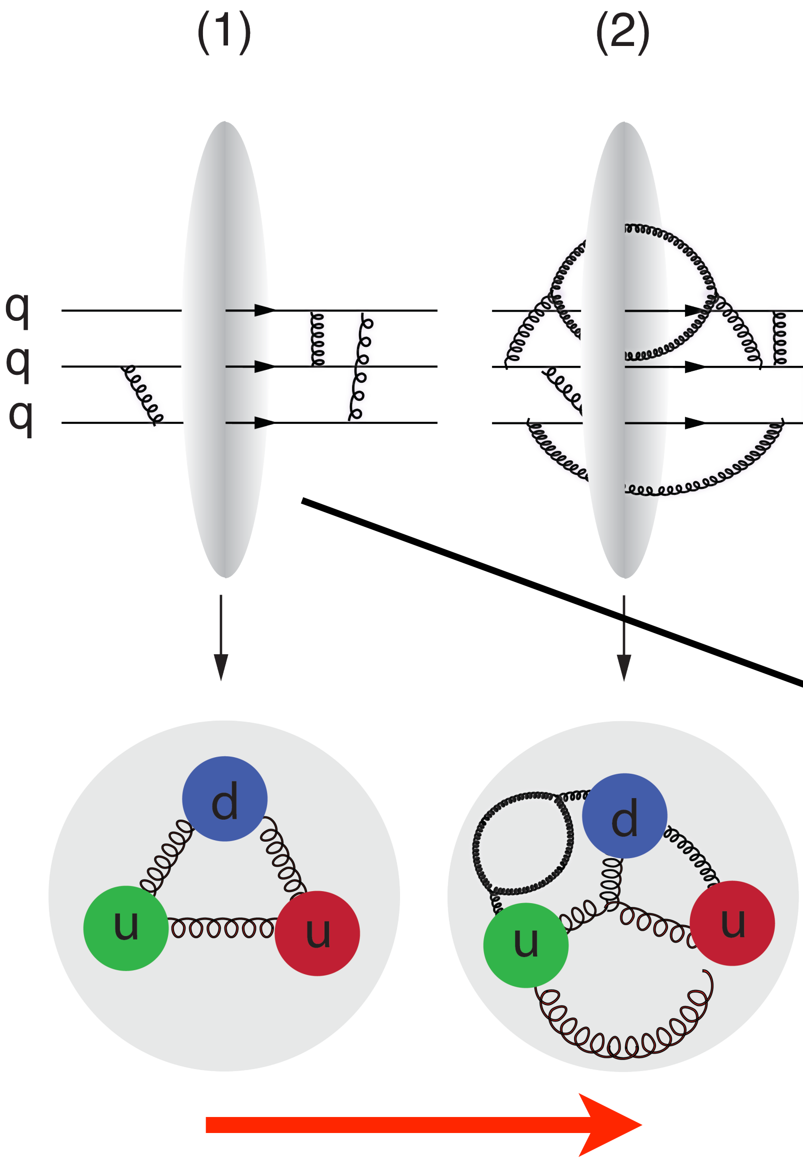




$$x \sim \frac{1}{\sqrt{s}}$$

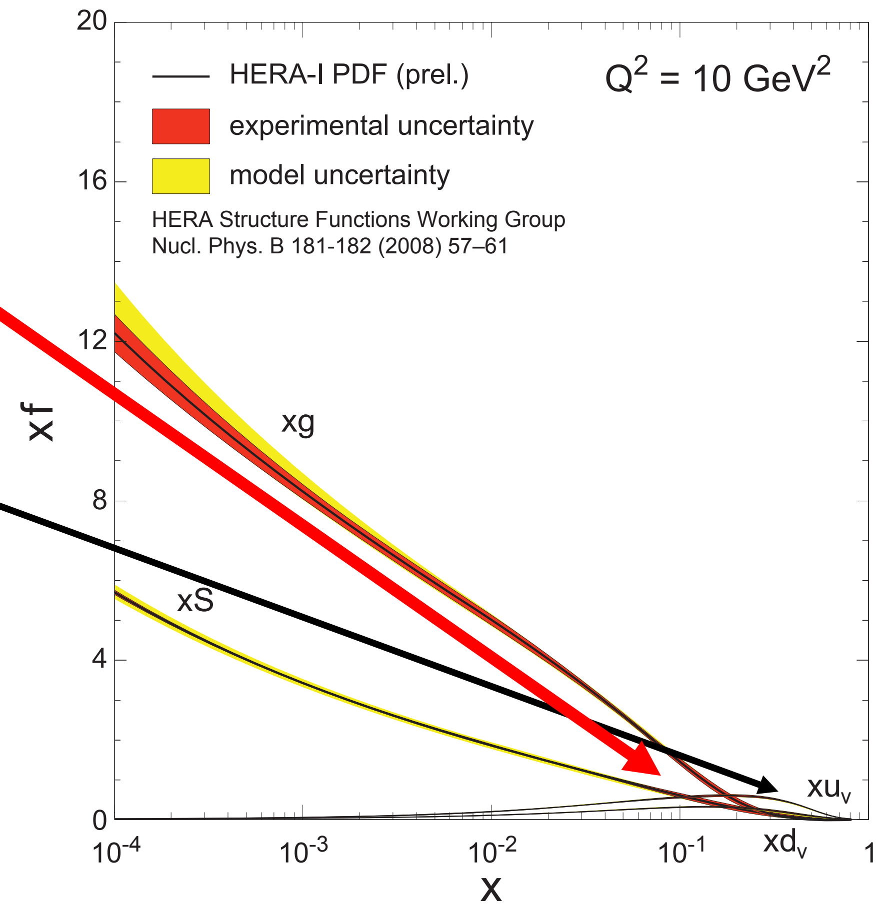
Increasing the energy of the collision means reaching lower values of x.

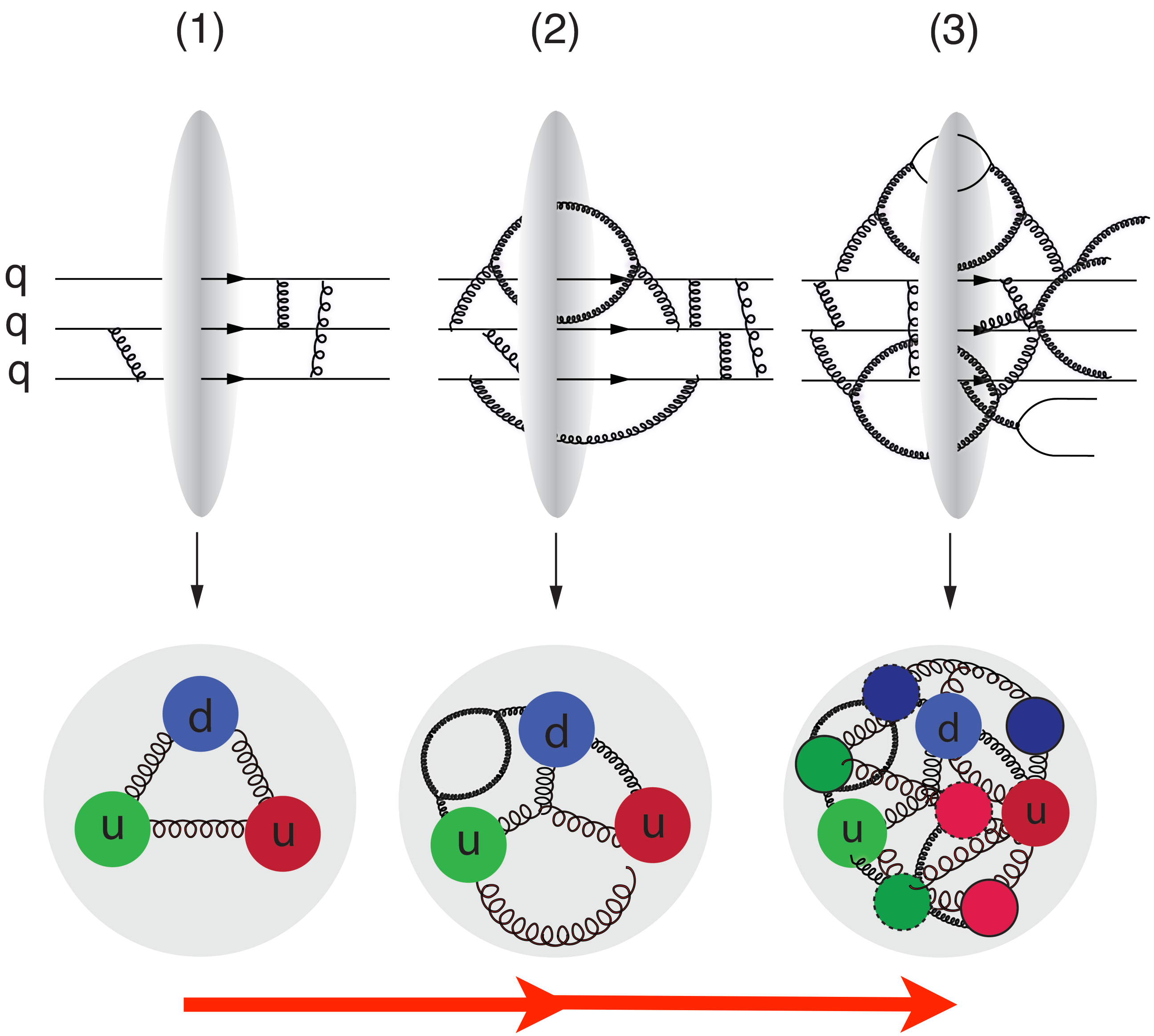
Energy of the collision



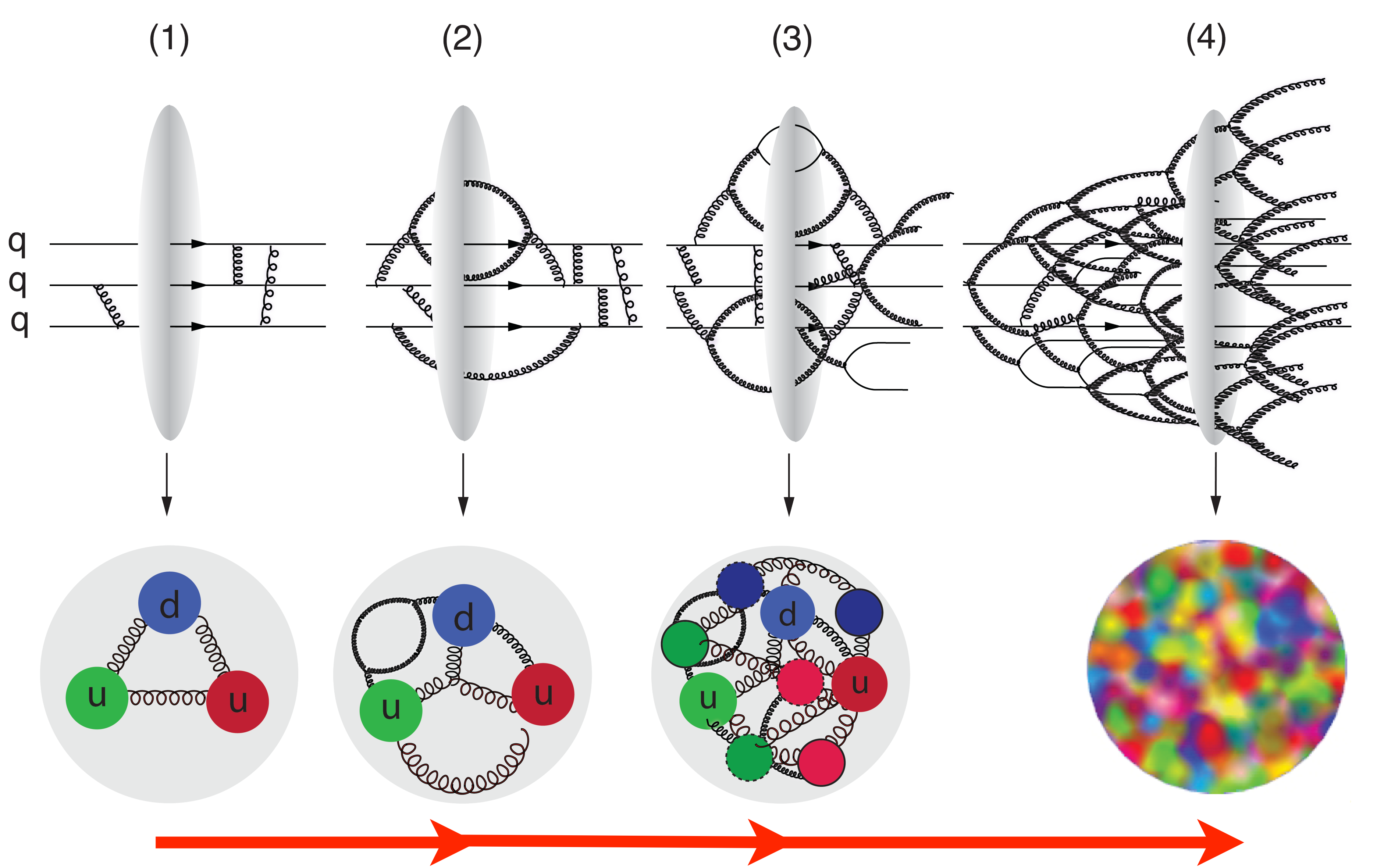
Energy of the collision

$$x \sim \frac{1}{\sqrt{s}}$$

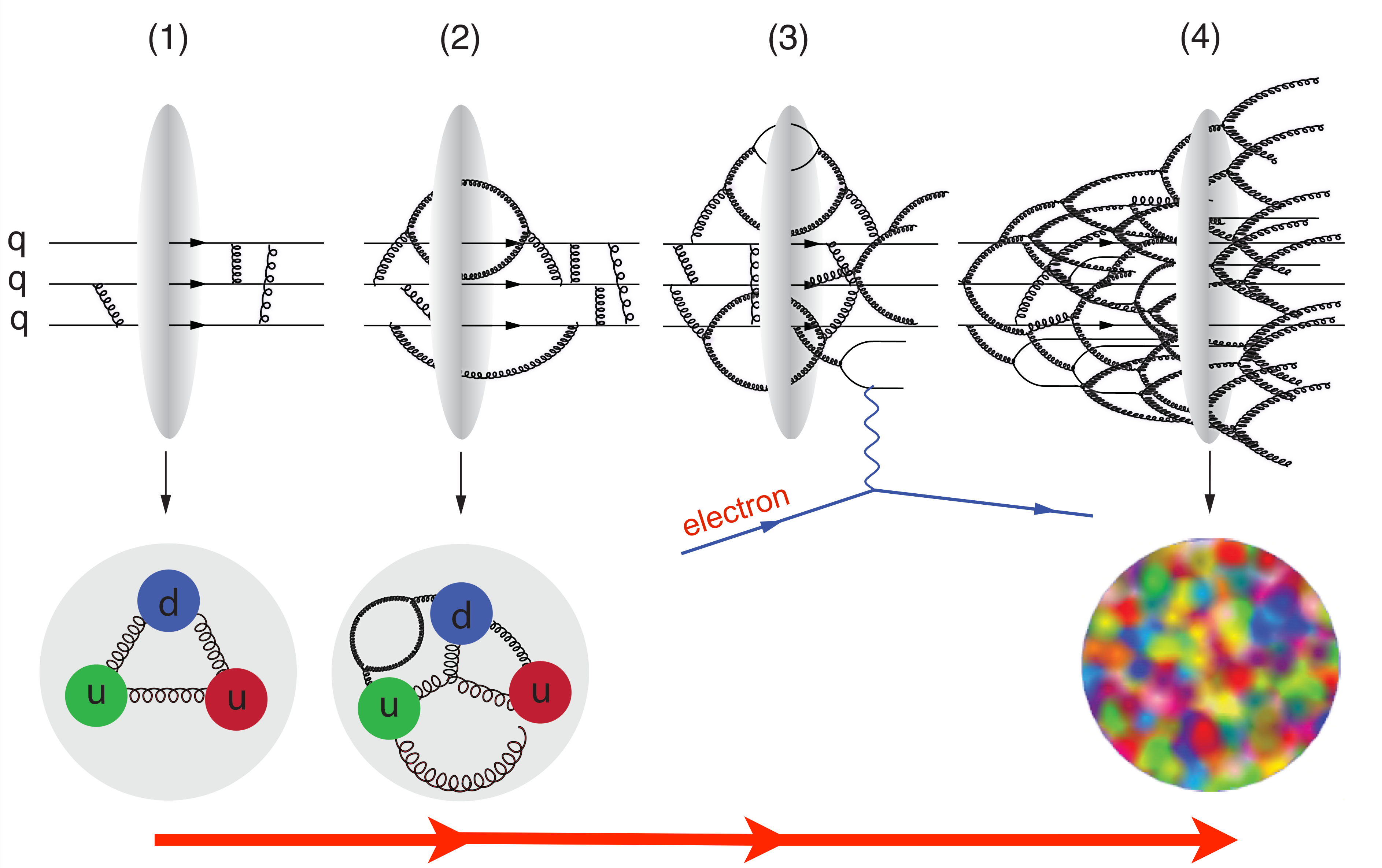




Energy of the collision



Energy of the collision



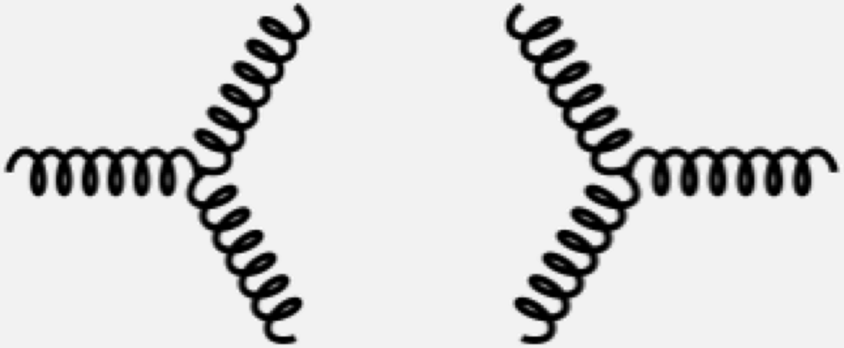
Energy of the collision

SATURATION

- If gluon numbers only grow toward region of low-x, the gluon distribution would diverge.
- This growth is governed by the BFKL equation.
- The rate of this growth is unphysical and gives us too high cross sections.
- Additional effects need to be taken into account!

SATURATION

- BFKL equation includes only the gluon radiation effects.
- Other non-linear evolution equation such as the BK equation takes gluon recombination into account.
- This slows down the evolution and tames the unphysical divergences.



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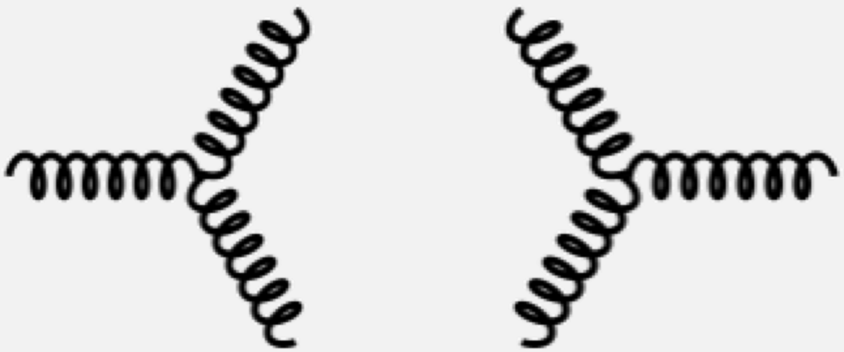

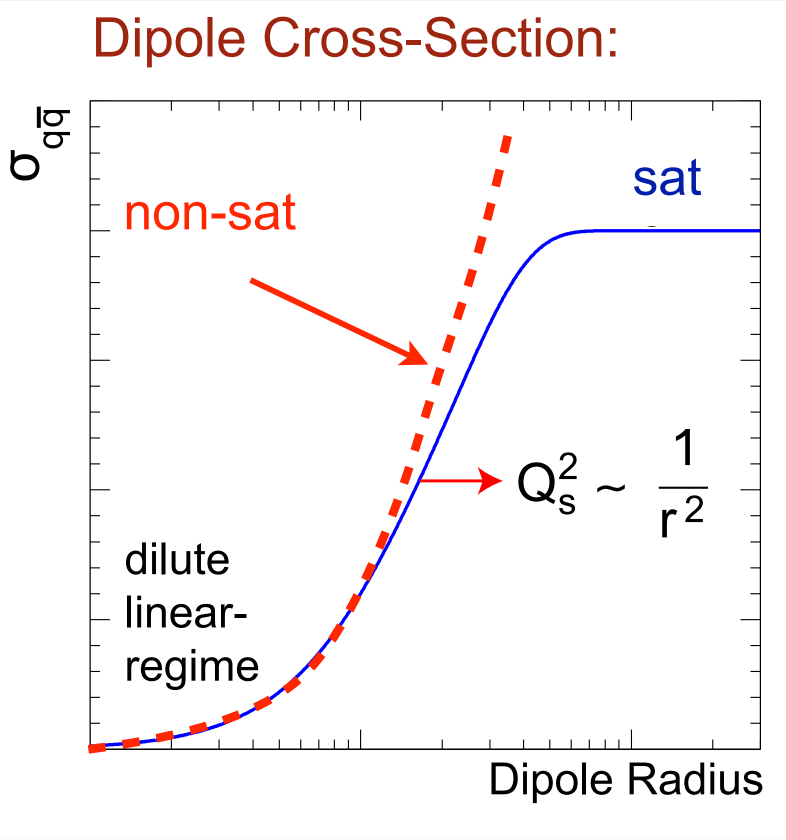

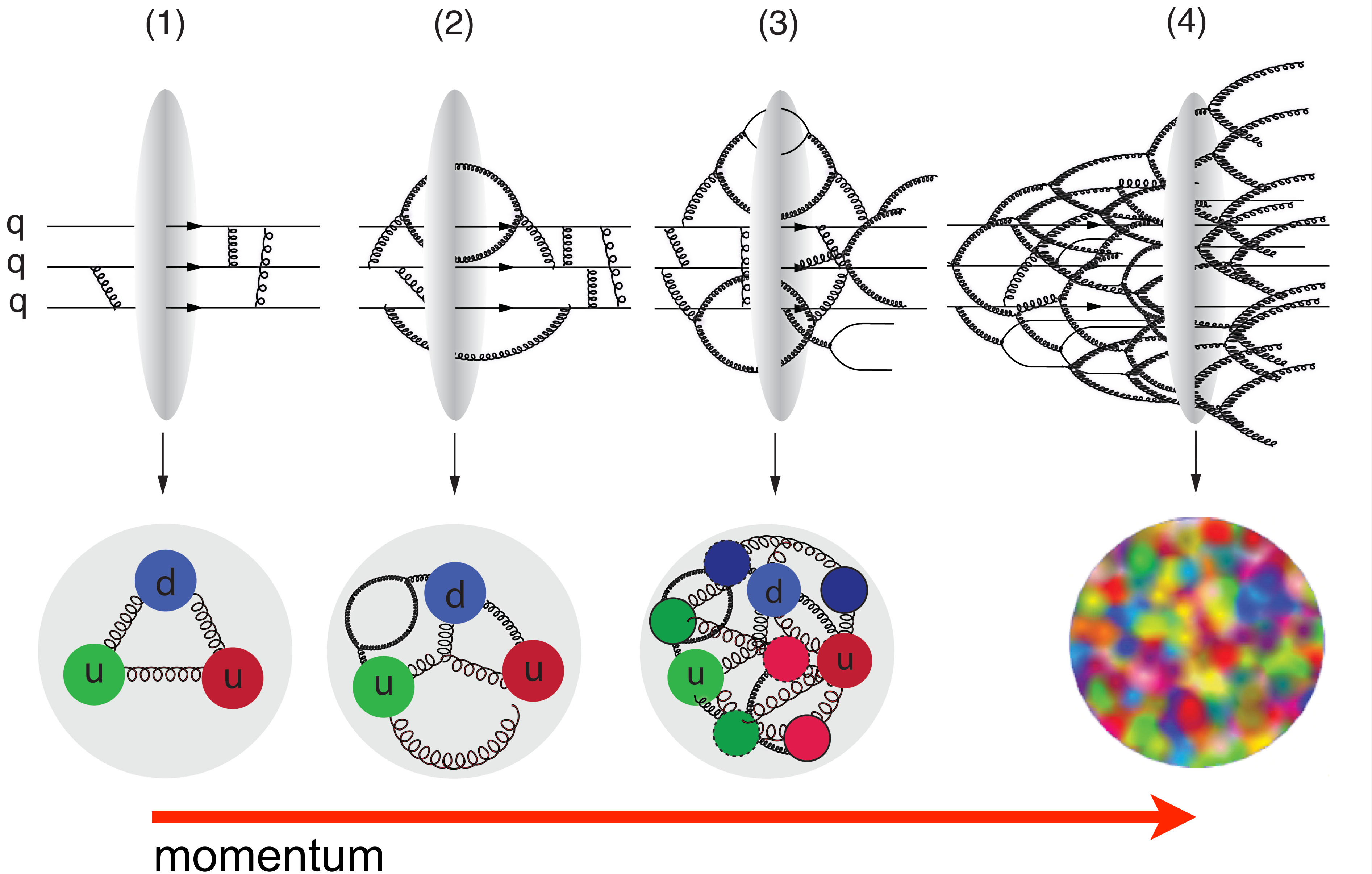

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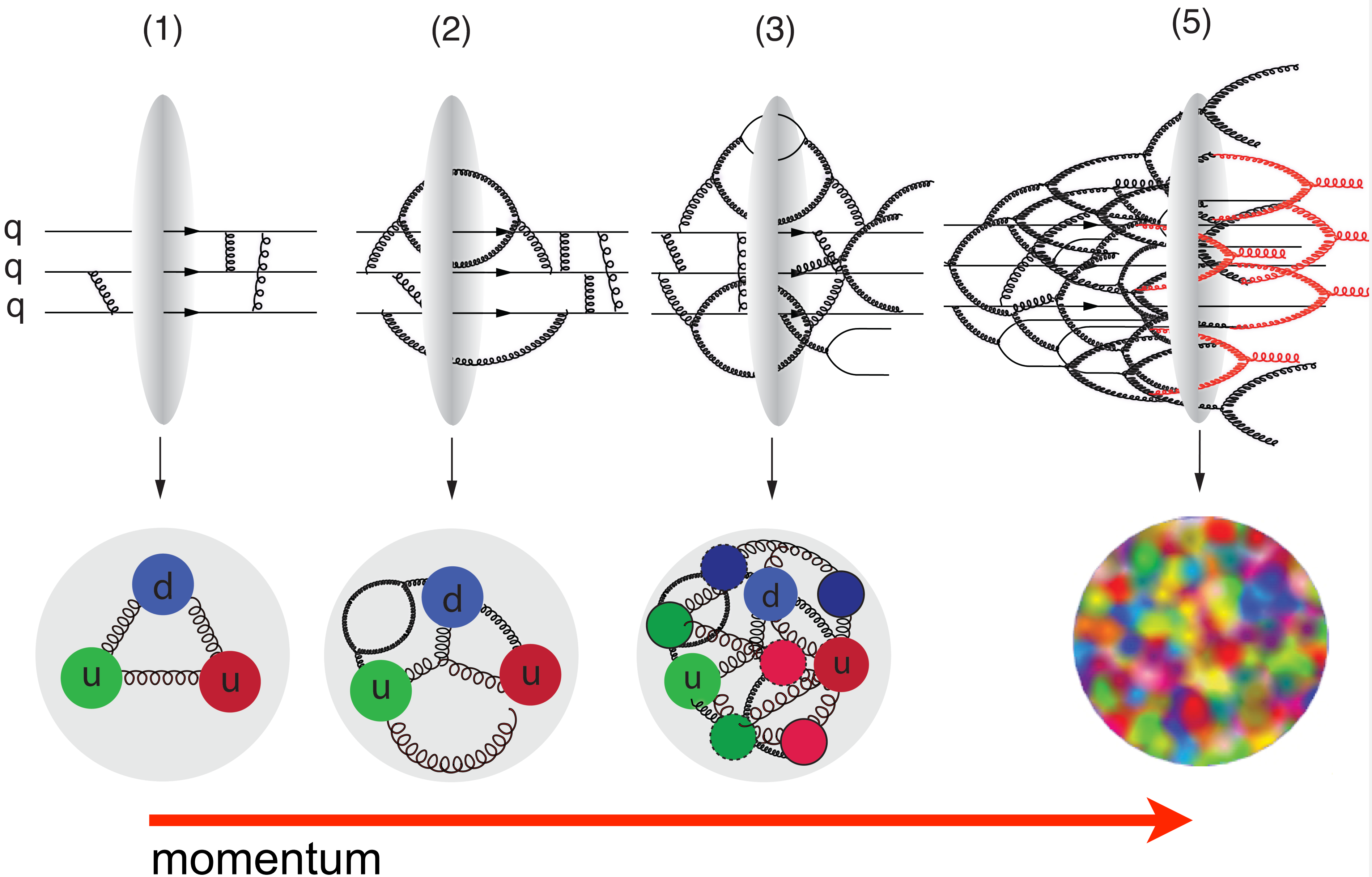

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momentum

IS SATURATION REAL?

The answer is „*We do not know.*“

There are strong hints, that it is, but it can be described by other phenomena.

CONCLUSIONS

- The BK evolution equation is one of the ways to solve the photon-hadron interactions.
- It describes nonlinear effects such as saturation.
- It can give us an insight on the internal dynamics of gluons inside nucleons as well as nuclei.

THANK YOU FOR YOUR ATTENTION

No matter what, don’t lose hope. We are all bombastic.

- Dan Nekonečný