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The Good-Walker approach to dissociation

J. G. Contreras
Czech Technical University

September 27, 2019, Děčín

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The original paper

PHYSICAL REVIEW VOLUME 120, NUMBER 5 DECEMBER 1, 1960

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M. L. Good and W. D. Walker
University of Wisconsin, Madison, Wisconsin
(Received May 26, 1960)

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https://books.google.com/books?id=FzYVAAAAQAAJ&pg=PA2

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Translation : It has illuminated for us another, fourth way, which we now make known and call "diffraction" [i.e., shattering], because we sometimes observe light break up; that is, that parts of the compound [i.e., the beam of light], separated by division, advance farther through the medium but in different [directions], as we will soon show.

https://en.wikipedia.org/wiki/Diffraction

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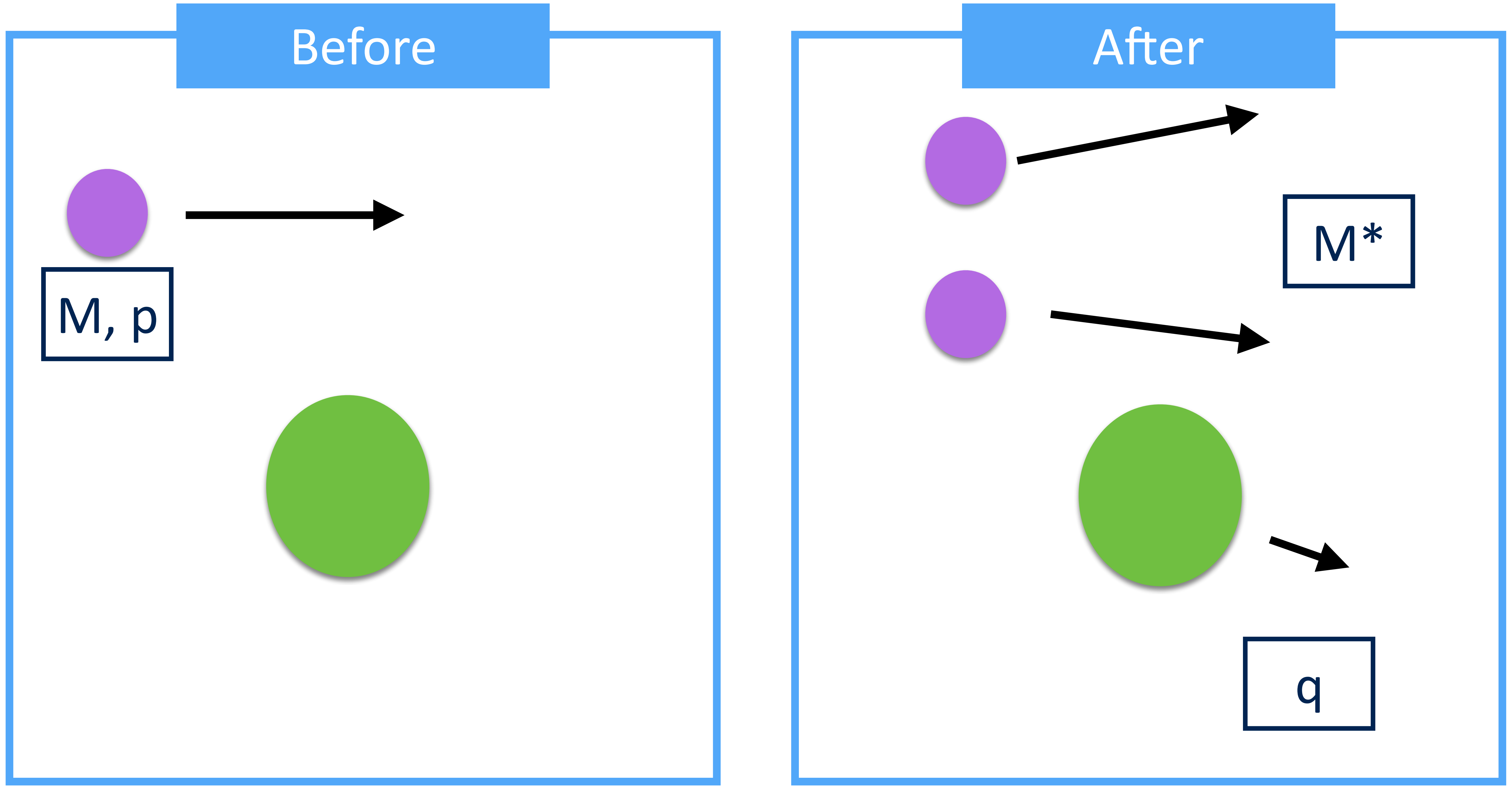
Energy-momentum arguments



Energy-momentum arguments

First we must establish that this is energetically possible. Suppose we have an incident particle A (rest mass M, momentum P) and consider the dissociation $A \rightarrow B+C$. Let the energy of $B+C$ in the rest frame of $B+C$ be M^*. We wish to consider a reaction in which the nucleus is left intact, and in its ground state. The nucleus will take up momentum $\mathbf{q}$ and essentially no energy. The requirement of energy and momentum conservation is then, for small transverse momenta,

$$q_{||} = (M^{*2} - M^2)/2p, \tag{1}$$

where $q_{||}$ is the component of $\mathbf{q}$ in the beam direction. $q_{||}$ may be very much less then $m_\pi/A^{\frac{1}{3}}$, thus justifying the assumption that the nucleus can hang together. We

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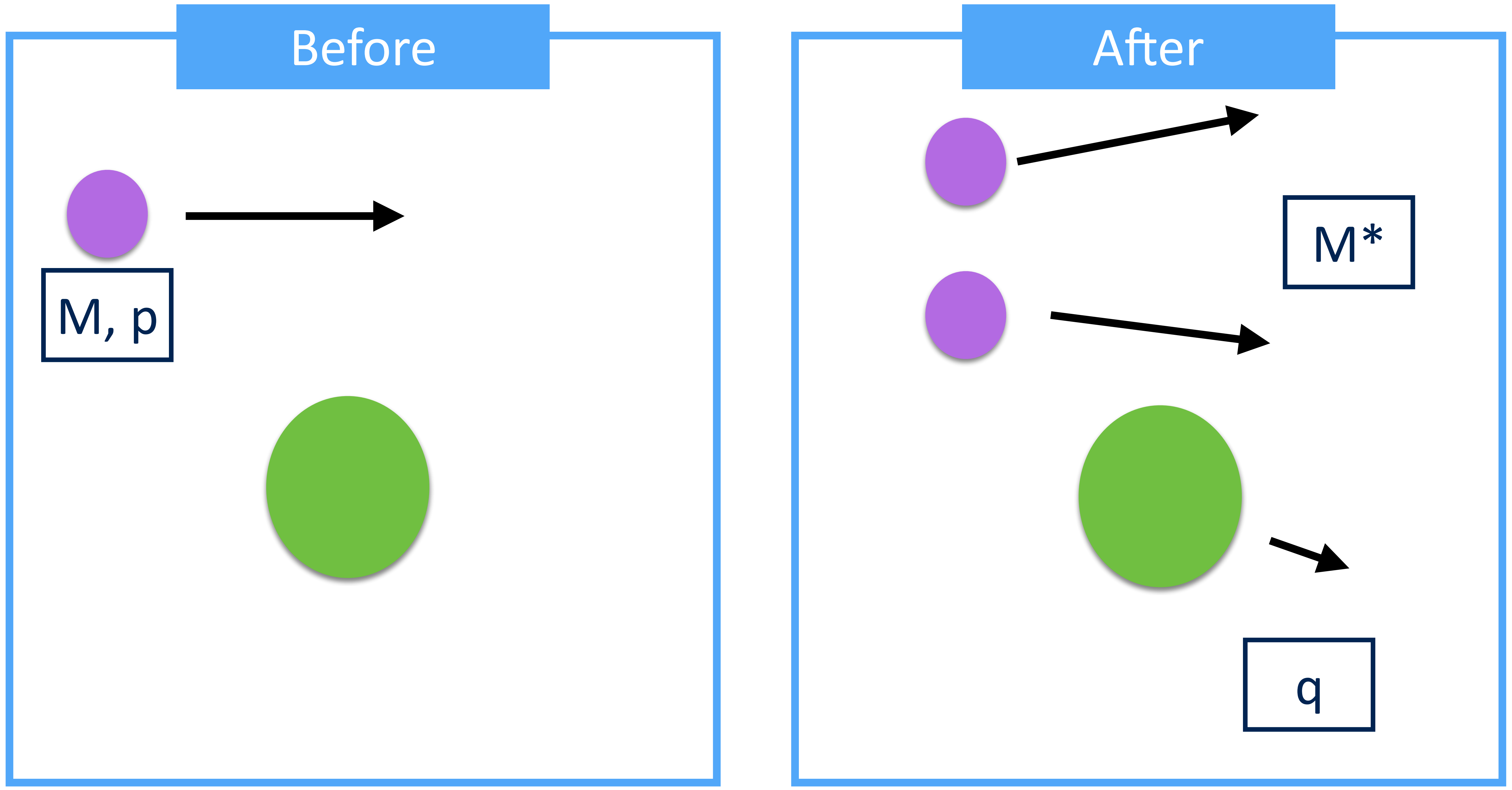
$$P_{\mathrm{th}} = \frac{M^{*2} - M^2}{2m_\pi} A^{\frac{1}{3}}. \tag{2}$$



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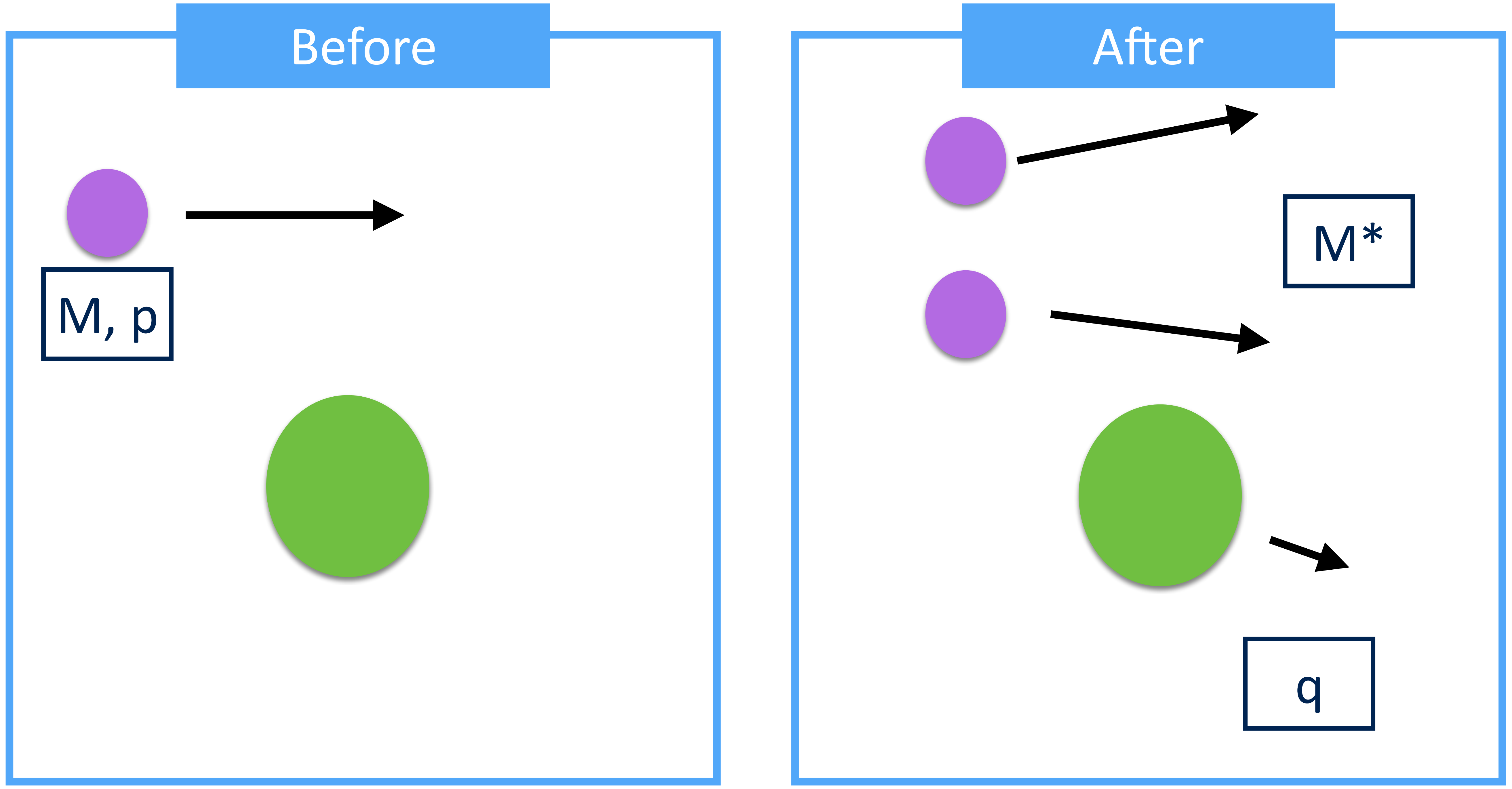

$$A + (\text{nucleus}) \to B + C + (\text{nucleus in ground state}), \tag{3}$$

is thus energetically possible if the beam energy is high enough.

Energy/time/momentum scale

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Note: A here is the mass number, not the incoming particle A, as in the second line

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Experimental observation about the nuclear density:

$$\frac{A}{(4/3)\pi R_A^3} \propto \text{constant} \longrightarrow R_A = r_0 A^{1/3}$$

$r_0 = 1.2$ fm

Note that $m_\pi \approx 1/1.4$ fm

Reasonable scale of standard nuclear scales

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States M and M* can be considered degenerated

The next question is whether the reaction actually happens. We do not know how to calculate its rate, in general, as the strong interactions are complicated, and as this is a many-body problem.

What we will do instead is to present a physical argument which shows how such reactions would be brought about, and makes apparent some interesting properties they would have.

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We now consider our nuclear reaction (3). First let us inquire whether the state $B+C$ of mass (or proper energy) M^* may be regarded as degenerate with the incident particle state, A. If the difference in frequency between $B+C$ and A, times the time of passage through the nucleus, is small compared to unity, then they are for all paractical purposes degenerate. The condition for this is

$$\boxed{(\gamma^* M^* - \gamma M) A^{\frac{1}{3}}/m_\pi \ll 1.}$$

If we take $\gamma^*\beta^*M^* = p = \gamma\beta M$, this becomes $\boxed{p \gg p_{\mathrm{th}}(M^*)}$, with P_{th} given by (2).

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For large momenta of the projectile there is enough energy for it to split and at the same time the new state can be considered degenerated with the original one

Dressed, bare and C bases

Let us now consider the incident particle to be a nucleon, for definiteness. It is a "dressed" or real nucleon, $|\tilde{N}\rangle$, in contradistinction to the "bare" nucleon, $|N\rangle$. Now we may expand any state in terms of any complete set of states. For example, we could expand $|\tilde{N}\rangle$ in terms of the states of "bare" particles:

$$\boxed{|\tilde{N}\rangle = \sum_i a_{Ni} |B_i\rangle}$$

$|B_i\rangle = |N\rangle, |N\pi\rangle, |N2\pi\rangle, \cdots, |\Lambda K\rangle, \cdots$, where the $|B_i\rangle$ are all the one, two, or more particle states (of "bare" nucleons and "bare" pions) with the same quantum numbers as the nucleon, i.e., the same charge, strangeness$=0$, intrinsic angular momentum$=J=\frac{1}{2}$, etc.[4]

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We now consider the case where $p \gg P_{\text{th}}(M^*)$ for the final state M^* of interest. We are then justified in neglecting mass differences in discussing the behavior of the nucleon wave as it penetrates the nucleus. We may therefore expand the state $|\tilde{N}\rangle$ into an appropriate complete set, and expect that the different terms in the expansion will be attenuated separately in passing through the nucleus. The set we want is clearly neither the bare-particle set $|B_i\rangle$ nor the dressed-particle set $|D_j\rangle$ but some third set, comprised of just those linear combinations of bare particle states which are the eigenstates inside nuclear matter. Call this set $|C_i\rangle$. The $|C_i\rangle$ have the property that each is attenuated with a simple exponential dependence in traversing the nucleus.

The formulation of the problem

The formulation of the problem is now simple. The incident wave is

$$\boxed{|I\rangle = e^{ikz}|\tilde{N}\rangle = e^{ikz}\sum_i c_{Ni}|C_i\rangle.}$$

After traversing the nucleus, the transmitted wave is[5]

$$\boxed{|T\rangle = \sum c_{Ni}\eta_i|C_i\rangle,}$$

where

$$|\eta_i| \leqslant 1.$$

The scattered wave is the difference between $|I\rangle$ and $|T\rangle$:

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angular distribution. But $|S\rangle$ is now in general no longer a pure nucleon state; rather the projections $\langle D_j|S\rangle$ represent the amplitude in $|S\rangle$ of the various two-or-more-particle states $|D_j\rangle$ of real particles of the same quantum numbers as $|\tilde{N}\rangle$. $|S\rangle$ may be written as

$$\boxed{\begin{aligned} |S\rangle &= \textstyle\sum_i c_{Ni}(1-\eta_i)|C_i\rangle \\ &= (1-\bar{\eta})|\tilde{N}\rangle + \textstyle\sum_i (\bar{\eta}-\eta_i)c_{Ni}|C_i\rangle, \end{aligned}}$$

where the first term is the scattered nucleon wave and the second represents the diffraction produced particles, i.e., not states involving only single nucleons. The

Discussion

(1) The outgoing wave $|S\rangle$ and its real-particle projections $\langle D_j|S\rangle$ have the angular dependence characteristic of diffraction scattering, since they are produced by differential absorption of the incident beam. What is meant by this is the following: one measures the momentum of each outgoing particle, and constructs, for each event, the vector sum of these momenta. The distribution in angle of this vector with respect to the incident beam should be that of a diffraction scattering. Since at high energy the diffraction pattern is very narrow, this represents a distinctive feature of the reaction, and could be used to identify it.

(2) The outgoing wave of other than incident particles will consist of two- (or more)-body systems having the same quantum numbers as the incident particle, i.e., the same charge, strangeness, nucleon number, isotopic spin, intrinsic angular momentum, and parity. Thus if $N \rightarrow N+\pi$ is observed in this way,