Hydrodynamics for heavy ion collisions: why, what and how?

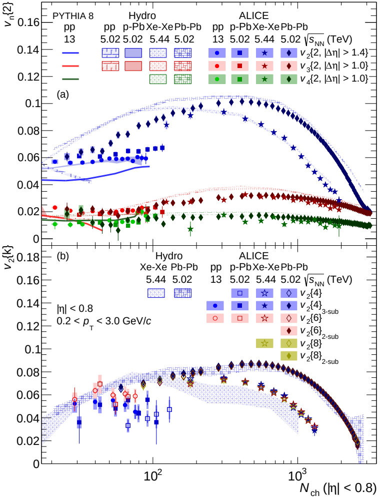
Iurii KARPENKO

FJFI CVUT

Outline

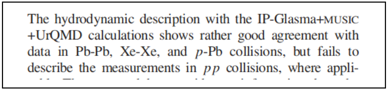
- **Why** hydrodynamic approach?
- **What** is hydrodynamic approach?
- Ingredients of the modelling (the "**how**" part)
- Making physics conclusions from it
- Summary

“Hydrodynamics works”



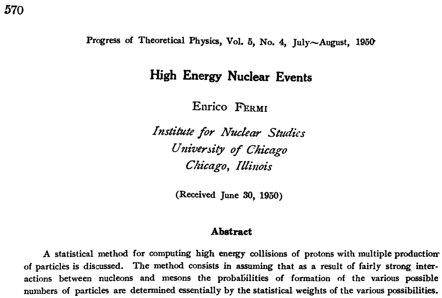
This is a typical plot showing a good reproduction of experimental flow measurements in the hydrodynamic approach, taken from:

ALICE collaboration, Phys. Rev. Lett. 123,142301 (2019)



The hydrodynamic description with the IP-Glasma+MUSIC +UrQMD calculations shows rather good agreement with data in Pb-Pb, Xe-Xe, and *p*-Pb collisions, but fails to describe the measurements in *pp* collisions, where appli-

The starting point: Fermi model for high energy NN collisions (1950)



570

Progress of Theoretical Physics, Vol. 5, No. 4, July~August, 1950

High Energy Nuclear Events

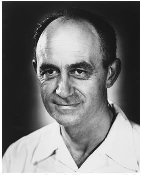
Enrico FERMI

Institute for Nuclear Studies
University of Chicago
Chicago, Illinois

(Received June 30, 1950)

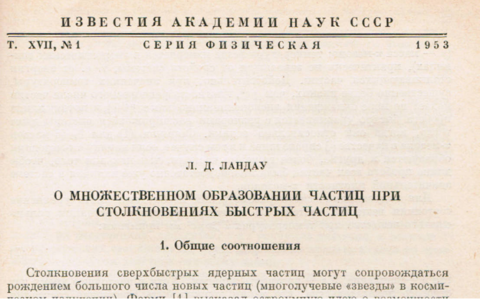
Abstract

A statistical method for computing high energy collisions of protons with multiple production of particles is discussed. The method consists in assuming that as a result of fairly strong interactions between nucleons and mesons the probabilities of formation of the various possible numbers of particles are determined essentially by the statistical weights of the various possibilities.



- Secondary particles are created in the Lorentz contracted volume $V = \frac{4\pi}{3} a^3 \frac{2Mc^2}{\sqrt{s}}$, where $a = \hbar/\mu c$ is the size of nucleon meson cloud.
- Particles are born in statistical equilibrium (because of strong interaction and small volume), and immediately escape in a “frozen state”.
- The angular distribution of produced pions and nucleons is essentially isotropic.

Hydro introduced: Landau model



ИЗВЕСТИЯ АКАДЕМИИ НАУК СССР

Т. XVII, № 1 СЕРИЯ ФИЗИЧЕСКАЯ 1953

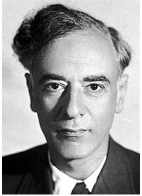
Л. Д. ЛАНДАУ

О МНОЖЕСТВЕННОМ ОБРАЗОВАНИИ ЧАСТИЦ ПРИ СТОЛКНОВЕНИЯХ БЫСТРЫХ ЧАСТИЦ

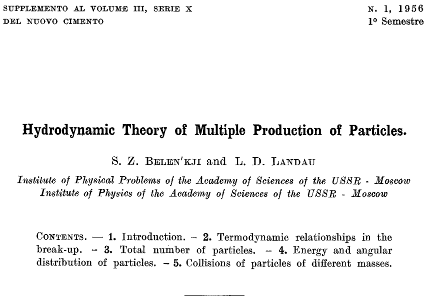
1. Общие соотношения

Столкновения сверхбыстрых ядерных частиц могут сопровождаться рождением большого числа новых частиц (многолучевые «звезды» в косми-

Izv. Ak. Nauk SSSR,
Ser. Fiz., **17**, 51 (1953)



Hydro introduced: Landau model



SUPPLEMENTO AL VOLUME III, SERIE X
DEL NUOVO CIMENTO

N. 1, 1956
1° Semestre

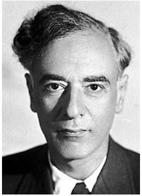
Hydrodynamic Theory of Multiple Production of Particles.

S. Z. Belen'kji and L. D. Landau

Institute of Physical Problems of the Academy of Sciences of the USSR - Moscow
Institute of Physics of the Academy of Sciences of the USSR - Moscow

Contents. — **1.** Introduction. – **2.** Termodynamic relationships in the break-up. – **3.** Total number of particles. – **4.** Energy and angular distribution of particles. – **5.** Collisions of particles of different masses.

Nuovo Cimento Suppl.,
Vol. **3**, 15 (1956)



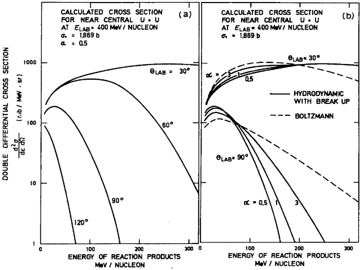
Massive critisizm of Fermi's model!

- Causality problems for noncentral collisions
- The initial volume V expands (hydrodynamically!)
- As the mean free path becomes comparable to the system size, it disintegrates into separate particles.
  This happens when $T \approx \mu$, the pion mass.

Some further developments

- Hagedorn '1968:
  Bootstrap model, "maximal temperature of hadronic matter"
- Chapline, Johnson, Teller, Weiss '1973
- Bondorf, Garpman, Zimanyi '1977:
  Fe+Ag, U+U reactions with $E_{\text{lab}} = 400$ MeV (LBNL),
  peaks in $d^2\sigma/d\varepsilon d\Omega$ which shift to higher energies for more forward angles (see Fig.).
  "As a final remark, one can conclude that a case for hydrodynamics, if any, will most likely be found in collisions between big and energetic heavy ions."

- Siemens, Rasmussen '1979:
  formulation of Blast-Wave (hydro-inspired) model
  Ne+NaF reactions with $E_{\text{lab}} = 800$ MeV
- ...and many more

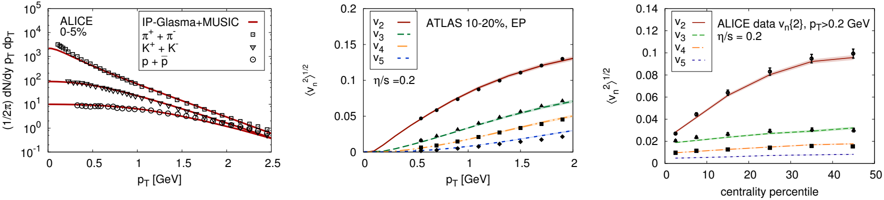


Why?

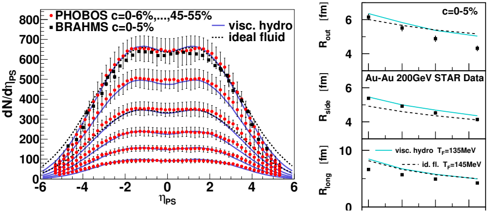
- Because it describes A LOT of observables in heavy ion collision experiments.

Some examples of “hydro vs experiment”

C. Gale et al, Phys.Rev.Lett. 110, 012302(2012)



P. Bozek, Phys. Rev. C 85, 034901 (2012)

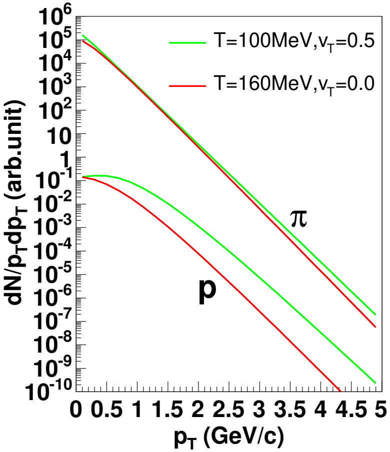


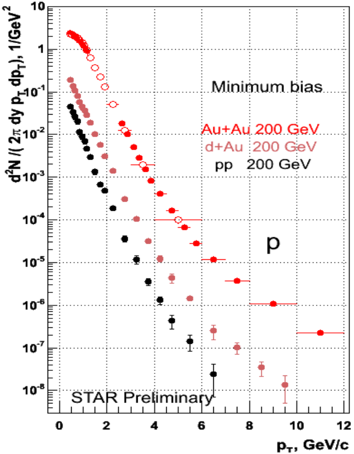
Also serves as a background for:

- photon, dilepton production
- heavy quark propagation
- fluid-jet interaction

“Radial flow”: the most basic signature of collective expansion

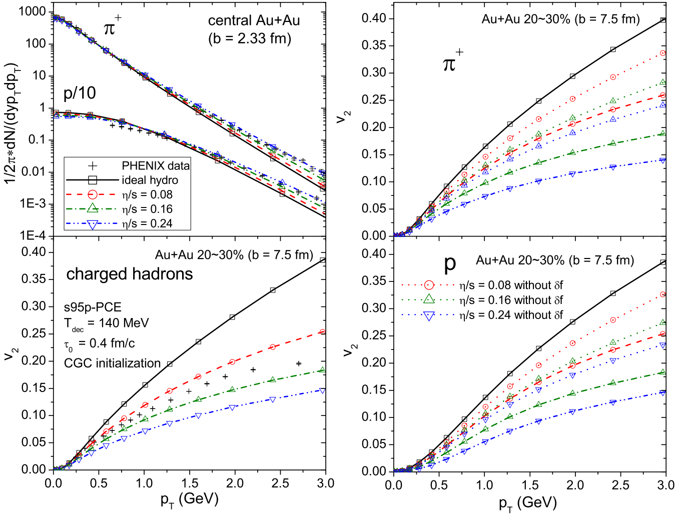
Notice the shoulder-like structure in the proton p_T spectrum.





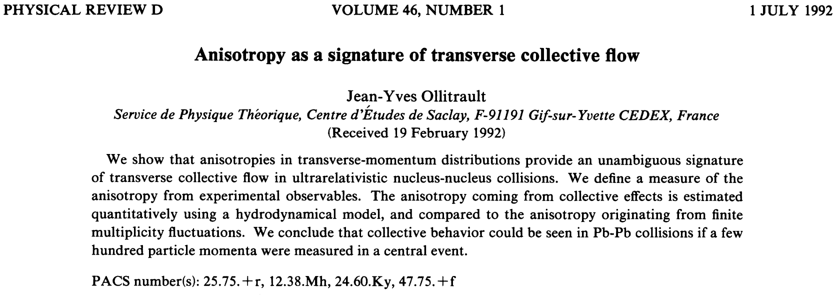
Radial flow in central gold-gold collisions at RHIC

Top left plot: transverse momentum distributions of pions and protons



Shen, Heinz, Huovinen, Song, Phys. Rev. C 82, 054904 (2010)

The elliptic flow (Ollitrault'92)



PHYSICAL REVIEW D VOLUME 46, NUMBER 1 1 JULY 1992

Anisotropy as a signature of transverse collective flow

Jean-Yves Ollitrault
Service de Physique Théorique, Centre d'Études de Saclay, F-91191 Gif-sur-Yvette CEDEX, France

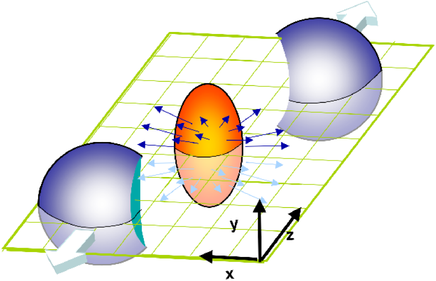
(Received 19 February 1992)

We show that anisotropies in transverse-momentum distributions provide an unambiguous signature of transverse collective flow in ultrarelativistic nucleus-nucleus collisions. We define a measure of the anisotropy from experimental observables. The anisotropy coming from collective effects is estimated quantitatively using a hydrodynamical model, and compared to the anisotropy originating from finite multiplicity fluctuations. We conclude that collective behavior could be seen in Pb-Pb collisions if a few hundred particle momenta were measured in a central event.

PACS number(s): 25.75.+r, 12.38.Mh, 24.60.Ky, 47.75.+f

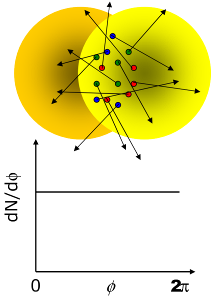
- Anisotropy in the transverse momentum distribution of produced hadons is an unambiguous signature of collective transverse flow.
- It is sensitive to equation of state (EoS) of the fluid, and to the presence of 1^{st} order phase transition between QGP and hadronic phases.

What is “elliptic” flow?

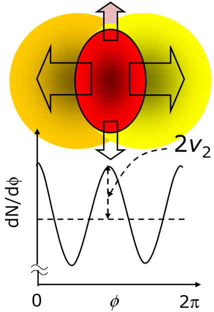


The participant region (orange ellipse on the sketch at the left) may behave as:

1) non-interacting system



2) strongly interacting system

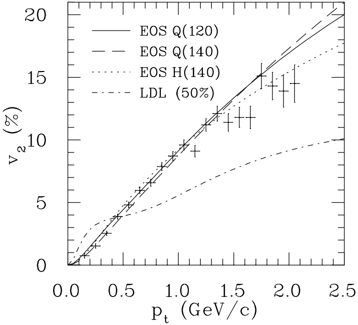


$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left[1 + 2v_1\cos(\phi) + 2v_2\cos(2\phi) + \cdots\right]$$

First $\sqrt{s_{NN}} = 130$ GeV RHIC results

Before RHIC start-up: two scenarios possible:

- asymptotic freedom ⇒ weakly interacting system
- stongly interacting system



Elliptic flow v_2 in $\sqrt{s_{NN}} = 130$ GeV Au-Au minimum-bias vs hydro calculation by Kolb,Huovinen,Heinz,Heiselberg, Phys.Lett.B500:232-240,2001

Home | Publications | APS News | June 2005 (Volume 14, Number 6) | **RHIC Detects Liquid State of Quark-Gluon Matter**

APS NEWS

RHIC Detects Liquid State of Quark-Gluon Matter

By Ernie Tretkoff

Analysis of the weird quark-gluon matter produced at RHIC shows that the substance is more like a liquid than a gas, researchers reported at the APS April Meeting. The researchers from the Relativistic Heavy Ion Collider at Brookhaven National Laboratory announced the results of recent analysis of the quark gluon matter they have been producing for years–a state many scientists expected would be the "quark gluon plasma."

"Theorists expected this phase to exist. The properties of this phase are surprising. The big surprise is that it's a liquid," said Brookhaven theorist Dmitri Kharzeev.

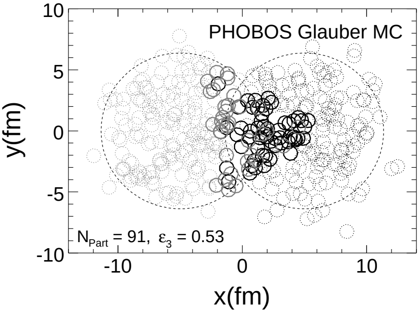
The RHIC collaborations made this announcement at a press conference during the APS April Meeting in Tampa. They will also publish a set of papers in the journal *Nuclear Physics A*.

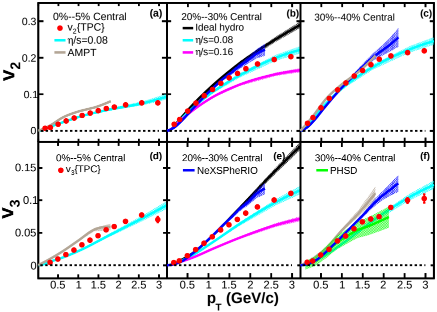
Researchers from all four RHIC collaborations–PHENIX, STAR, PHOBOS, and BRAHMS–participated in the announcement. The new results are based on analysis of data from the 2000-2003 run.

RHIC creates the blob of quark-gluon matter by smashing gold nuclei together at very high energies. Under these extreme conditions, the quarks and gluons normally bound in nucleons can become unbound. The quark gluon matter is extremely hot and dense, nearly 150,000 times as hot as the sun's core and 100 times the density of a nucleus, said the researchers. The blob lasts for only about 10^{-23} seconds.

Triangular flow

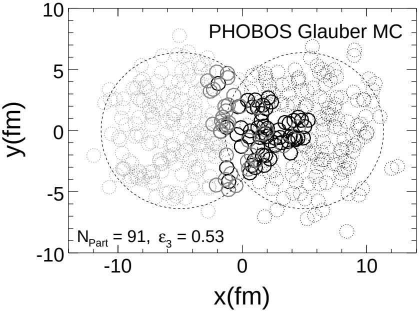
Somewhat different argument:
initial nucleon distribution is irregular ⇒ initial state triangularity ε_3 ⇒ hydrodynamics ⇒ final state v_3.

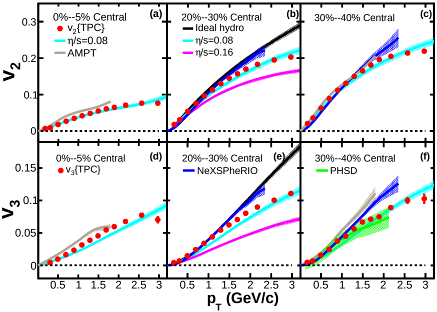




Note in the middle column of the right figure: the splitting between "ideal hydro" and "$\eta/s = 0.16$" cases is stronger for v_3 than for v_2 ⇒ "triangular flow is more sensitive to shear viscosity".

Triangular flow (2)





Even more impressive consequence:

hydrodynamic works not only for average but for **for each initial nuclear configuration**

What is hydrodynamic approach?

A 2-slide introduction to relativistic hydrodynamics

1) hydrodynamics of ideal fluid (a.k.a. "ideal hydrodynamics")

- every small element of the medium is in a complete thermal equilibrium
- mean free path is zero (collision rate is infinite)
- equations of motion come from local energy-momentum conservation:

$$\partial_\mu T^{\mu\nu} = 0$$

Energy-momentum tensor of ideal fluid

Let's derive it Landau way:

In the *rest frame* of the fluid element, for ideal fluid the Pascal law applies:

$$T_{ij} = p \cdot \delta_{ij}$$

At the same time, the momentum density $T^{0i} = 0$, and the energy density $T^{00} = \varepsilon$.

$$T^{\mu\nu}_{\mathsf{LRF}} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

Then ... in any arbitrary frame the expression for the energy-momentum tensor is:

$$\boxed{T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - p \cdot g^{\mu\nu}}$$

ε, p, u^x, u^y, u^z are 5 variables whereas there are 4 hydrodynamic equations, $\Rightarrow$ $\Rightarrow$ one needs one more relation to enclose the system, e.g. $p = p(\varepsilon)$. It is called the Equation of State (EoS).

Relativistic viscous hydrodynamics

A 1-slide introduction

- Now the mean free path is not zero: there is non-equilibrium energy-momentum transfer etc.

$$T^{\mu\nu} = \underbrace{(\varepsilon + p + \Pi)u^\mu u^\nu - (p + \Pi)\cdot g^{\mu\nu}}_{\text{ideal part}} + \underbrace{W^\mu u^\nu + W^\nu u^\mu + \pi^{\mu\nu}}_{\text{viscous corrections}}$$

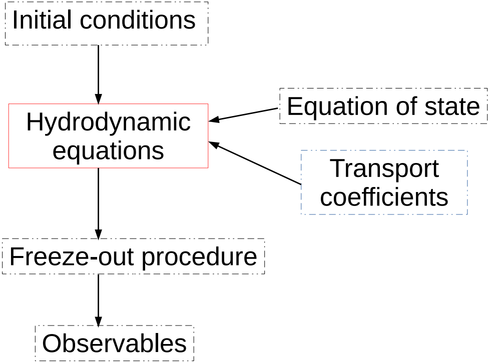
- in a limiting case (Navier-Stokes limit) $\pi^{\mu\nu} = \eta \nabla_{\langle\mu} u_{\nu\rangle}, \quad \Pi = -\zeta \nabla_\alpha u^\alpha$, where η and ζ are shear and bulk viscosities, respectively.

- There is a microscopic scale $l_{\text{micro}} \propto \tau_\pi$ (relaxation time of the local shear stress tensor to its Navier-Stokes value), an usual ansatz is $\tau_\pi = 5\eta/(\varepsilon + p)$

- because of the microsopic scale, now there is a so-called Knudsen number: $\text{Kn} = \tau_\pi / L$, where L is a macroscopic scale (e.g. inverse gradient).

- Viscous corrections must not be large.
  Viscous hydrodynamics is applicable when $\text{Kn} \ll 1$.

Equations of relativistic hydrodynamics is a set of PDE, which require the following ingredients:



Initial conditions

Hydrodynamic equations

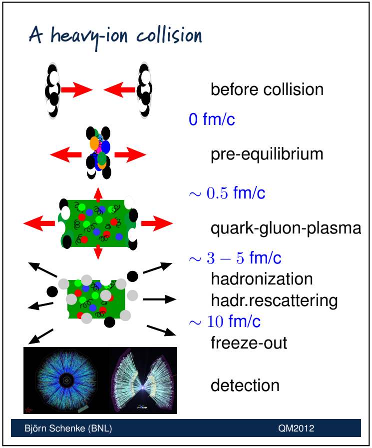
Equation of state

Transport coefficients

Freeze-out procedure

Observables

Our understanding of heavy ion collision dynamics



A heavy-ion collision

before collision

0 fm/c

pre-equilibrium

~ 0.5 fm/c

quark-gluon-plasma

$\sim 3-5$ fm/c

hadronization

hadr.rescattering

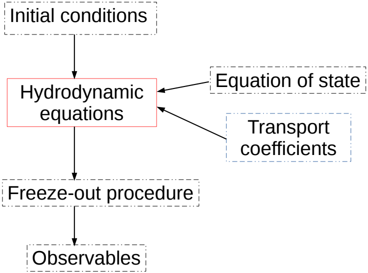
~ 10 fm/c

freeze-out

detection

Björn Schenke (BNL) QM2012

source: B. Schenke



Initial conditions

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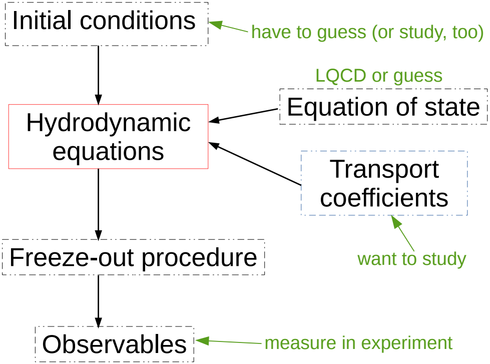

Initial conditions

have to guess (or study, too)

LQCD or guess

Equation of state

Hydrodynamic equations

Transport coefficients

want to study

Freeze-out procedure

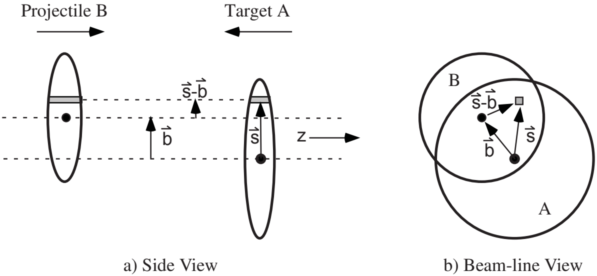
Observables

measure in experiment

The ingredients

Simplest idea for the initial state: Glauber model

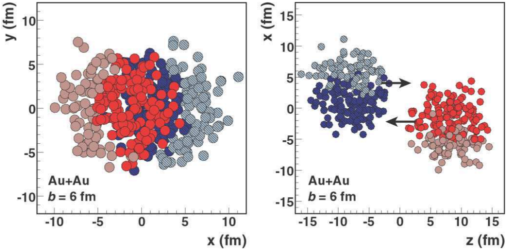
- Impose longitudinal boost invariance: system is infinite and uniform in rapidity.
- Zero initial transverse flow: $v_x = v_y = v_\eta = 0$, zero initial $\pi^{\mu\nu}$.
- Nuclear geometry implies that density is not uniform



Miller *et al.*, Ann.Rev.Nucl.Part.Sci. 57, 205 (2007)

“Monte Carlo Glauber” model

- Nuclear geometry implies that density varies event-by-event



Miller *et al.*, Ann.Rev.Nucl.Part.Sci. 57, 205 (2007)

There are two options:

- evaluate average initial state, and evolve it
- evolve many initial state ⇒ event-by-event hydro

Models for initial conditions

- **Glauber:** geometric model determining wounded nucleons based on the inelastic nucleon-nucleon cross section (whole family of variants).
  - Wounded quark model.
- **MC-KLN:** Color-Glass-Condensate (CGC) based model using kT-factorization
- **IP-Glasma:** CGC based model using classical Yang-Mills evolution of early-time gluon fields.
- **pQCD+saturation:** calculate minijets using pQCD to get energy deposited in the collision region.
- **Event generators:** UrQMD (string/hadron), BAMPS and AMPT (partonic), HIJING (pQCD/string), NEXUS or EPOS (pomeron picture).

So far none of these reaches equilibrium, but it has to be dialed in by hand.

Energy-momentum tensor of ideal hydrodynamics in the local rest frame (LRF):

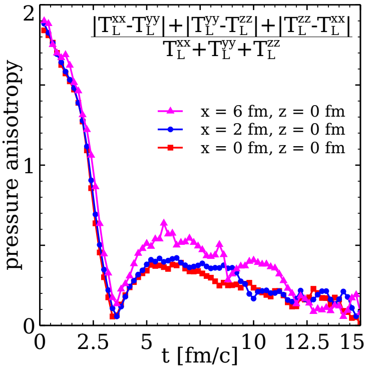
$$T^{\mu\nu}_{\mathsf{LRF}} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

LRF, with shear viscous corrections:

$$T^{\mu\nu}_{\mathsf{LRF}} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p+\pi^{xx} & \pi^{xy} & \pi^{xz} \\ 0 & \pi^{yx} & p+\pi^{yy} & \pi^{yz} \\ 0 & \pi^{zx} & \pi^{zy} & p+\pi^{zz} \end{pmatrix}$$

With shear viscous corrections, T^{xx}, T^{yy} and T^{zz} in LRF can differ from each other, but not a lot.

Initial state from UrQMD

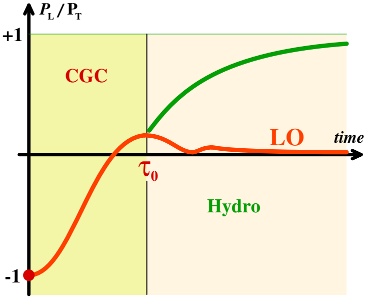


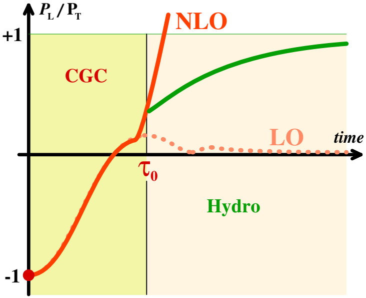
An example: pressure anisotropy in the initial state from UrQMD hadron/string cascade just before switching to hydro (thermalizing)

Oliinychenko, Petersen, Phys. Rev. C 93, 034905 (2016)

Initial state from CGC

Pressure anizotropy is a serious problem here, too!





F. Gelis, Nucl. Phys. A 931, 73 (2014)

Some (inconsistent) recipes to match initial state to hydrodynamics:

- Take only $T^{00} = \varepsilon$ from initial state, assume zero initial velocities
  (and ignore the rest of $T^{\mu\nu}$ from the initial state)
- Take T^{00}, T^{0i} and recalculate initial energy density ε and initial velocities u^{μ}_{ini}
  assuming hydrodynamical EoS $p = p(\varepsilon)$
  (and ignore T^{ij} from the initial state)
- Take $T^{\mu\nu}$, find Landau frame and extract $\varepsilon = T$ and u^{μ}
  (and ignore that T^{ii} differ from equilibrium pressure)

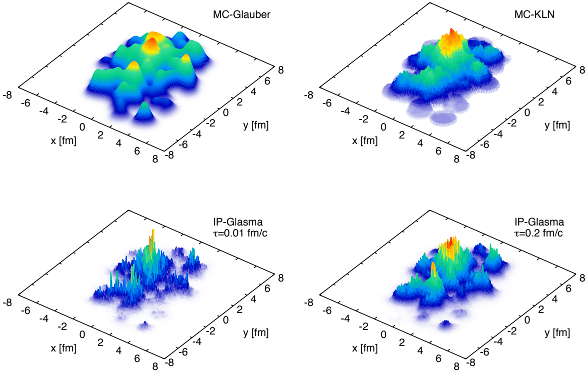
Some combinations of initial state + recipe to match to hydro

Table taken from: Oliinychenko, Petersen, Phys. Rev. C 93, 034905 (2016)

Model	Initial condition	Hydro	Switching criterion	Smearing kernel	Getting $T^{\mu\nu}_{ideal}$
UrQMD hybrid [18]	UrQMD cascade	ideal 3+1D, SHASTA	t_{CM}[fm/c] = $max(2R\sqrt{\frac{E_{lab}}{2m_N}}, 1.0)$	Gaussian z-contracted	$T^{\mu 0}$, j^0
Skokov-Toneev hybrid [19]	Quark-Gluon-String-Model	ideal 3+1D, SHASTA	t_{CM} such that $S/Q_B = \text{const}$	not mentioned	$T^{\mu 0}$, j^0
EPOS [21]	Strings (Regge-Gribov model)	ideal 3+1D	τ	Gaussian z-contracted	Landau frame
NeXSPheRIO hybrid [22, 23]	Strings (Regge-Gribov model)	ideal 3+1D, SPH	$\tau = 1$ fm [24]	Gaussian in x, y, $\tau\eta$	Landau frame
Gale et al [25]	IP-glasma	viscous 3+1D, MUSIC	$\tau = 0.2$ fm/c ($\sqrt{s_{NN}} = 2.76$ TeV)	not mentioned	Landau frame
Karpenko hybrid [26]	UrQMD cascade	viscous 3+1D	τ_{geom}	Gaussian with $\sigma_\perp$ and σ_η	$T^{\mu 0}$, j^0
Pang et al hybrid [27]	AMPT	ideal 3+1D, SHASTA	τ	Gaussian with $\sigma_\perp$ and σ_η	$T^{\mu 0}$, j^0

Table I. Fluidization features in different hybrid approaches. Each of these models, including those using viscous hydrodynamics, neglects viscous corrections at fluidization.

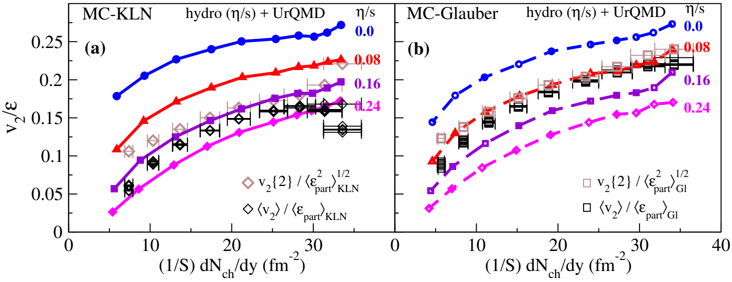
Different models provide different shapes of initial state



C. Gale et al, Int. J. of Mod. Phys. A, Vol. 28, 1340011 (2013)

...and it affects observables (from the model)

C. Shen et al., J. Phys. G38, 124045 (2011)



Different initial eccentricity in from Glauber and KLN models

$\Downarrow$

Different resulting elliptic flow

$\Downarrow$

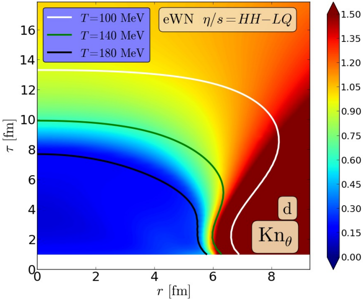
Factor 2 difference in 'extracted' η/s from comparison to experimental data!

How well is the viscous hydrodynamic applicable in terms of Kn?

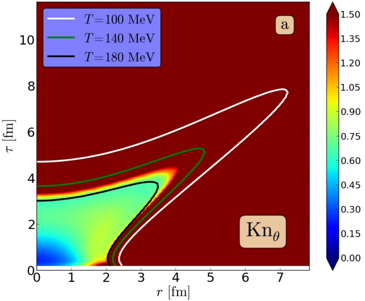
recap: Kn=micro/macro

It largely depends on the $\eta/s(T)$, initial state, a definition of Kn, and a particular space-time region,
but here are two plots to have some impression:

20-30% central Pb-Pb collsion at LHC



pPb with $dN_{\mathrm{ch}}/d\eta = 270$ at LHC

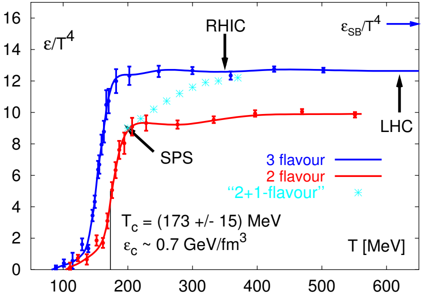


Blue=good (small Kn), red=bad (not small Kn).

H. Niemi, G.S. Denicol, arXiv:1404.7327

Equation of state

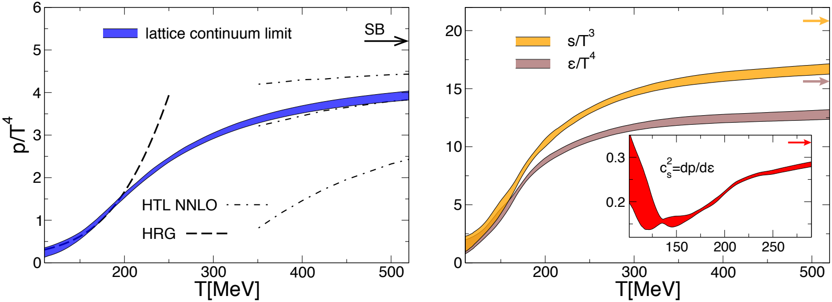
Equation of state of stronly interacting matter in full equilibrium in a box using numerical lattice QCD calculations was available already in early noughties.



Karsch and Laermann, hep-lat/0305025

Lattice QCD calculations are constantly improving, and as for now

- there is a good agreement with hadron resonance gas at low temperatures

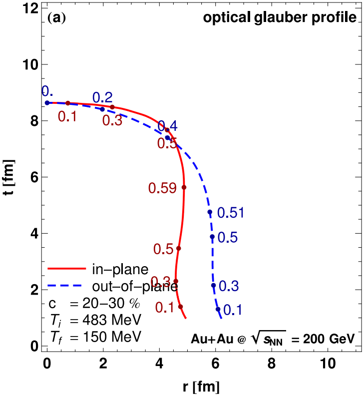


Borsányi et al, Phys. Lett. B 370 (2014) 99-104

Freezing out

Particles are observed. Not a fluid. ⇒ How to convert fluid into particles?

- kinetic equilibrium requires $\underbrace{\text{(scattering rate)}}_{\tau_{\mathrm{sc}}^{-1}\propto\sigma n\propto\sigma T^3} \gg \underbrace{\text{(expansion rate)}}_{\theta=\partial_\mu u^\mu}$
- Fluid description breaks down when $\tau_{\mathrm{sc}}^{-1} \approx \theta$
- Approximation: decoupling takes place on a hypersurface of constant temperature or constant energy density. At this hypersurface, momentum distributions of hadrons freeze out.



A. Kisiel et al, Phys.Rev. C79, 014902 (2009).

Cooper-Frye prescription

Approximation: decoupling takes place on a hypersurface of constant temperature or constant energy density. At this hypersurface, momentum distributions of hadrons freeze.

Number of particles emitted = number of particles crossing Σ:

$$N = \int_\Sigma d\Sigma_\mu J^\mu$$

From kinetic theory:

$$J^\mu(x) = \int \frac{d^3p}{p^0} p^\mu f(x,p)$$

$$N = \int d\Sigma_\mu \int \frac{d^3p}{p^0} p^\mu f(x,p) = \int \frac{d^3p}{p^0} \int d\Sigma_\mu p^\mu f(x,p)$$

$$\Rightarrow \boxed{p^0 \frac{d^3N}{d^3p} = \int_\Sigma d\Sigma_\mu p^\mu f(x,p)}$$

F. Cooper, G. Frye, Phys.Rev. D 10, 186 (1974)

- Present paradigm: not freeze-out, but fluid→particle transition, or 'particlization' (momentum distributions can still change in post-hydro evolution).

- At the particlization (from the particle side), in most of the models the matter is described as a mixture of gases of free hadrons (hadron resonance gas, HRG) in or close to thermal and chemical eqilibrium:

$$f_i(x,p) = f_{i,\mathrm{eq}} = \frac{g_i}{(2\pi)^3} \frac{1}{\exp\left(\frac{p^\nu u_\nu - \mu_i(x)}{T(x)}\right) \pm 1},$$

where $g_i = (2J+1)$ is degeneracy factor, $\mu_i(x)$ is chemical (local) potential $T(x)$ is (local) temperature.

- If viscous hydro is used (corrections due to shear viscosity):

$$f_i = f_{i,\mathrm{eq}} + \delta f_i = f_{i,\mathrm{eq}} + f_{i,\mathrm{eq}}(1 \pm f_{i,\mathrm{eq}}) \frac{p^\mu p^\nu}{2(\varepsilon + p)T^2}$$

such choice of δf is an 'industry standard', but it is not unique.
Also, for multi-component system the δf_i should be in principle not the same for all sorts of hadrons.

- Hadron resonance gas contains many hadron states (usually > 50), most of which have short lifetimes.

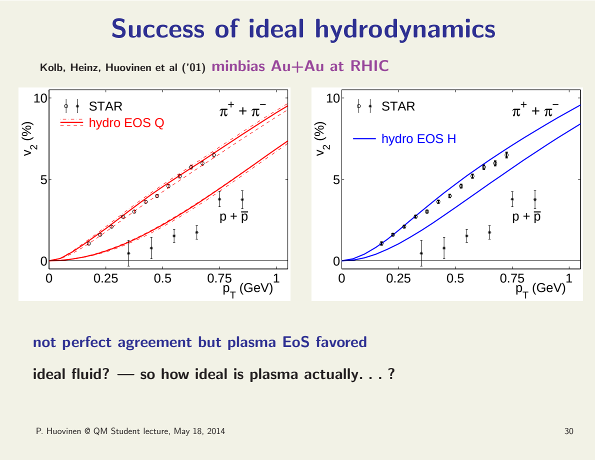
- Dominating fraction of pions are products of resonance decays. Therefore resonance contributions to 'stable' hadrons should be calculated.

$$N_i = N_{i,\mathrm{thermal}} + \sum_{R \to i} b(R \to i + X) N_R$$

- A more advanced approach is to generate ensemble of final hadrons via Monte Carlo, and input them to a hadronic cascade (**afterburner**), for example UrQMD. This naturally generates hadronic viscosity and less extreme transition than hydro→free streaming

Making physics conclusions from the modelling

Making physics conclusions from the modelling (early 2000s)



Success of ideal hydrodynamics

Kolb, Heinz, Huovinen et al ('01) **minbias Au+Au at RHIC**

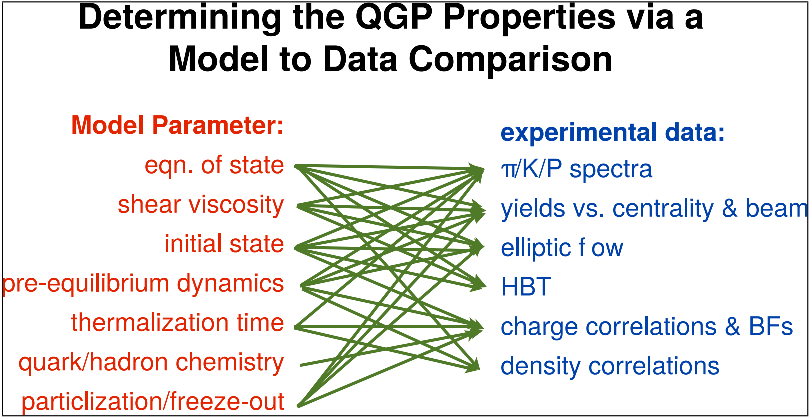
not perfect agreement but plasma EoS favored

ideal fluid? — so how ideal is plasma actually. . . ?

P. Huovinen @ QM Student lecture, May 18, 2014 30

… and in late 2010s

S. Bass, Quark Matter 2017



Determining the QGP Properties via a Model to Data Comparison

Model Parameter:

- eqn. of state
- shear viscosity
- initial state
- pre-equilibrium dynamics
- thermalization time
- quark/hadron chemistry
- particlization/freeze-out

experimental data:

- π/K/P spectra
- yields vs. centrality & beam
- elliptic flow
- HBT
- charge correlations & BFs
- density correlations

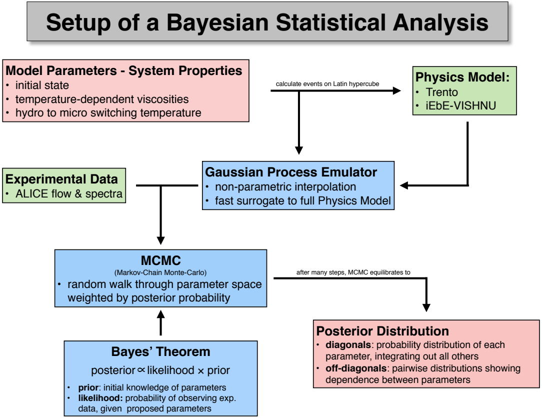
- evaluating model output for single point in the parameter space requires many CPU hours.

A problem?

Emulator + MCMC technique

Hydrodynamic model ⇒ Gaussian process emulator ⇒

Markov Chain Monte Carlo (MCMC)



Setup of a Bayesian Statistical Analysis

Model Parameters - System Properties
- initial state
- temperature-dependent viscosities
- hydro to micro switching temperature

calculate events on Latin hypercube

Physics Model:
- Trento
- iEbE-VISHNU

Experimental Data
- ALICE flow & spectra

Gaussian Process Emulator
- non-parametric interpolation
- fast surrogate to full Physics Model

MCMC

(Markov-Chain Monte-Carlo)
- random walk through parameter space weighted by posterior probability

after many steps, MCMC equilibrates to

Posterior Distribution
- **diagonals**: probability distribution of each parameter, integrating out all others
- **off-diagonals**: pairwise distributions showing dependence between parameters

Bayes' Theorem

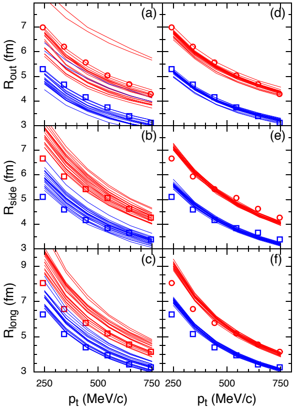
posterior ∝ likelihood × prior

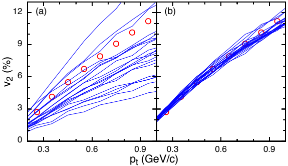
- **prior**: initial knowledge of parameters
- **likelihood:** probability of observing exp. data, given proposed parameters

S. Bass, QM2017

Constraining the EoS of QGP matter

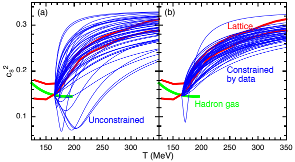
S. Pratt, E. Sangaline, P. Sorensen, H. Wang, Phys. Rev. Lett. 114, 202301 (2015)





$$c_s^2(\varepsilon) \quad = \quad c_s^2(\varepsilon_h) + \left(\frac{1}{3} - c_s^2(\varepsilon_h)\right) \frac{X_0 x + x^2}{X_0 x + x^2 + X'^2}, \quad (1)$$

$$X_0 \quad = \quad X' R c_s(\varepsilon)\sqrt{12}, \quad x \equiv \ln \varepsilon/\varepsilon_h,$$



Summary

- Hydrodynamic approach interprets successfully *most of* the measurements of hadrons at low $p_\perp$ in heavy ion collisions:
  lead-lead, xenon-xenon at all LHC energies, copper-copper, gold-gold at RHIC etc.

- we understand very well how the shape of colliding nuclei translate into final momentum distributions of produced hadrons

Challenges:

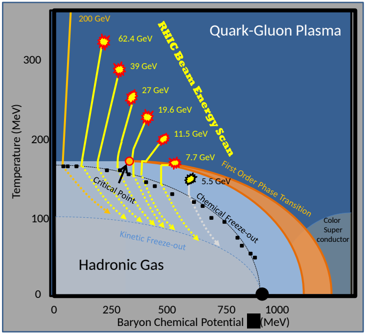
- **Why the hydrodynamic approach works where it should not?**
  Why does the hydrodynamic approach work also for proton-nucleus collisions?
  ...proton-proton collisions ???

- **Does it still work if we decrease collision energy?** (RHIC Beam Energy Scan)
  spoiler: yes, it does but it's a whole new challenge to set it up properly

- **Can we agree on the inputs and parameters?**
  We must agree on the exact initial state physics, medium properties etc.

Projects I've been working on

Hydrodynamics at “lower” energies and QCD phase diagram



Picture taken from: G. Odyniec, Acta Phys. Polon. B 43, 627 (2012).

- Initial state: thick pancakes
  - boost ivariance is not a good approximation
    $\rightarrow$ need for 3 dimensional evolution
  - CGC picture does not work as well anymore
- Fluctuations in initial state, viscosity, afterburner
- Baryon and electric charges
  - obtained from the initial state
  - included in hydro phase
  - taken into account at particlization

A model for RHIC BES: UrQMD + vHLLE (+ UrQMD)

Pre-thermal evolution: UrQMD cascade

until $\tau = \tau_0 = const,\ \tau_0 = \frac{2R}{\gamma v_z}$

Fluctuating initial state, event-by-event hydrodynamics

Hydrodynamic phase:

$$\partial_{;\nu} T^{\mu\nu} = 0, \quad \partial_{;\nu} N^{\nu} = 0 \qquad < u^\gamma \partial_{;\gamma} \pi^{\mu\nu} > = -\frac{\pi^{\mu\nu} - \pi^{\mu\nu}_{\text{NS}}}{\tau_\pi} - \frac{4}{3} \pi^{\mu\nu} \partial_{;\gamma} u^\gamma$$

* Bulk viscosity $\zeta = 0$, charge diffusion=0

`vHLLE` code: free and open source. Comput. Phys. Commun. 185 (2014), 3016

`https://github.com/yukarpenko/vhlle`

Fluid→particle transition and hadronic phase

Cooper-Frye prescription at $\varepsilon = \varepsilon_{\text{sw}}$:

$$p^0 \frac{d^3 n_i}{d^3 p} = \sum f(x,p) p^\mu \Delta\sigma_\mu$$

$$f(x,p) = f_{\text{eq}} \cdot \left(1 + (1 \mp f_{\text{eq}}) \frac{p_\mu p_\nu \pi^{\mu\nu}}{2T^2(\varepsilon + p)} \right)$$

- $\Delta\sigma_i$ using Cornelius subroutine*
- Hadron gas phase: back to UrQMD cascade

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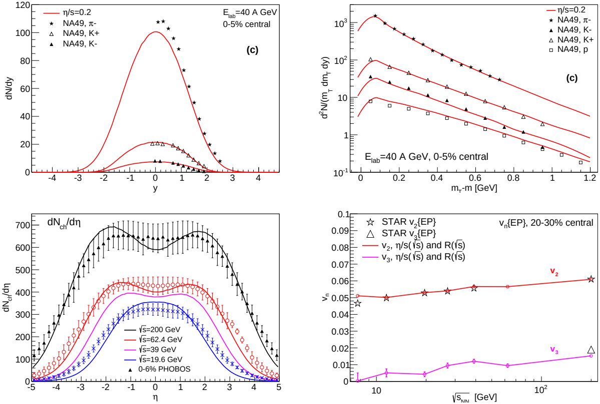
*Huovinen and Petersen, *Eur.Phys.J.* A **48** (2012), 171

First application of 3D viscous hydro + cascade model at RHIC BES energies

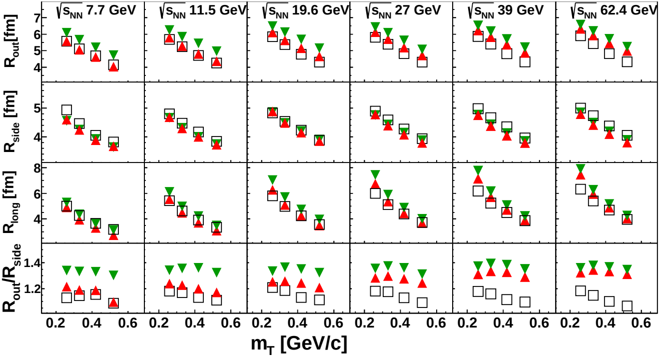


IK, Huovinen, Petersen, Bleicher,

Phys.Rev. C91 (2015) no.6, 064901

Femtoscopic radii from `vHLLE+UrQMD` (project with JINR Dubna)

0-5% central Au-Au: vHLLE+UrQMD using crossover EoS (chiral EoS), 1PT EoS (EoS Q)



P.Batyuk, R.Lednicky, L.Malinina, K.Mikhailov, O.Rogachevsky, IK, D.Wielanek, Phys. Rev. C 96, 024911 (2017)

Polarization of fermions from the fluid

F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338 (2013) 32

Also: Ren-hong Fang, Long-gang Pang, Qun Wang, Xin-nian Wang, Phys. Rev. C 94 (2016), 024904

Mechanism: spin-vorticity coupling at local thermodynamic equilibrium.

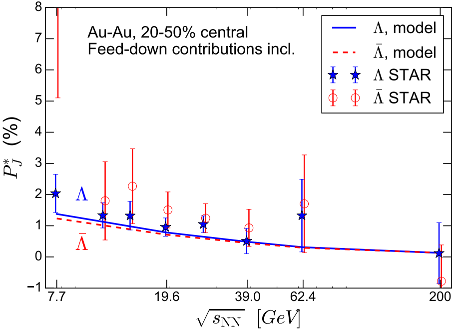
- Cooper-Frye prescription: $p^0 \frac{d^3N}{d^3p} = \int d\Sigma_\lambda p^\lambda \frac{1}{\exp\left(\frac{p\cdot u-\mu}{T}\right)\pm 1}$
- For the spin $1/2$ particles at the particlization surface:
  $\langle S(x,p)\rangle = \frac{1}{8m}(1-f(x,p))\varepsilon^{\mu\nu\rho\sigma}p_\sigma\partial_\nu\beta_\rho$,
  where $\beta_\mu = \frac{u_\mu}{T}$ is the inverse four-temperature field.

$$\boxed{S^\mu(p) = \frac{\int d\Sigma_\lambda p^\lambda f(x,p)\langle S(x,p)\rangle}{\int d\Sigma_\lambda p^\lambda f(x,p)}}$$

Polarization depends on the the thermal vorticity $\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu)$.

- polarization is close or equal for particles and antiparticles
- caused not only by velocity, but also temperature gradients

Λ polarization for RHIC BES in `UrQMD+vHLLE` (project at INFN Firenze)

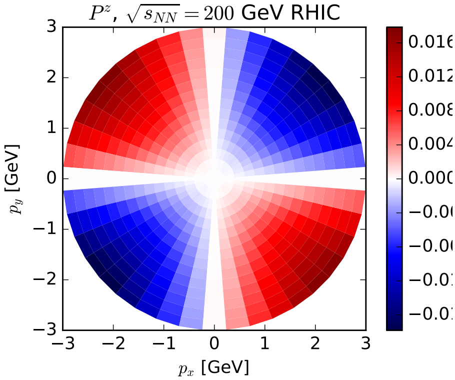


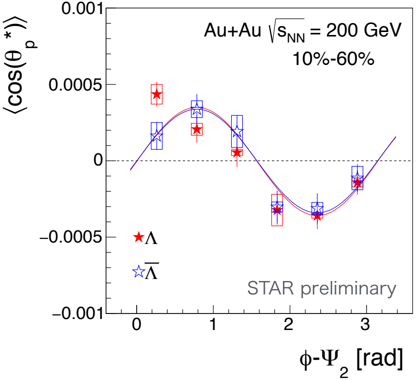
IK, F. Becattini, Eur. Phys. J. C 77, 213 (2017)
F. Becattini, IK, M. Lisa, I. Upsal, S. Voloshin, Phys. Rev. C 95, 054902 (2017)

Quadrupole polarization in beam direction P^z: hydro versus recent STAR data

vHLLE+Glissando IS

Preliminary STAR data: Takafumi Niida, talk at Chirality workshop 2018



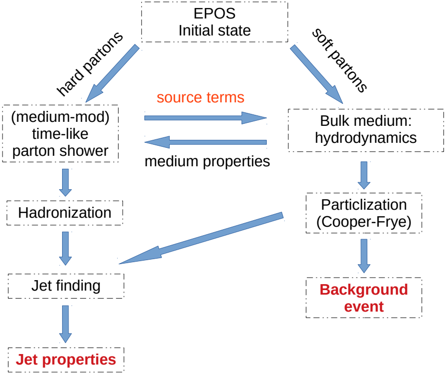


Similar $\sin(2\phi)$ structure is observed, with opposite sign!

F. Becattini, IK, Phys. Rev. Lett. 120, 012302 (2018)

Jets+hydro at the LHC energies (project with SUBATECH Nantes)

To get both hydrodynamic IS and initial hard partons from EPOS3.



EPOS
Initial state

hard partons

soft partons

(medium-mod)
time-like
parton shower

source terms

medium properties

Bulk medium:
hydrodynamics

Hadronization

Particlization
(Cooper-Frye)

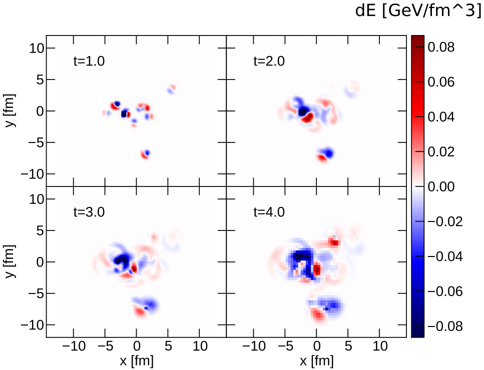
Jet finding

Background event

Jet properties

Medium recoil

Mach cones.



IK, Aichelin, Gossiaux, Rohrmoser, Werner, arXiv:1911.04155

Hydrodynamics has a solid niche in the heavy ion community: about 6% talks each Quark Matter (2011-2019) contain ‘hydro’ or ‘fluid’ in the title.

Interesting/relevant projects to do:

- hydrodynamic modelling at high baryon densities / lower energies (RHIC BES, FAIR, NICA etc):
  dynamical fluidization, multi-fluid approach
- search for the critical point of QCD (one of the main reasons for RHIC BES and NICA/FAIR):
  critical dynamics in hydro
- observables: femtoscopy (HBT), flow, hadron chemistry
- cluster (light nuclei) production, also relevant at the lower energies
- high energies (LHC): hydro plus jets!