Workshop EJCF 2020

B-DEPENDENT BK IN ALL ITS BEAUTY

Marek Matas (matas.marek1@gmail.com)
Czech Technical University in Prague,
Faculty of Nuclear Sciences and Physical Engineering, Department of Physics

OUTLINE

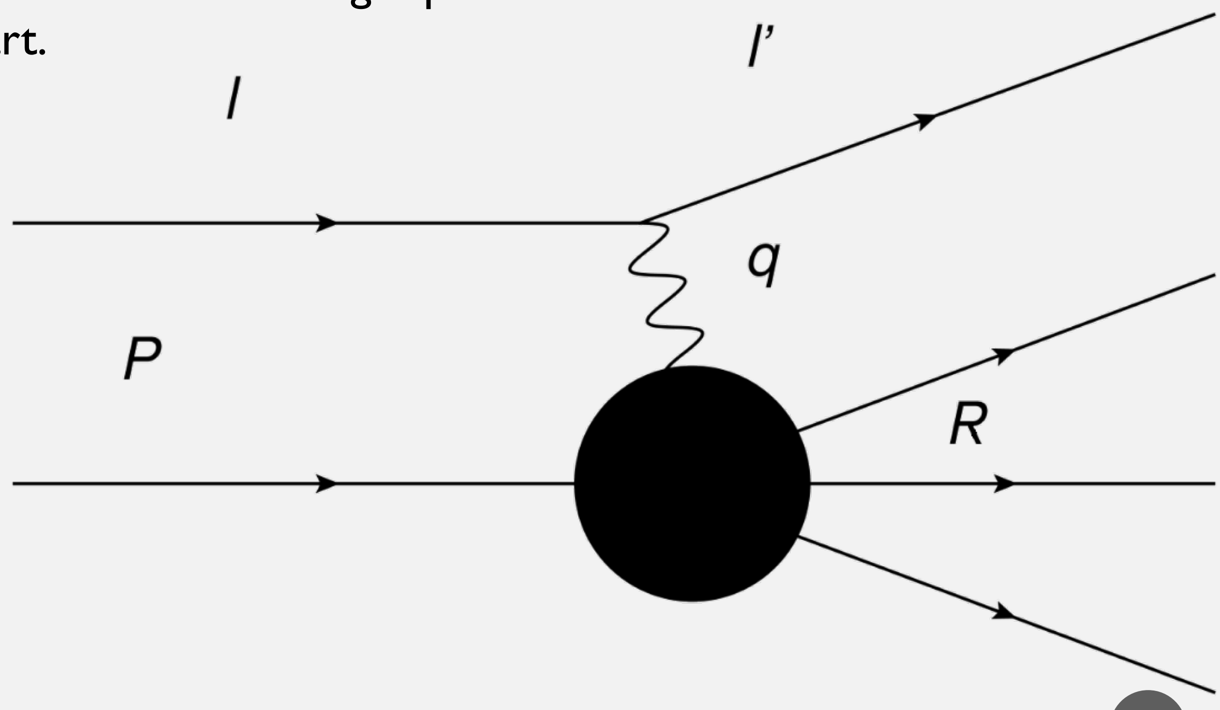
I. Deep inelastic scattering
II. What is BK equation and the dipole model
III. Saturation and its role in nucleons
IV. Impact parameter dependence

DEEP INELASTIC SCATTERING

DEEP INELASTIC SCATTERING

The electron-proton collisions are considered to happen as:

1. The incoming electron emits a virtual photon.
2. The virtual photon interacts with the target proton
3. The proton breaks apart.

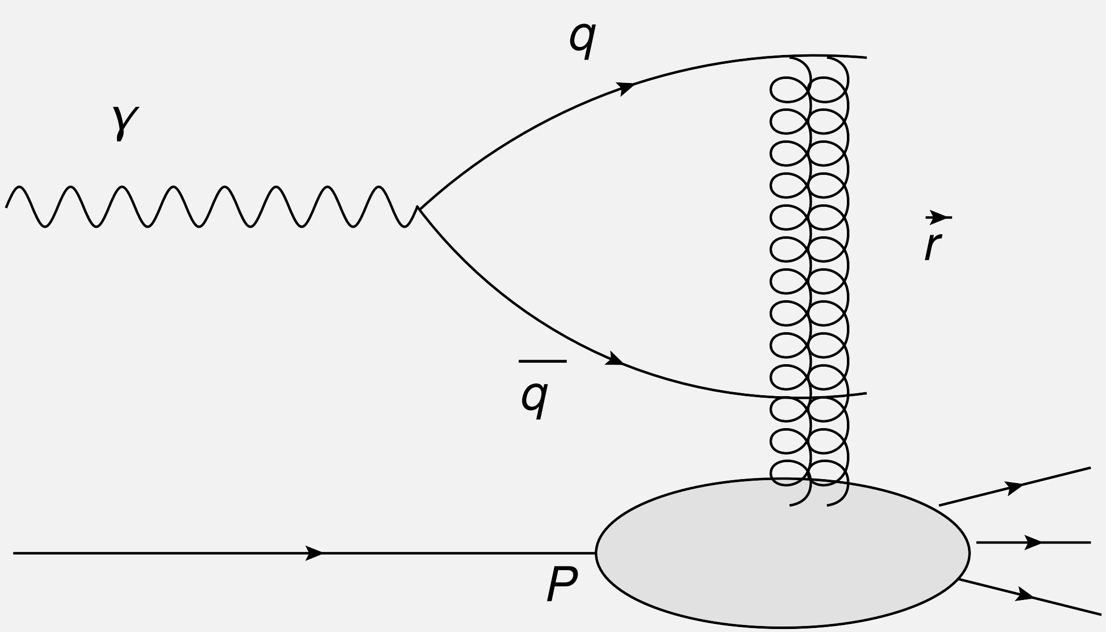


HOW DOES A PHOTON INTERACT WITH A PROTON?

DIPOLE MODEL

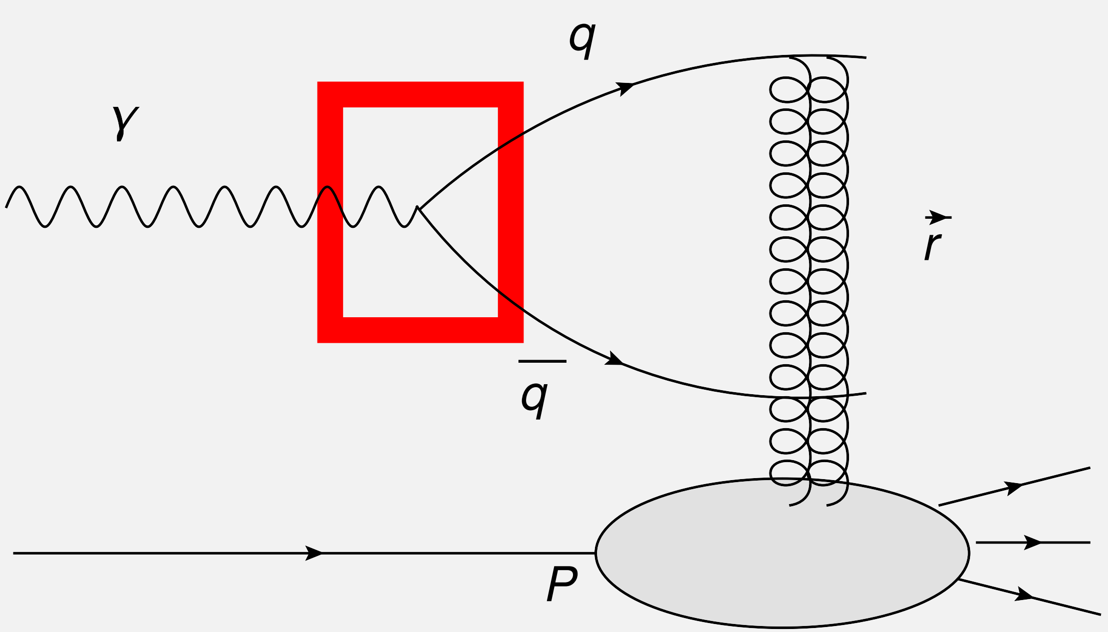
The photon must interact strongly with the target proton, how is that possible?

1. The virtual photon first fluctuates into a quark-antiquark pair
2. Then it exchanges an object with vacuum quantum numbers with the proton



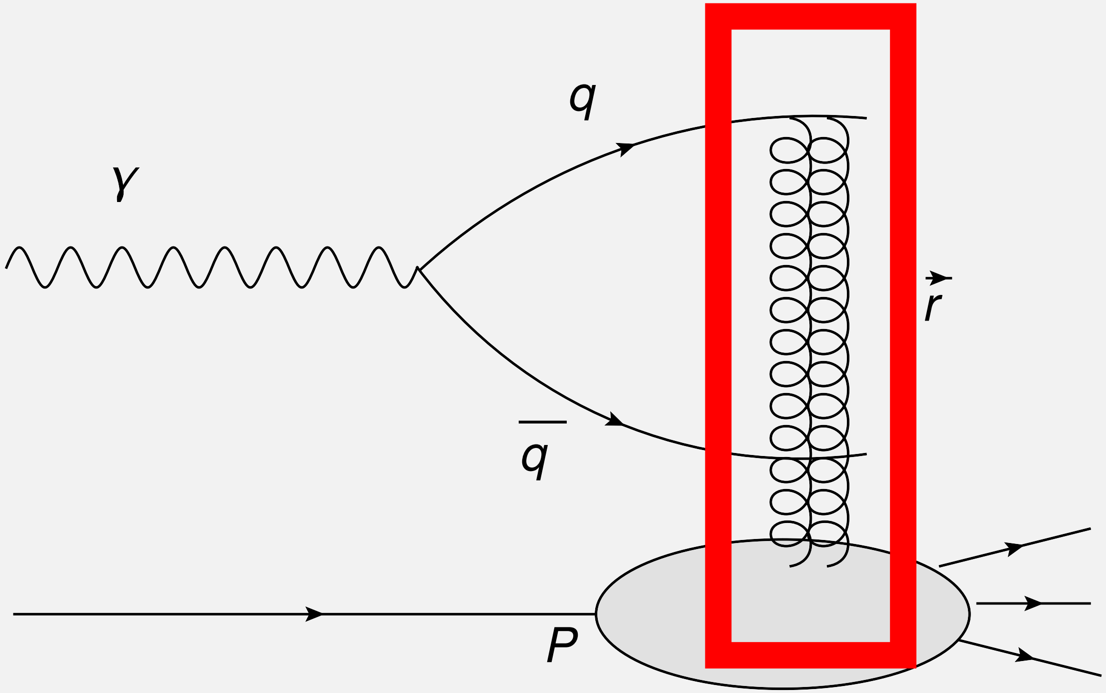
DIPOLE MODEL

The probability of a photon splitting to a quark-antiquark pair is computed from QFT.



DIPOLE MODEL

To compute the cross section of the interaction, we are missing the $\sigma_{\text{dipole-proton}}$



HOW DO WE OBTAIN THE DIPOLE-PROTON CROSS SECTION?

MV MODEL

You approximate the target as a dense gluonic field, that interacts with the passing quark strongly.

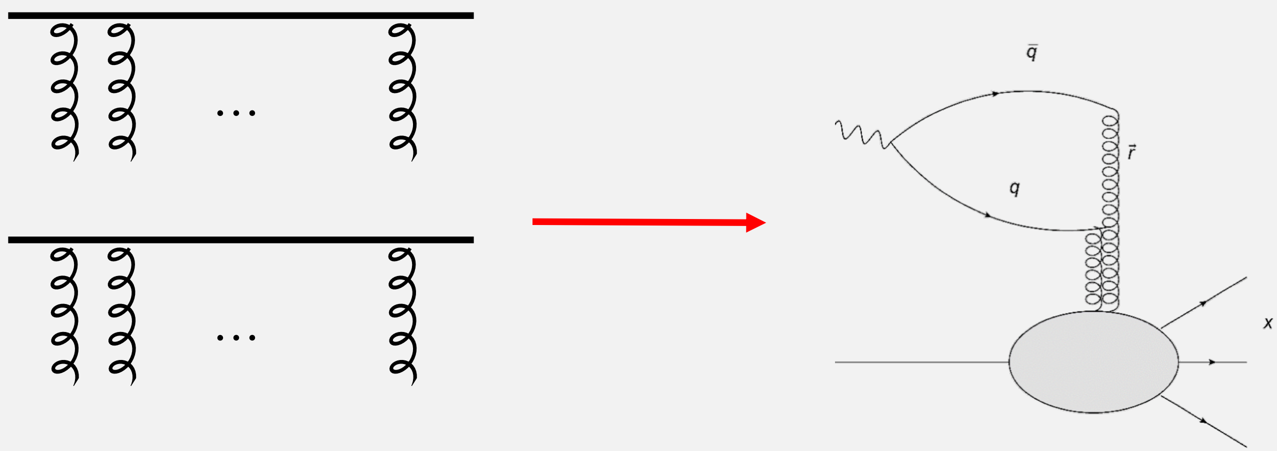
$$S \sim \int dx_T \, P \, exp\left[ig \int dx^+ A^{c-}(x^+, x_T) t^c\right]_{ab}$$

This object is called a Wilson line, it works under the approximation that no momentum is exchanged and can be viewed as a rotation in the color space.

arXiv:hep-ph/9309289

MV MODEL

- You can use two Wilson lines to compute, what effect will there be on a bare dipole passing through such medium.
- They find, that the scattering amplitude then is proportional to: $\mathrm{N} \sim exp\left[-\frac{r_T^2 Q_S^2}{4} log \frac{1}{r_T \Lambda}\right]$



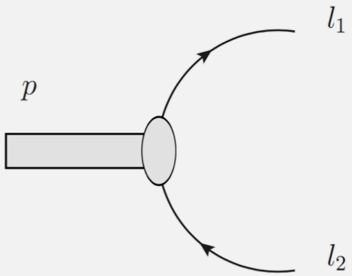
- Where r_T is the transversal size of the dipole, Q_s is called the saturation scale (a parameter, that is fit to data) and Λ is the QCD scale.

arXiv:hep-ph/9309289

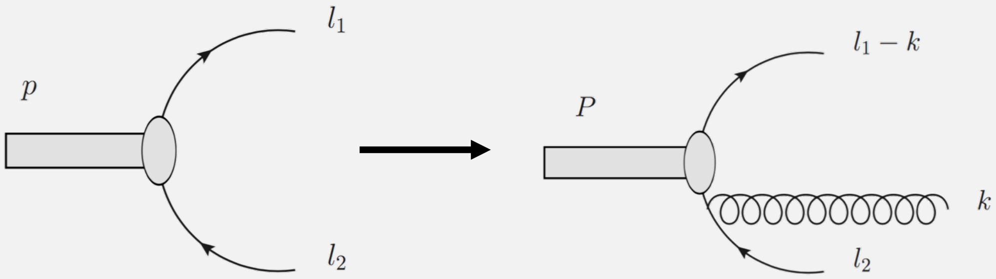
IS THE DIPOLE ALWAYS BARE?

BK EQUATION

Boost to a frame, where dipole is at rest

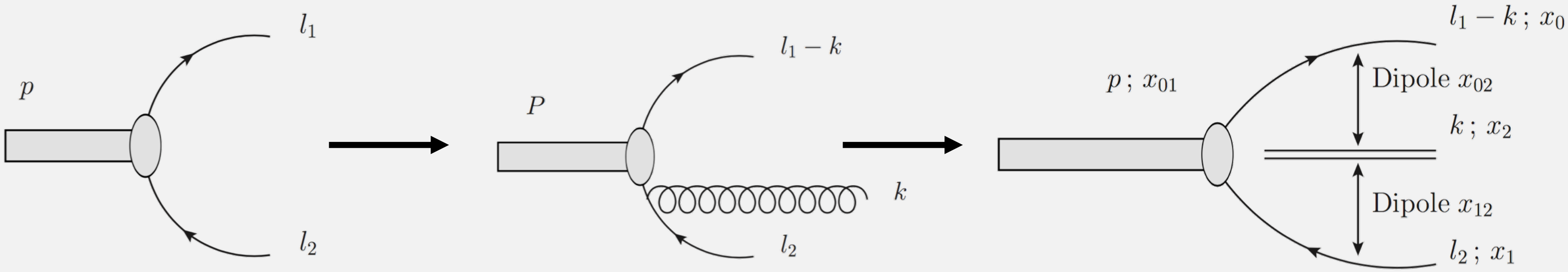


BK EQUATION



Add a bit of energy

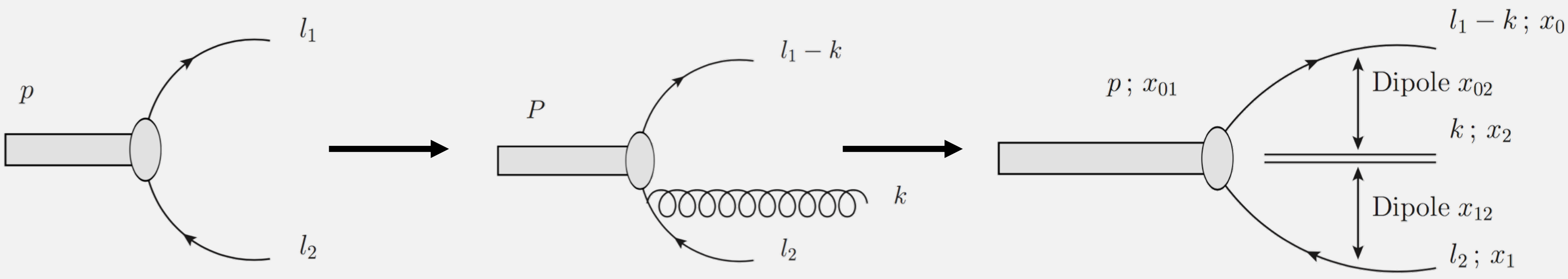
BK EQUATION



Add a bit of energy

High N_c limit

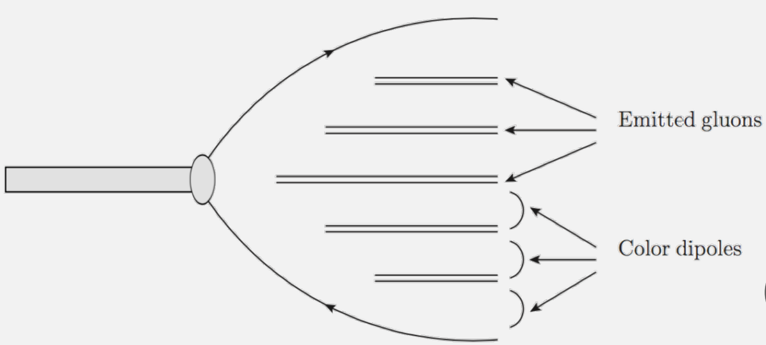
BK EQUATION



Add a bit of energy

High N_c limit

After some time, the initial dipole becomes dressed.



BK EQUATION

Mathematically, this realates to:

$$\frac{\partial N(r,Y)}{\partial \ln Y} = \int d\vec{r}_1 K(\vec{r},\vec{r}_1,\vec{r}_2)(N(\vec{r}_1,Y) + N(\vec{r}_2,Y) - N(\vec{r},Y) - N(\vec{r}_1,Y)N(\vec{r}_2,Y))$$

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This is the change of the scattering amplitude, when we add a bit of energy into the system.

BK EQUATION

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$$\frac{\partial N(r,Y)}{\partial \ln Y} = \int d\vec{r}_1 K(\vec{r},\vec{r}_1,\vec{r}_2)(N(\vec{r}_1,Y) + N(\vec{r}_2,Y) - N(\vec{r},Y) - N(\vec{r}_1,Y)N(\vec{r}_2,Y))$$

Kernel is computed from QCD to reflect the probability of the gluon emission.

BK EQUATION

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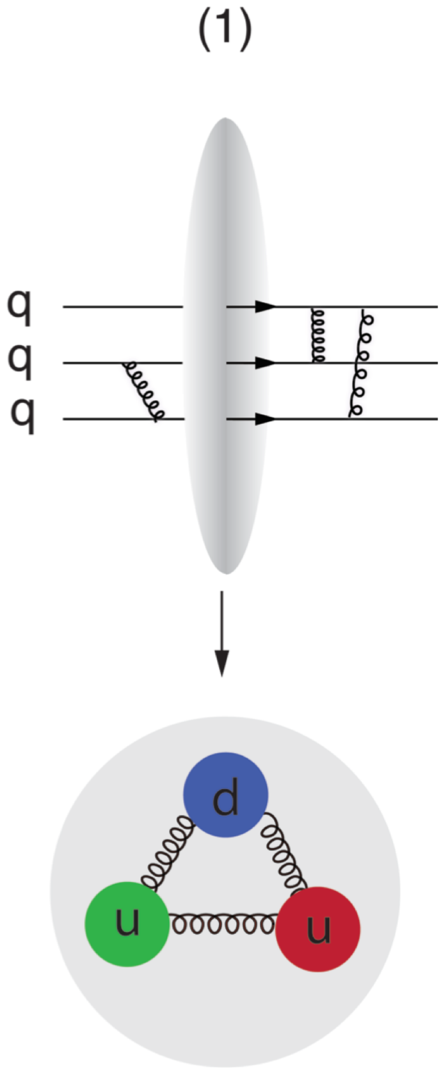
$$\frac{\partial N(r,Y)}{\partial \, lnY} = \int d\vec{r}_1 K(\vec{r},\vec{r}_1,\vec{r}_2)(N(\vec{r}_1,Y) + N(\vec{r}_2,Y) - N(\vec{r},Y) - N(\vec{r}_1,Y)N(\vec{r}_2,Y))$$

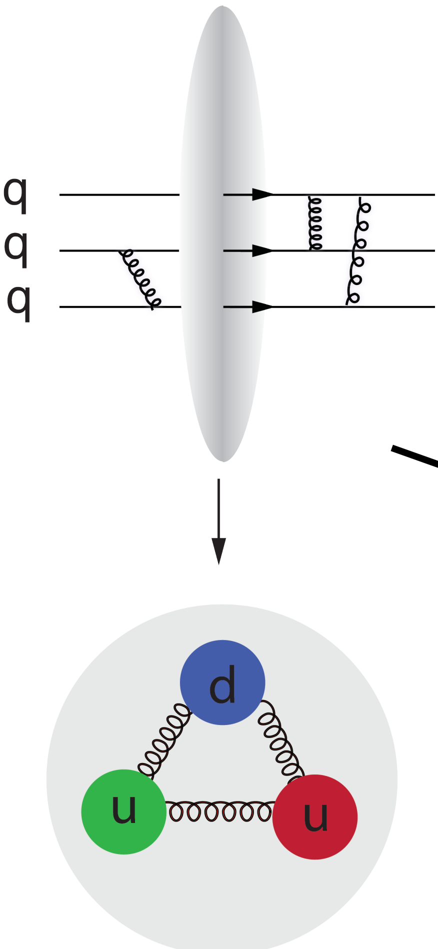
Dipole-proton scattering amplitudes.

WHAT DOES THE BK TELL US ABOUT THE PROTON?



At large values of x (carried momentum fraction), the proton is made of valence quarks

(1)



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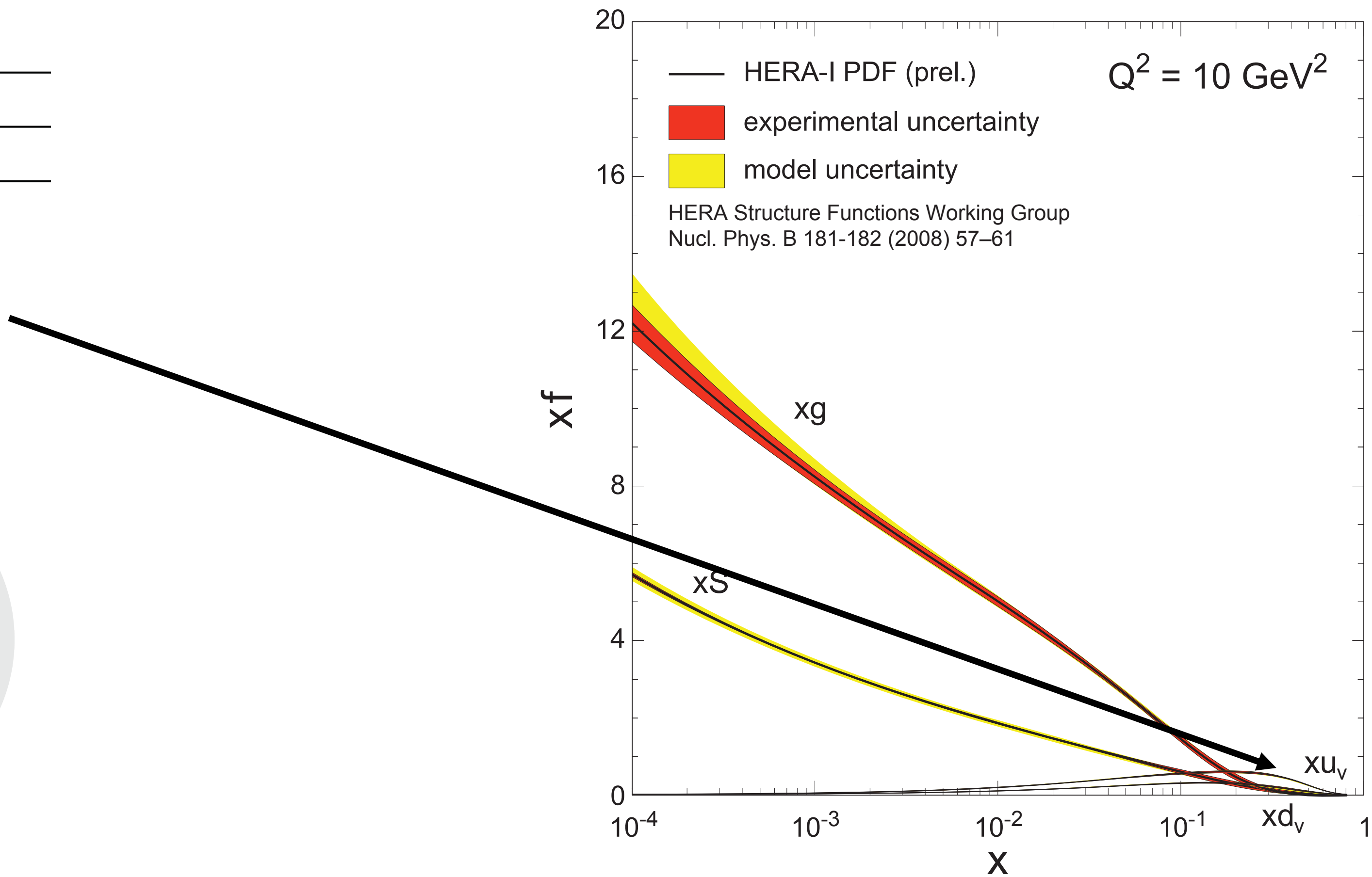
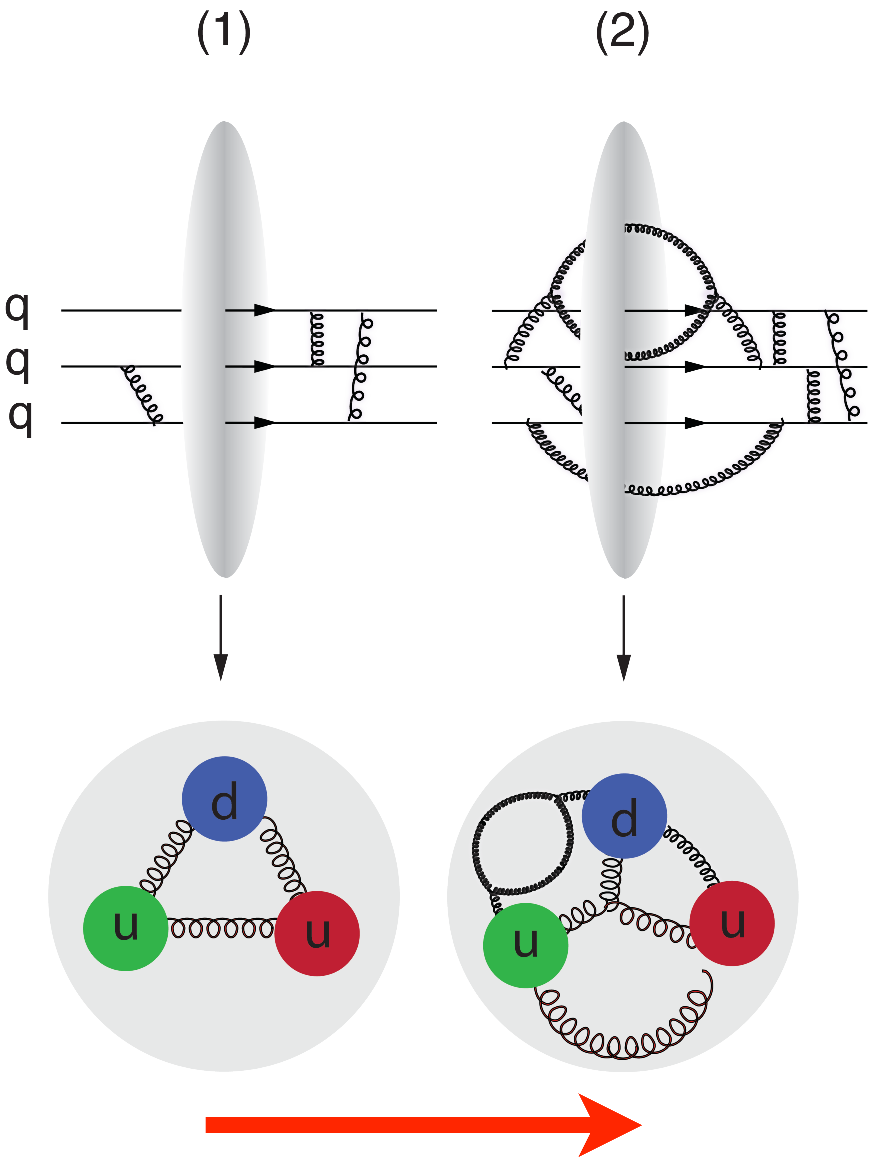
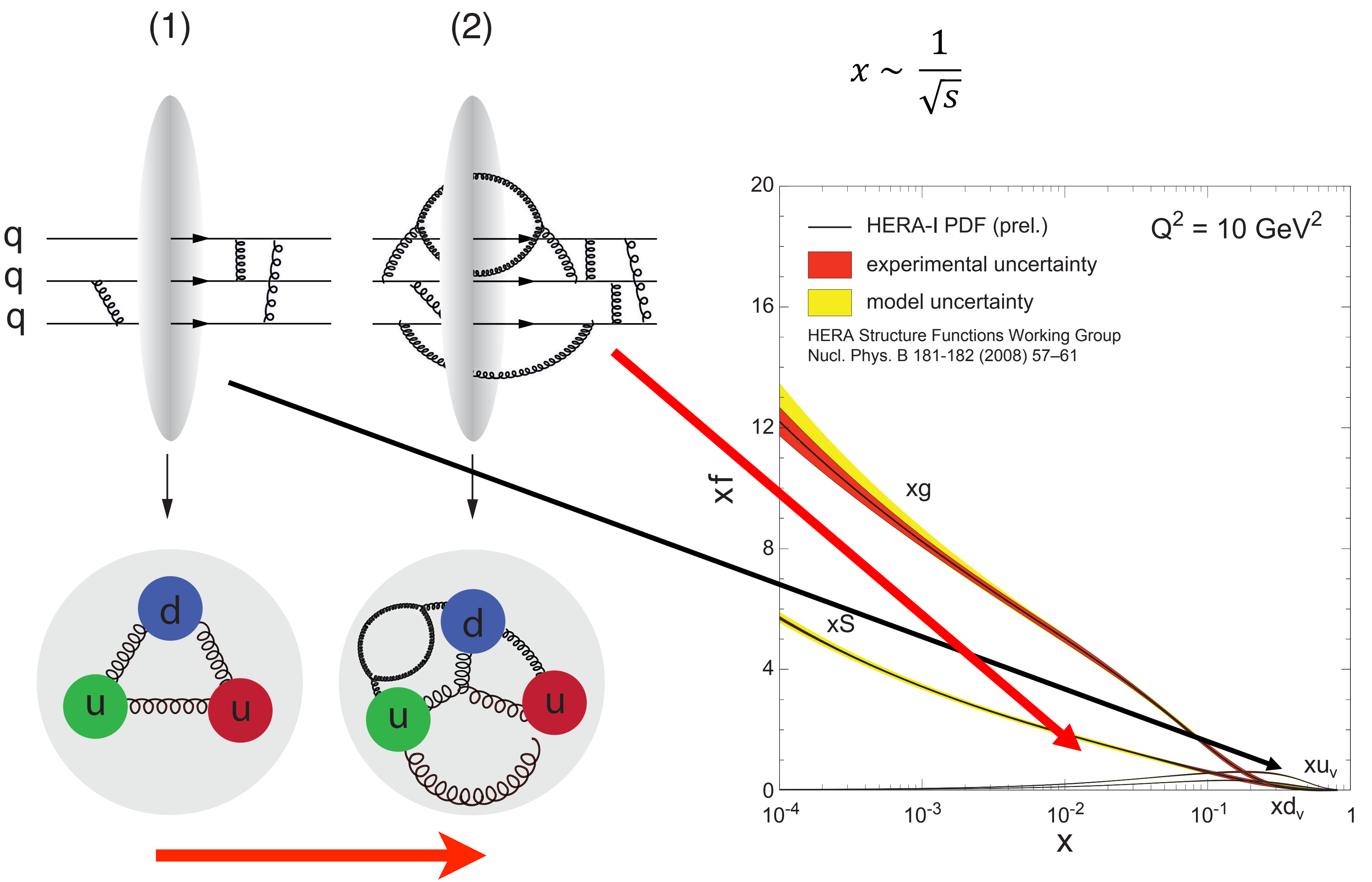




Energy of the collision

$$x \sim \frac{1}{\sqrt{s}}$$

Increasing the energy of the collision means reaching lower values of x.



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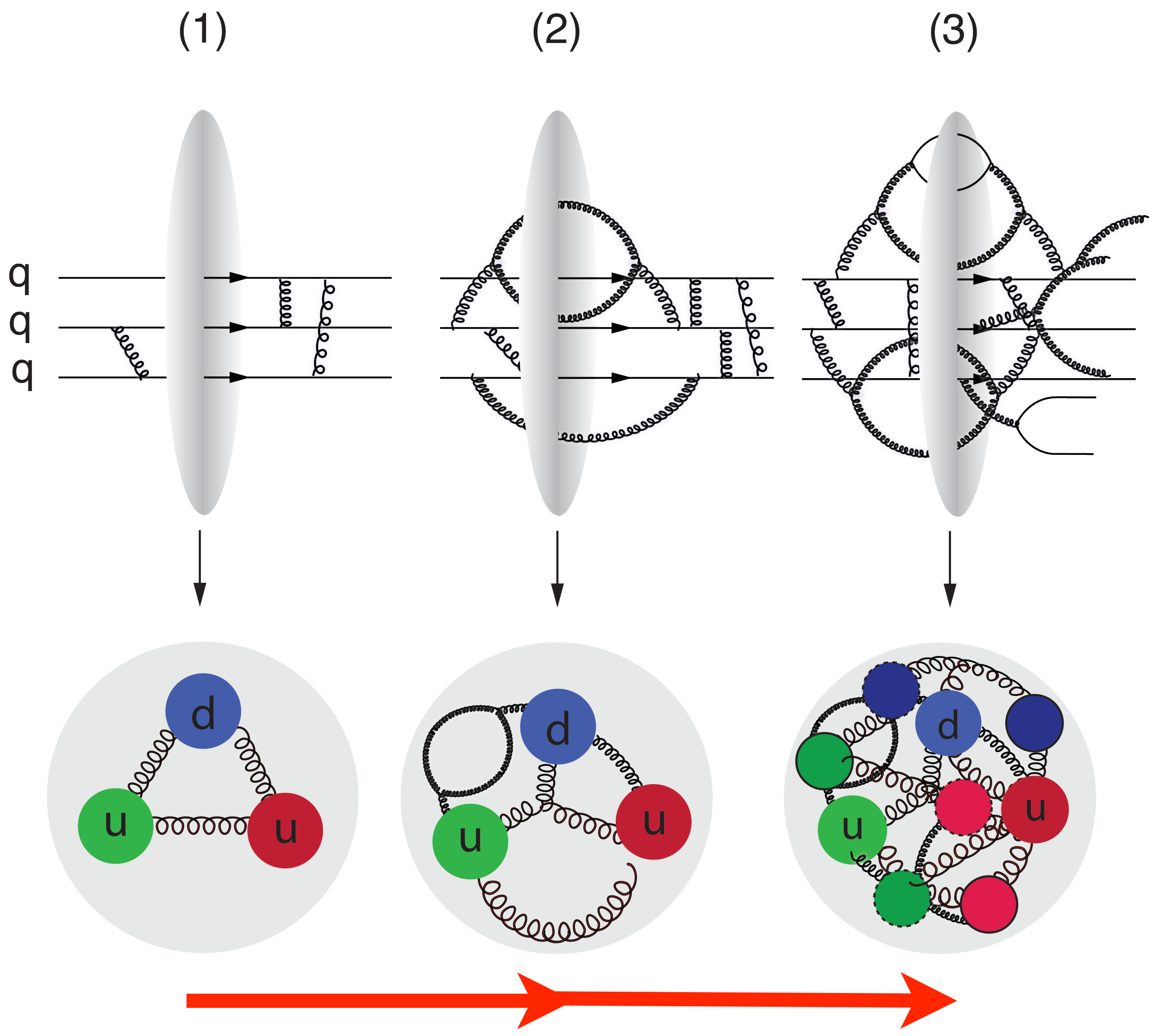

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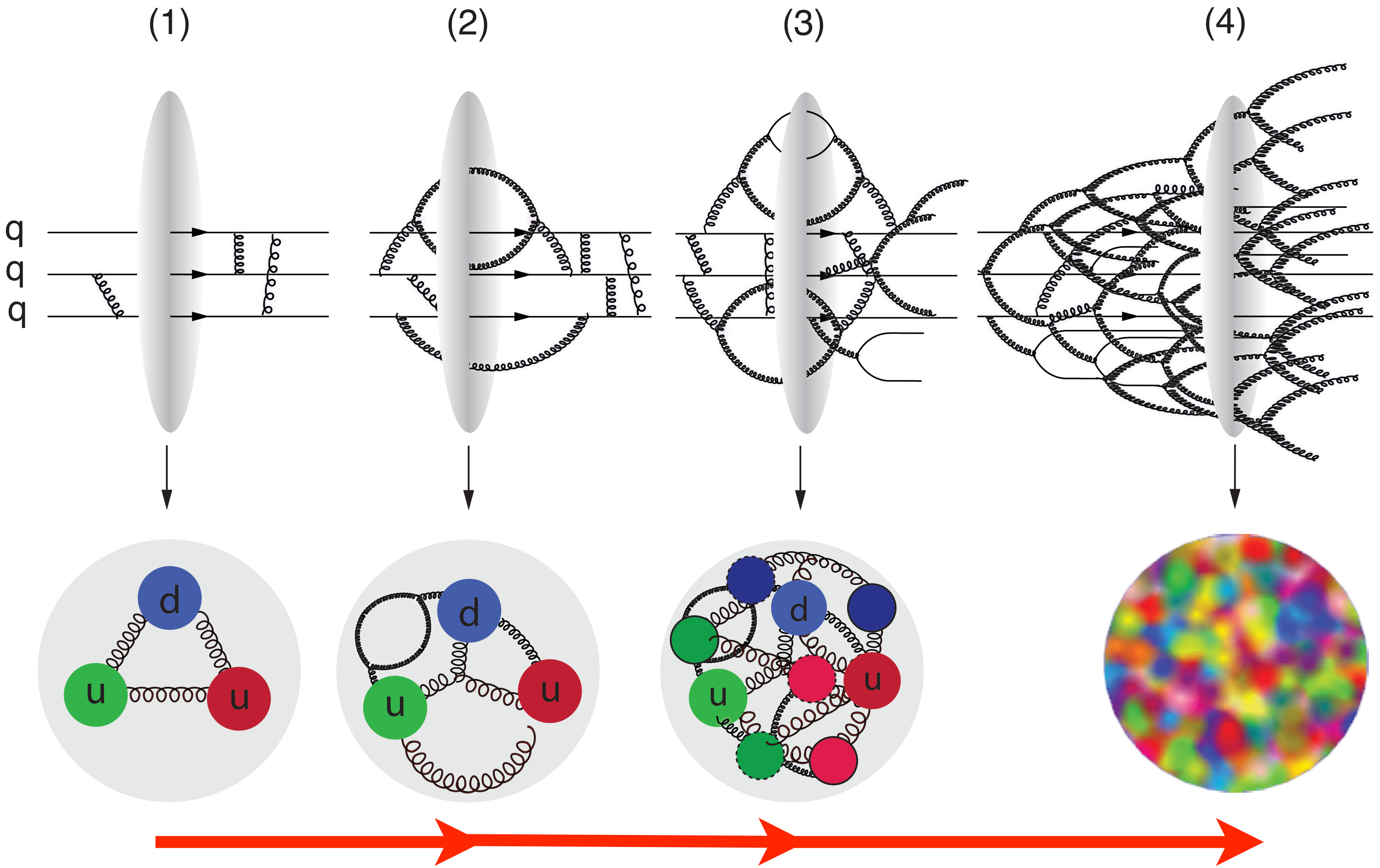
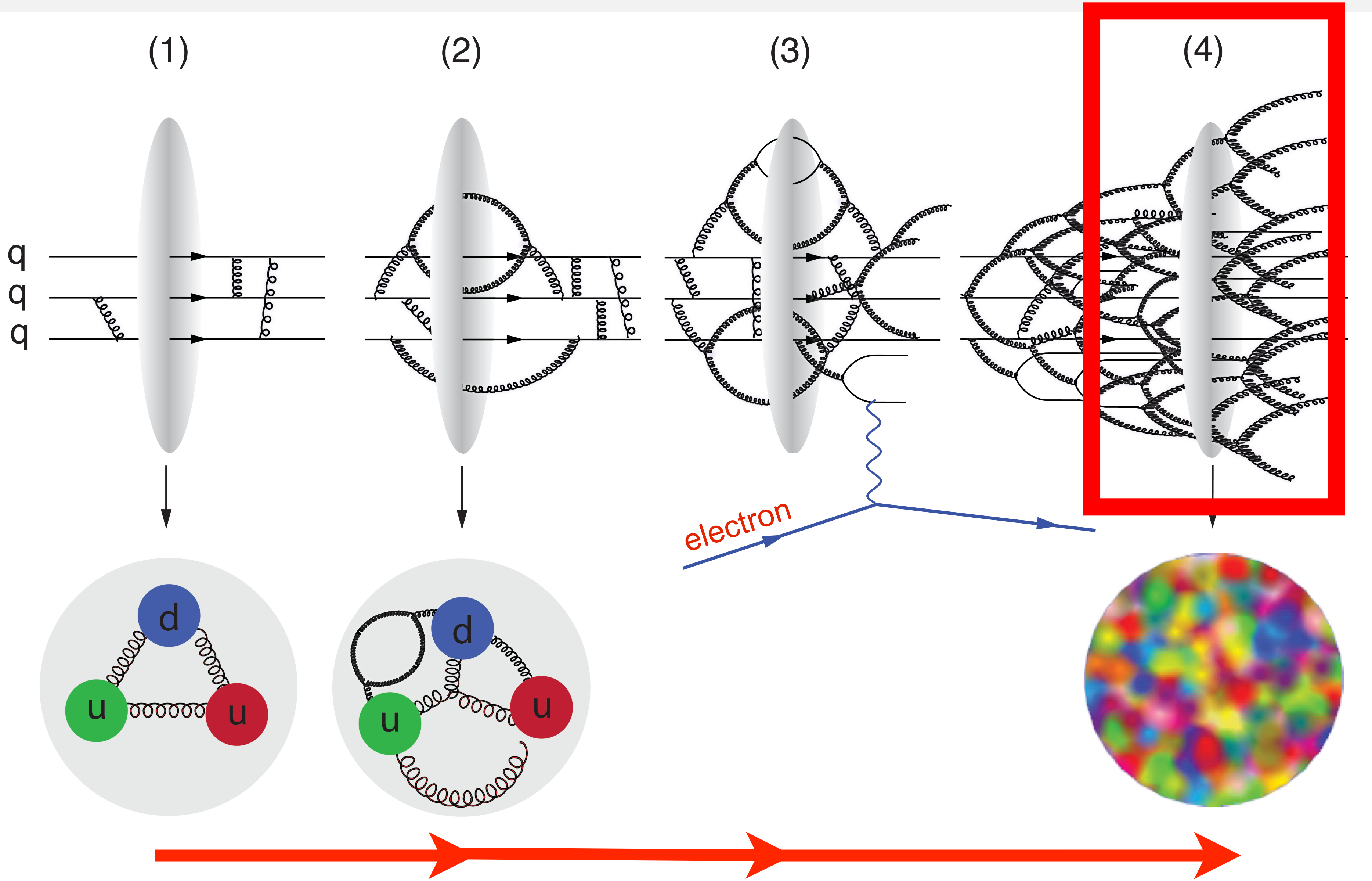


Energy of the collision

What is happening here?



Energy of the collision

SATURATION

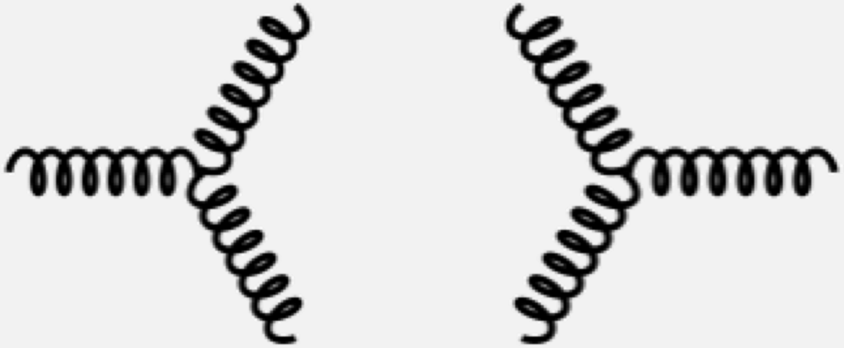
- If gluon numbers only grow toward region of low-x, the gluon distribution would diverge.
- This growth is governed by the BFKL equation.

$$\frac{\partial N(r,Y)}{\partial \ln Y} = \int d\vec{r}_1 K(\vec{r},\vec{r}_1,\vec{r}_2)(N(\vec{r}_1,Y) + N(\vec{r}_2,Y) - N(\vec{r},Y) - N(\vec{r}_1,Y)N(\vec{r}_2,Y))$$

- The rate of this growth is unphysical and gives us too high cross sections.
- Additional effects need to be taken into account!

SATURATION

- BFKL equation includes only the gluon radiation effects.
- Other non-linear evolution equation such as the BK equation takes gluon recombination into account.
- This slows down the evolution and tames the unphysical divergences.



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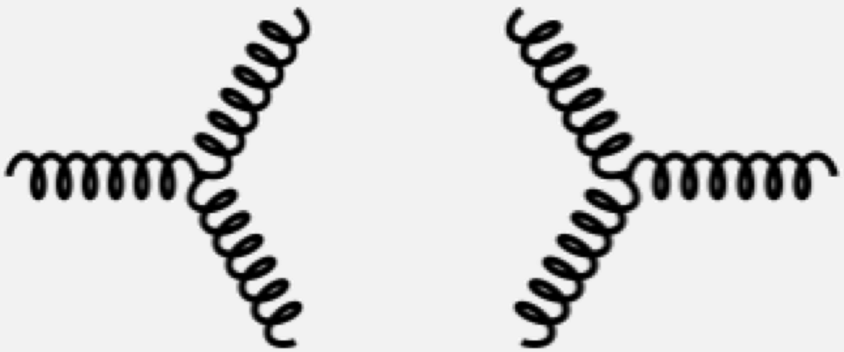

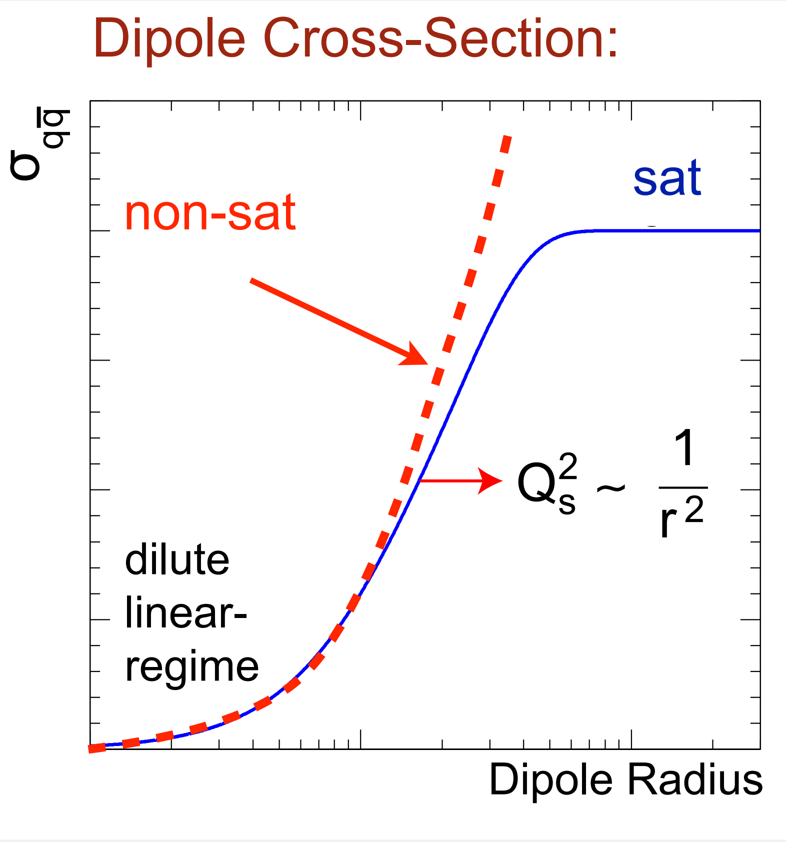

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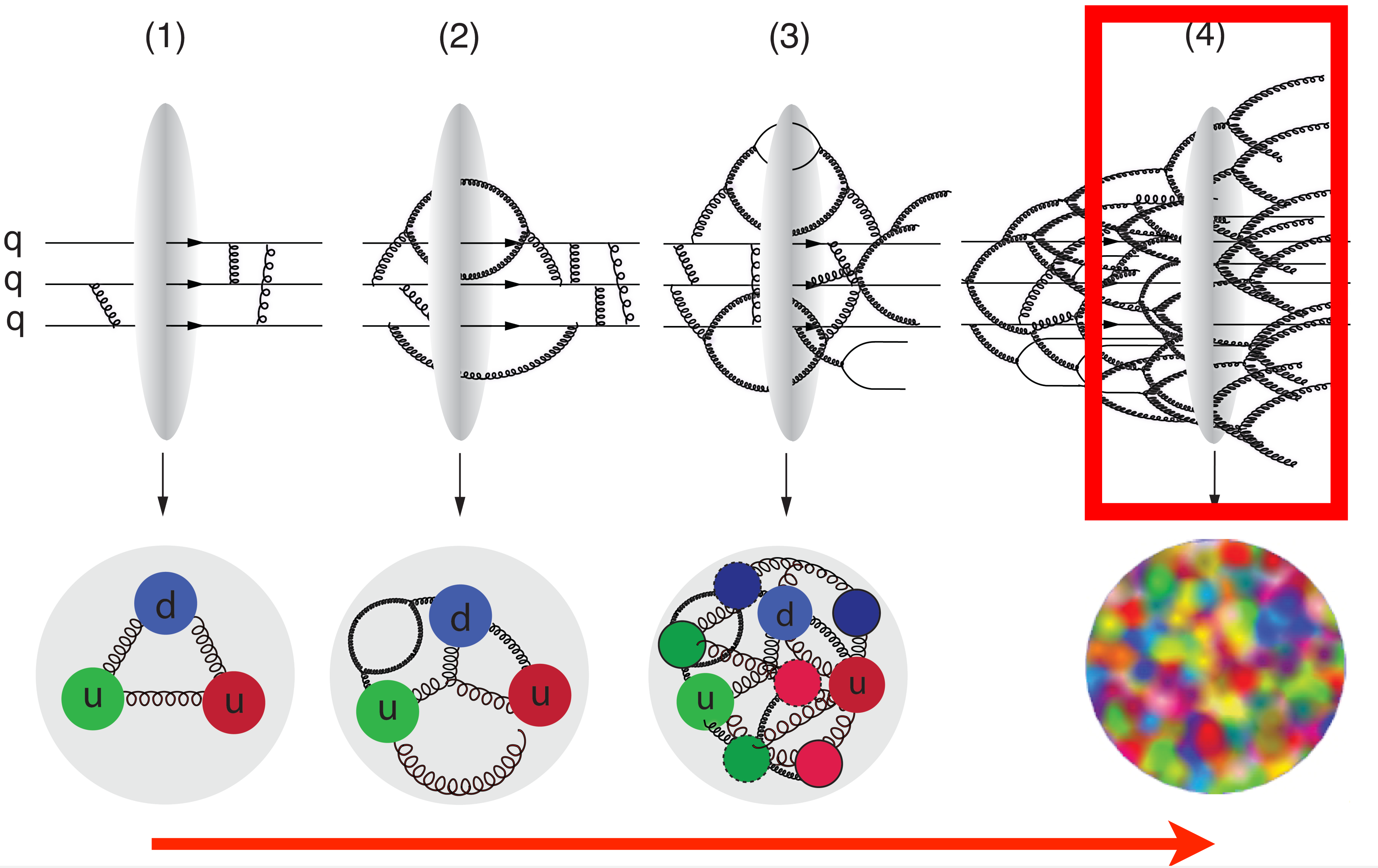
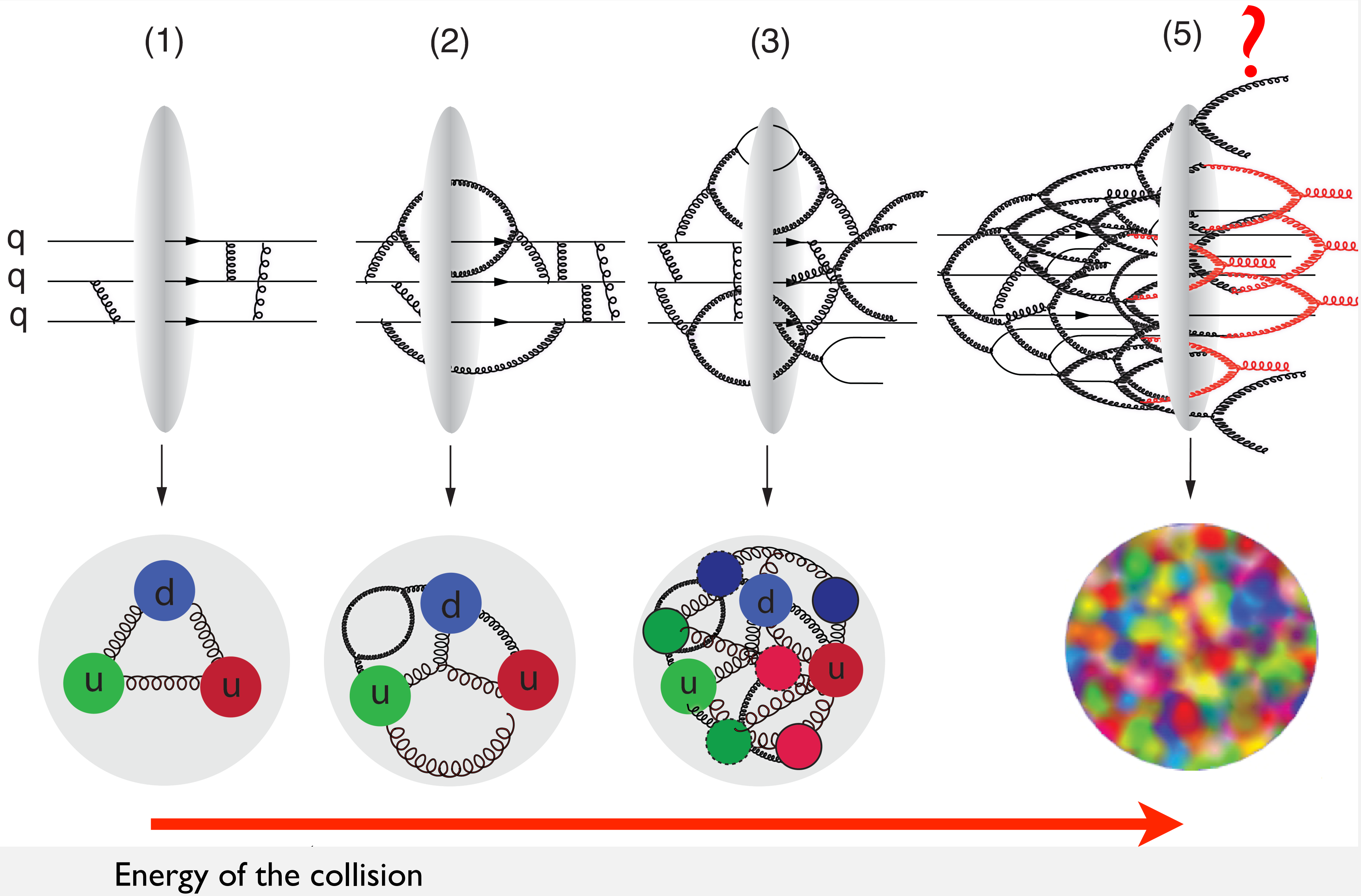


Energy of the collision


(1)
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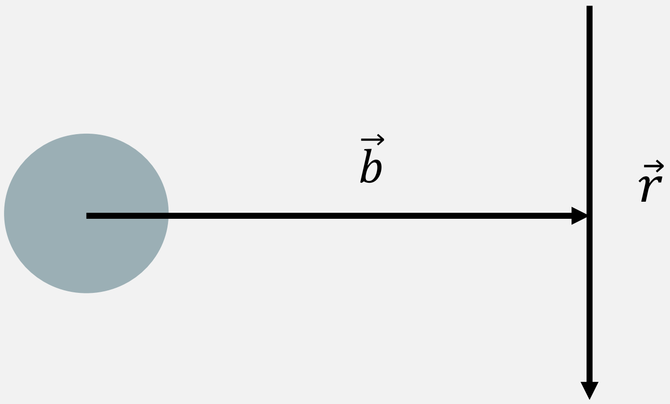
Energy of the collision

IMPACT-PARAMETER DEPENDENCE OF THE BK EQUATION

b-BK EQUATION

Impact parameter is the distance of an interacting dipole from the center of the target.



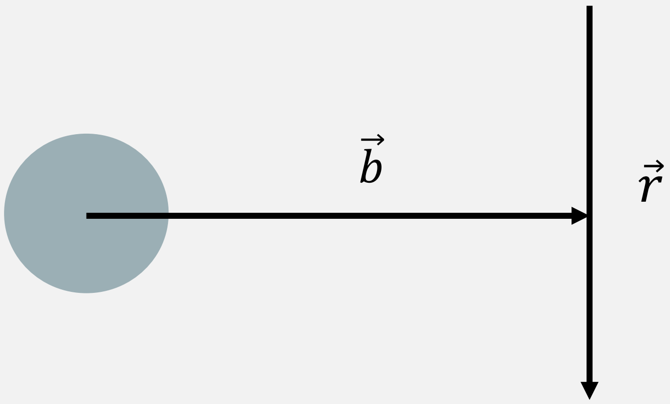
b-BK EQUATION

The Balitsky-Kovchegov equation describes the evolution of a color dipole scattering amplitude $N(\vec{r},\vec{b},Y)$ in rapidity

$$\frac{\partial N(\vec{r},\vec{b},Y)}{\partial Y} = \int d\vec{r_1} K(r,r_1,r_2)(N(\vec{r_1},\vec{b_1},Y) + N(\vec{r_2},\vec{b_2},Y) - N(\vec{r},\vec{b},Y) - N(\vec{r_1},\vec{b_1},Y)N(\vec{r_2},\vec{b_2},Y))$$

given by $Y = \ln\frac{x_0}{x}$.

Impact parameter dependence enters the equation.



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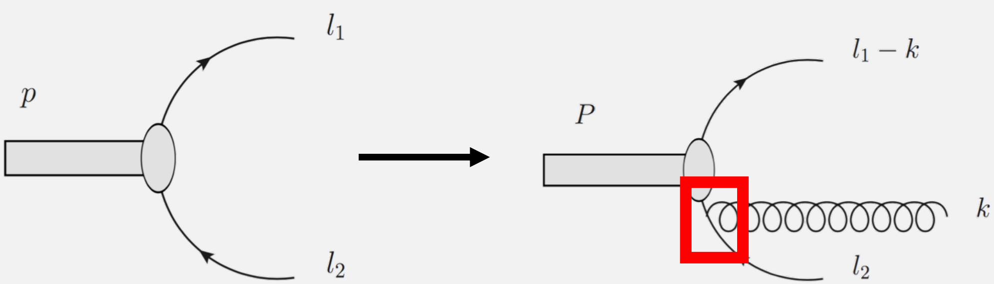

I. Balitsky, Nucl. Phys. B 463, 99 (1996)
Y. V. Kovchegov, Phys. Rev. D 60, 034008 (1999)
J. L. Albacete at al, Phys.Rev. D75 (2007) 125021
E. Iancu at al, Phys. Let. B. (2015) 1507.03651

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given by $Y = \ln \frac{x_0}{x}$.

Since the process of gluon emission can be computed under different approximations, we have a number of kernels derived such as

Running coupling kernel:

$$K^{run}(r,r_1,r_2) = \frac{N_c \alpha_s(r^2)}{2\pi^2} \left(\frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_1^2}\left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1\right) + \frac{1}{r_2^2}\left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1\right)\right)$$

Collinearly improved kernel:

$$K^{col}(r,r_1,r_2) = \frac{\overline{\alpha}_s}{2\pi} \frac{r^2}{r_1^2 r_2^2} \left[\frac{r^2}{\min(r_1^2,r_2^2)}\right]^{\pm\overline{\alpha}_s A_1} K_{DLA}(\sqrt{L_{r_1 r} L_{r_2 r}})$$

I. Balitsky, Nucl. Phys. B 463, 99 (1996)
Y. V. Kovchegov, Phys. Rev. D 60, 034008 (1999)
J. L. Albacete at al, Phys.Rev. D75 (2007) 125021
E. Iancu at al, Phys. Let. B. (2015) 1507.03651

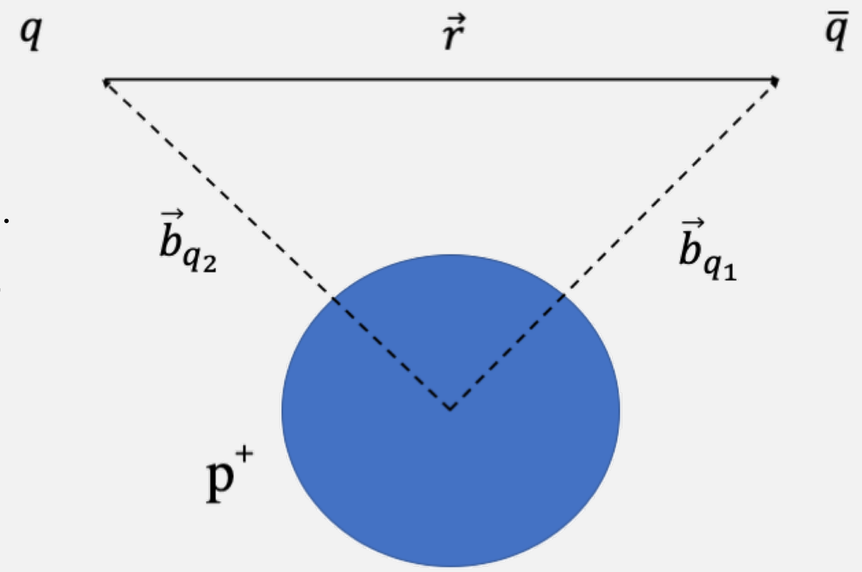
b-BK EQUATION

For solving this equation numerically, we choose an initial condition

$$N(r, b, Y = 0) = 1 - \exp\left(-\frac{1}{2}\frac{Q_s^2}{4}r^2 T(b_{q_1}, b_{q_2})\right), \qquad \text{where} \quad T(b_{q_1}, b_{q_2}) = \left[\exp\left(-\frac{b_{q_1}^2}{2B}\right) + \exp\left(-\frac{b_{q_2}^2}{2B}\right)\right].$$

There are two free parameters; saturation scale $Q_s^2 = 0.49\ \mathrm{GeV}^2$ and variance of the profile distribution $B_G = 3.22\ \mathrm{GeV}^{-2}$.

- The *r* behavior mimics that of the GBW model.
- The *b* behavior exhibits the exponential fall-off calculated for the individual quarks.

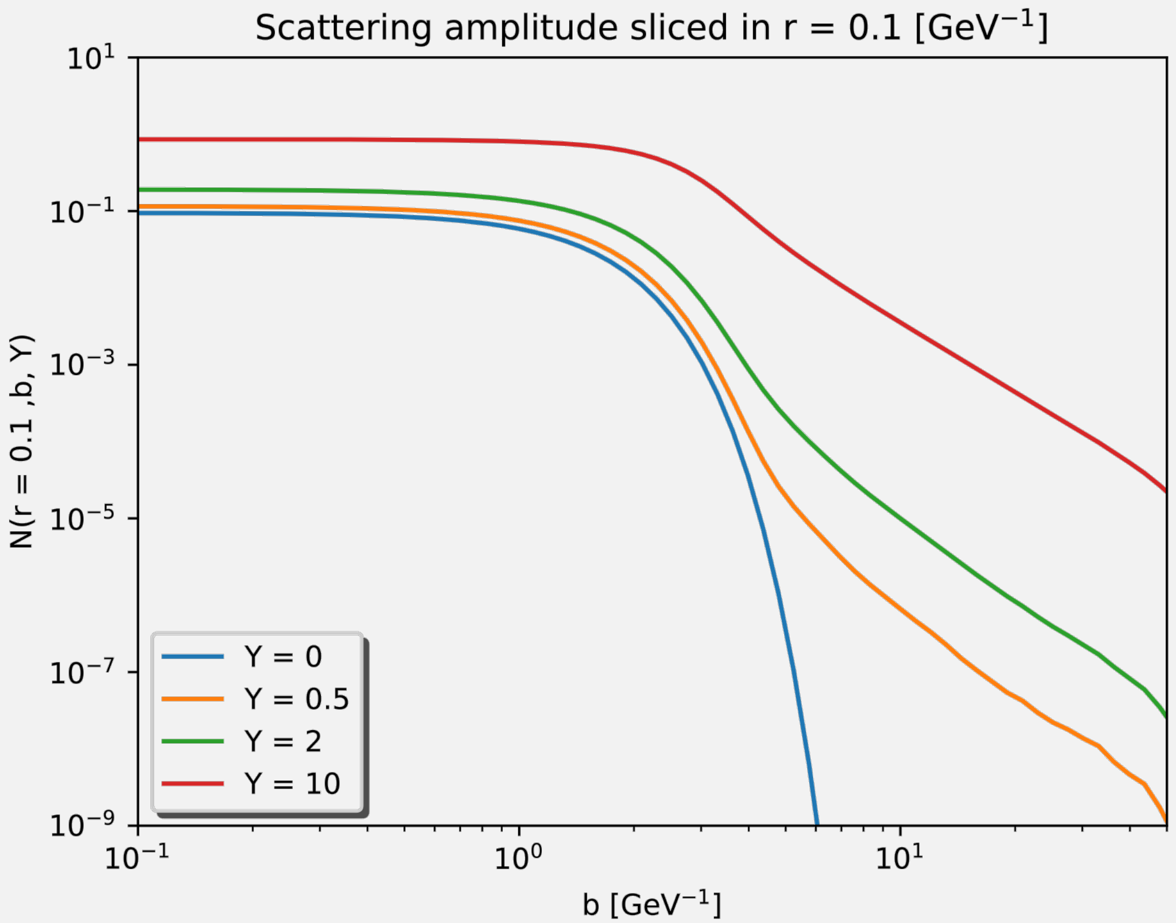


K. J. Golec-Biernat and M. Wusthoff, Phys. Rev. D59, 014017 (1998)
J. Cepila, J. G. Contreras, M. Matas; Phys. Rev. D 99, 051502

THE PROBLEM OF COULOMB TAILS

THE PROBLEM

If we start with an exponentially falling initial condition and the usual running coupling kernel.



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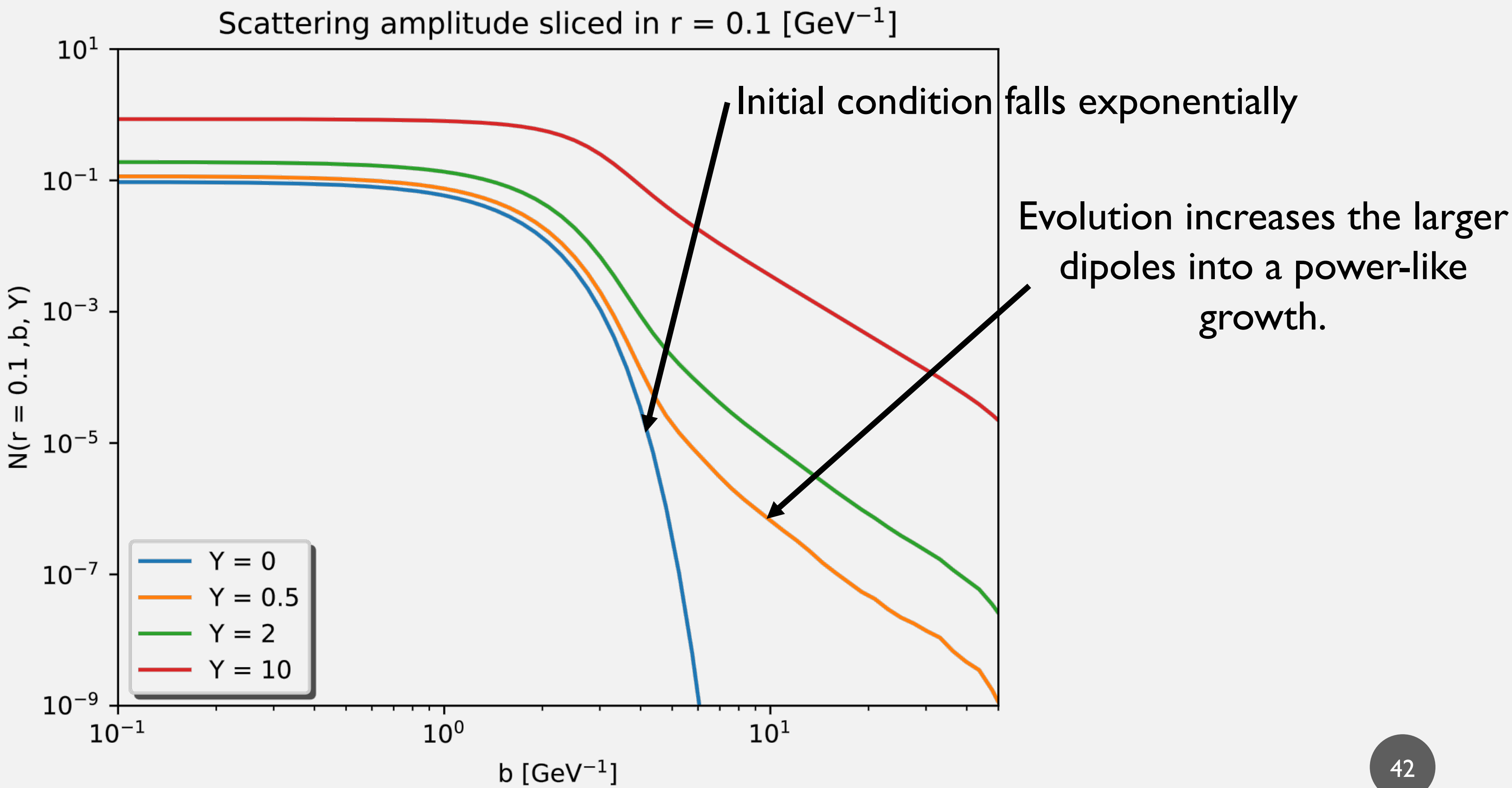

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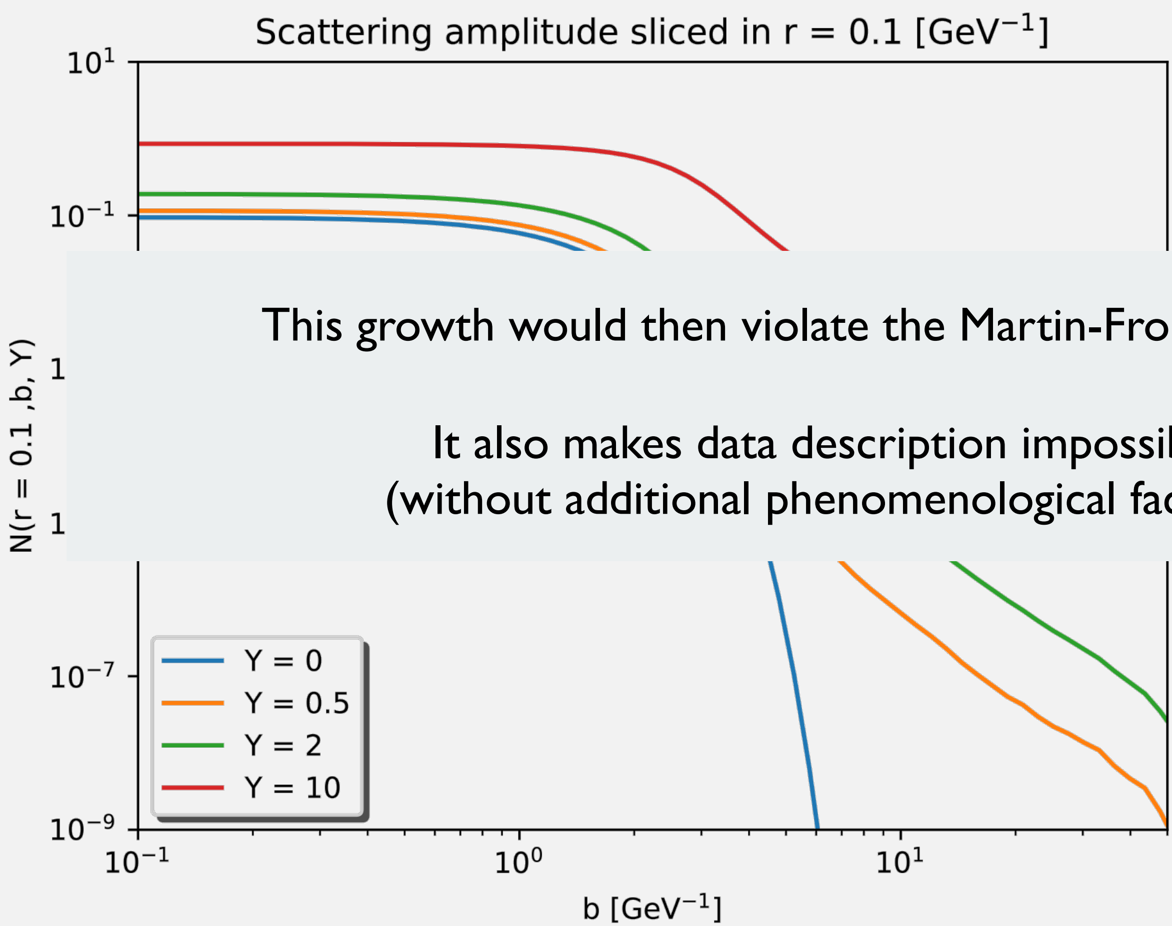


This growth would then violate the Martin-Froisart bound.

It also makes data description impossible.
(without additional phenomenological factors)

HIGH-b SUPPRESSION

The kernel itself does not depend on b. We can however tame the growth in b by suppressing evolution at big sizes of daughter dipoles.

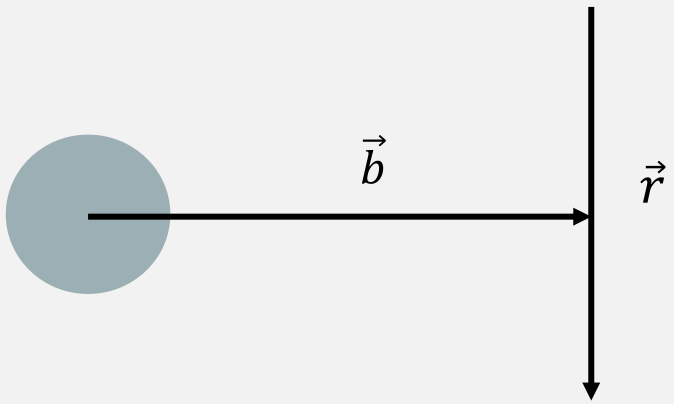
Why?

HIGH-b SUPPRESSION

For high-*b*, the scattering amplitude is exponentially suppressed at the initial condition.

~0

$$\frac{\partial N(\vec{r},\vec{b},Y)}{\partial Y} = \int d\vec{r_1} K(r,r_1,r_2)(N(\vec{r_1},\vec{b_1},Y) + N(\vec{r_2},\vec{b_2},Y) - N(\vec{r},\vec{b},Y) - N(\vec{r_1},\vec{b_1},Y)N(\vec{r_2},\vec{b_2},Y))$$



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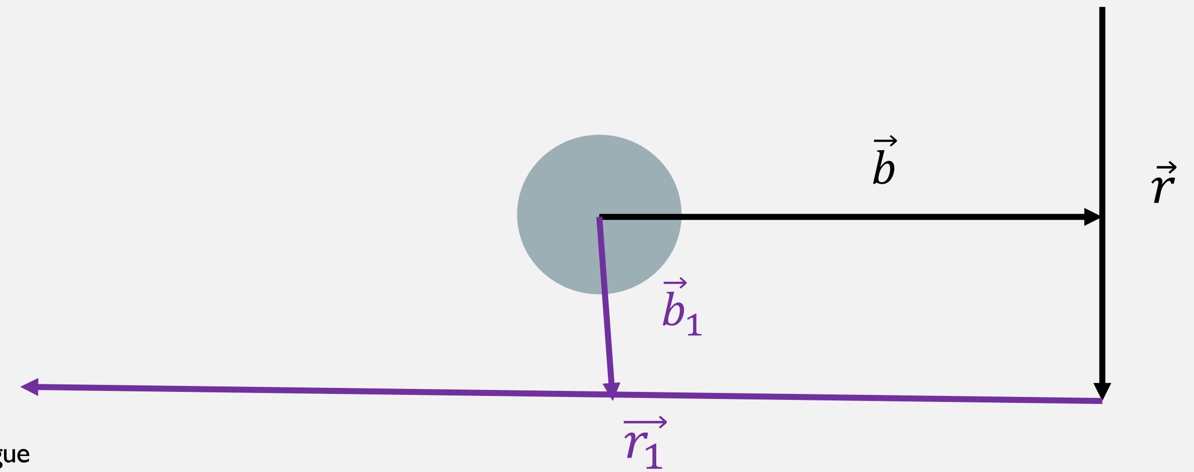


HIGH-b SUPPRESSION

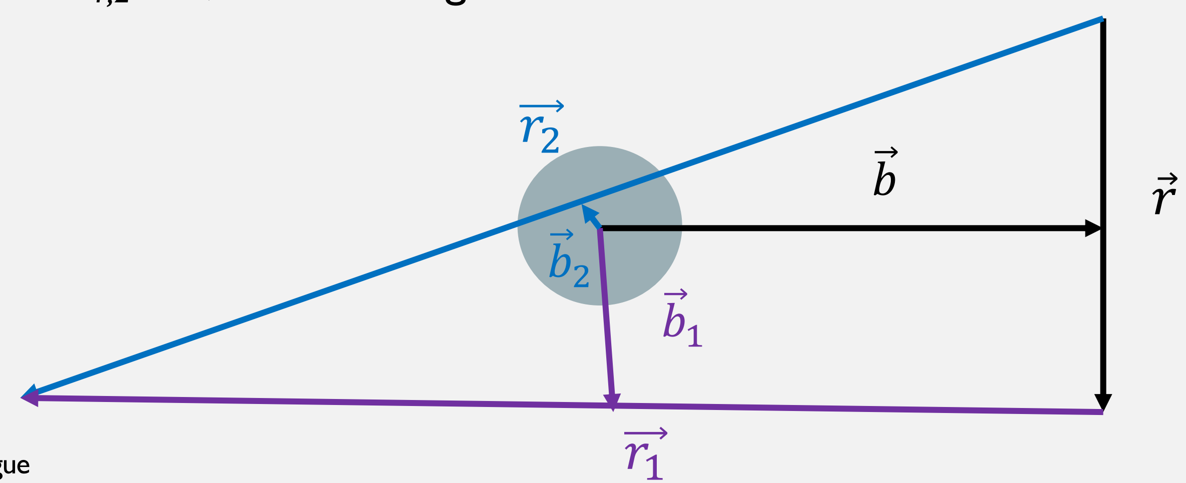
For high-*b*, the scattering amplitude is exponentially suppressed at the initial condition.

~0 ~1 ~0 ~0 ~1

$$\frac{\partial N(\vec{r},\vec{b},Y)}{\partial Y} = \int d\vec{r_1} K(r,r_1,r_2)(N(\vec{r_1},\vec{b_1},Y) + N(\vec{r_2},\vec{b_2},Y) - N(\vec{r},\vec{b},Y) - N(\vec{r_1},\vec{b_1},Y)N(\vec{r_2},\vec{b_2},Y))$$

The only amplitudes that could be non-zero are those with small impact parameter.

These have $r_{1,2}$~$2b$, which is large.



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~0 ~1 ~0 ~0 ~1

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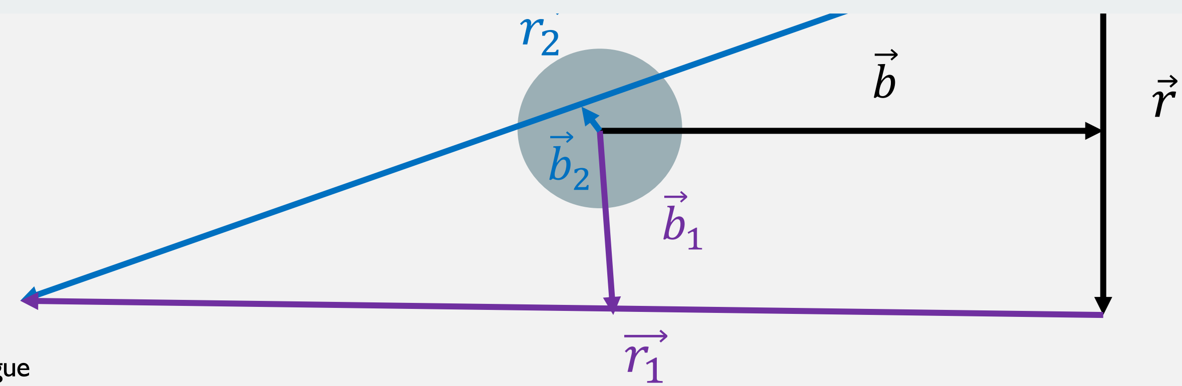
T ...er.

Therefore if we suppress kernel at high r_1 and r_2, we suppress the evolution at high-*b* and maintain the exponential falloff of the scattering amplitude.

T



HOW TO SUPPRESS LARGE DAUGHTER DIPOLES

KERNEL CUTOFF

One possible solution to this problem is that we can cut the kernel, so that dipoles, that are too big would not contribute to the evolution.

$$\frac{\partial N(r,\vec{b},Y)}{\partial Y} = \int d\vec{r_1} K^{run}(r,r_1,r_2)\Theta\left(\frac{1}{m^2} - r_1^2\right)\Theta\left(\frac{1}{m^2} - r_2^2\right)$$
$$(N(r_1,\vec{b_1},Y) + N(r_2,\vec{b_2},Y) - N(r,\vec{b},Y) - N(r_1,\vec{b_1},Y)N(r_2,\vec{b_2},Y))$$

Jeffrey Berger, Anna M. Stasto, Phys.Rev. D84 (2011) 094022

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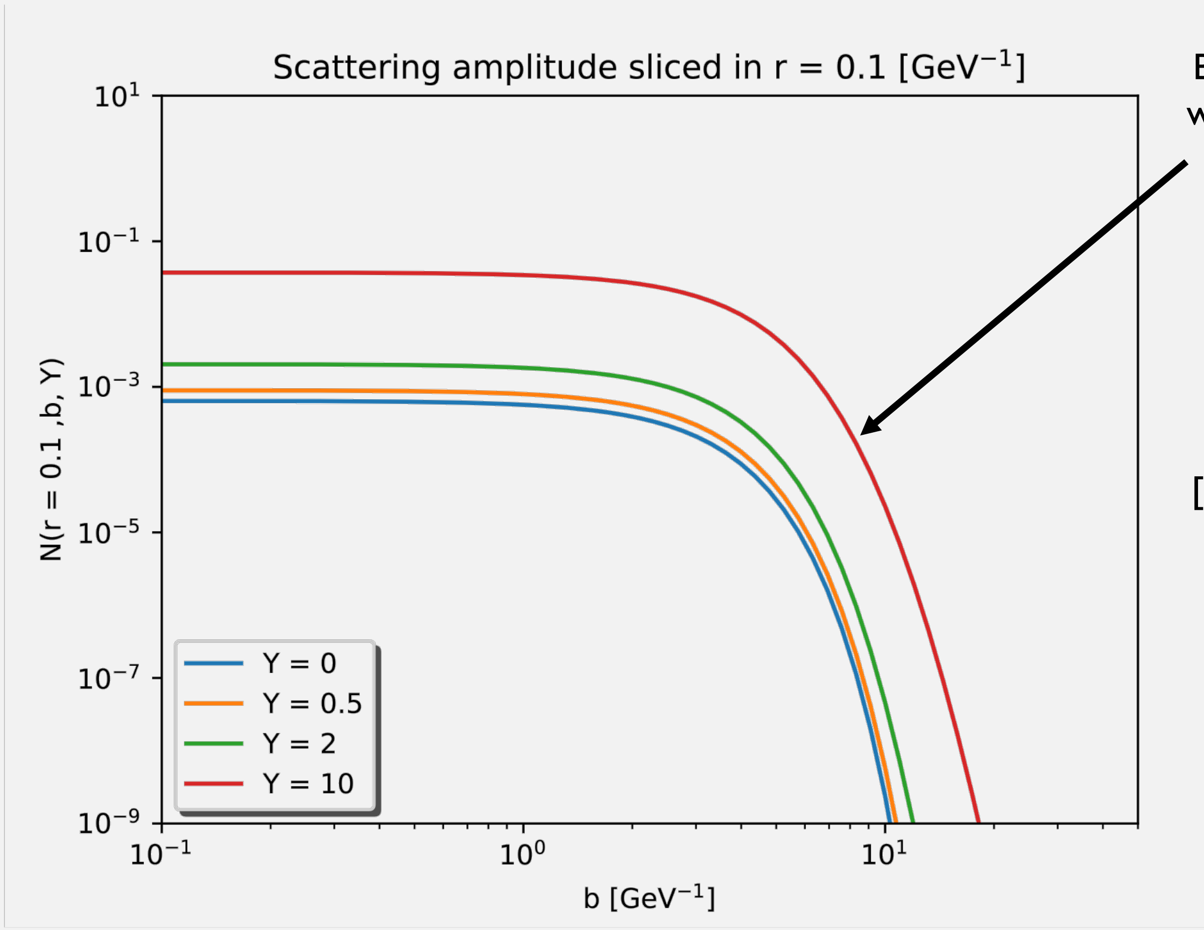
$$\frac{\partial N(r,\vec{b},Y)}{\partial Y} = \int d\vec{r_1} K^{run}(r,r_1,r_2)\Theta\left(\frac{1}{m^2}-r_1^2\right)\Theta\left(\frac{1}{m^2}-r_2^2\right)$$
$$(N(r_1,\vec{b_1},Y)+N(r_2,\vec{b_2},Y)-N(r,\vec{b},Y)-N(r_1,\vec{b_1},Y)N(r_2,\vec{b_2},Y))$$

Mass of the emitted gluon is a free parameter, that is fitted to data.

Jeffrey Berger, Anna M. Stasto, Phys.Rev. D84 (2011) 094022

KERNEL CUTOFF



By imposing the cutoff of the kernel, we maintain the exponential falloff of the scattering amplitude.

However, as was shown in [Phys. Rev. D84(2011)094022], we still cannot describe the data, since the cutoff is too strong and we need to impose new phenomenological constants to cure this.

Jeffrey Berger, Anna M. Stasto, Phys.Rev. D84 (2011) 094022

KERNEL CUTOFF

The recently proposed collinearly improved kernel is by its nature suppressed at high $r_{1,2}$ and does not require additional dimensional parameters.

$$K^{col}(r, r_1, r_2) = \frac{\overline{\alpha}_s}{2\pi} \frac{r^2}{r_1^2 r_2^2} \left[\frac{r^2}{\min(r_1^2, r_2^2)}\right]^{\pm\overline{\alpha}_s A_1} K_{DLA}(\sqrt{L_{r_1 r} L_{r_2 r}})$$

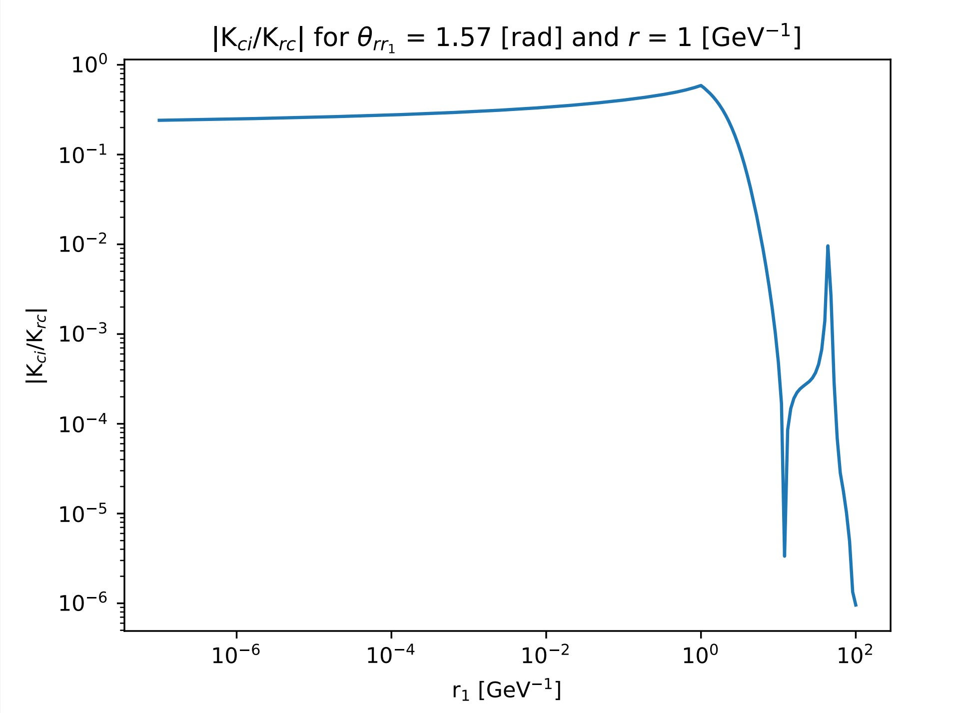
The collinearly improved kernel imposes a time ordering in the lifetime of the consequent dipoles.

It is a consequence of resumming collinear logarithms in the derivation of the kernel.

E. Iancu at al, Phys. Let. B. (2015) 1507.03651

KERNEL CUTOFF

Here we compare the value of the collinearly improved kernel with the running coupling kernel versus r_l.



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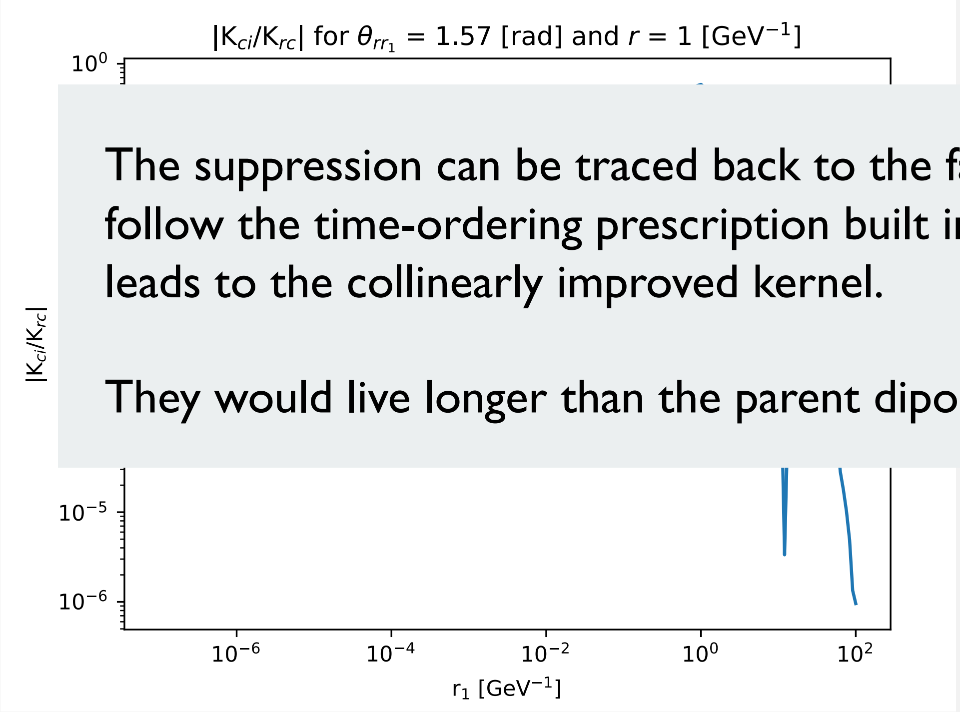

The suppression can be traced back to the fact that large daughter dipoles do not follow the time-ordering prescription built in when setting up the resummation that leads to the collinearly improved kernel.

They would live longer than the parent dipole.

KERNEL CUTOFF

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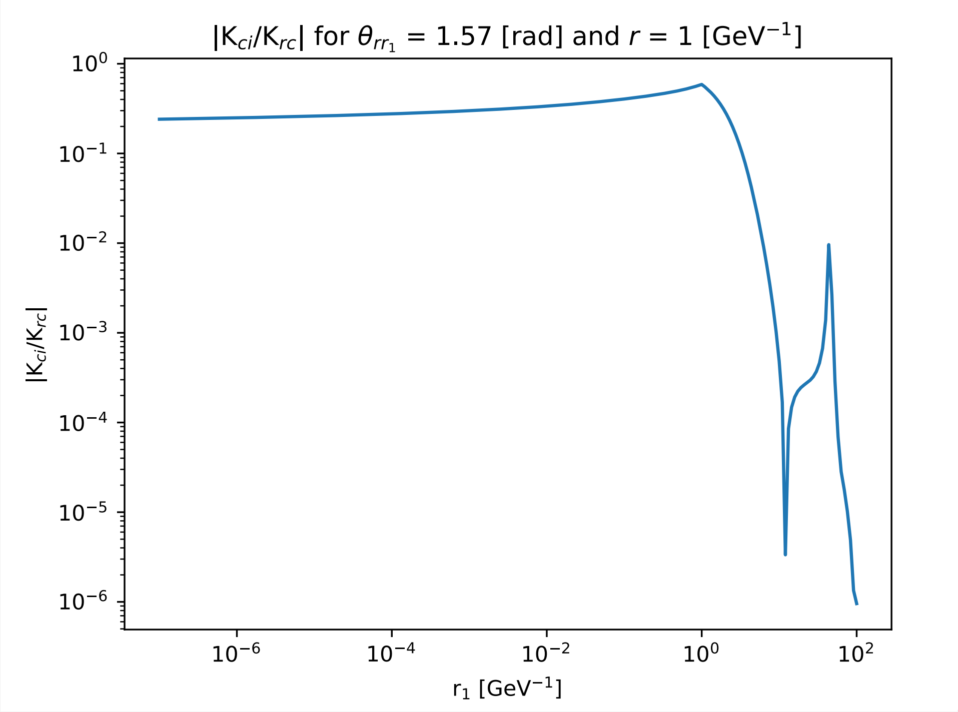
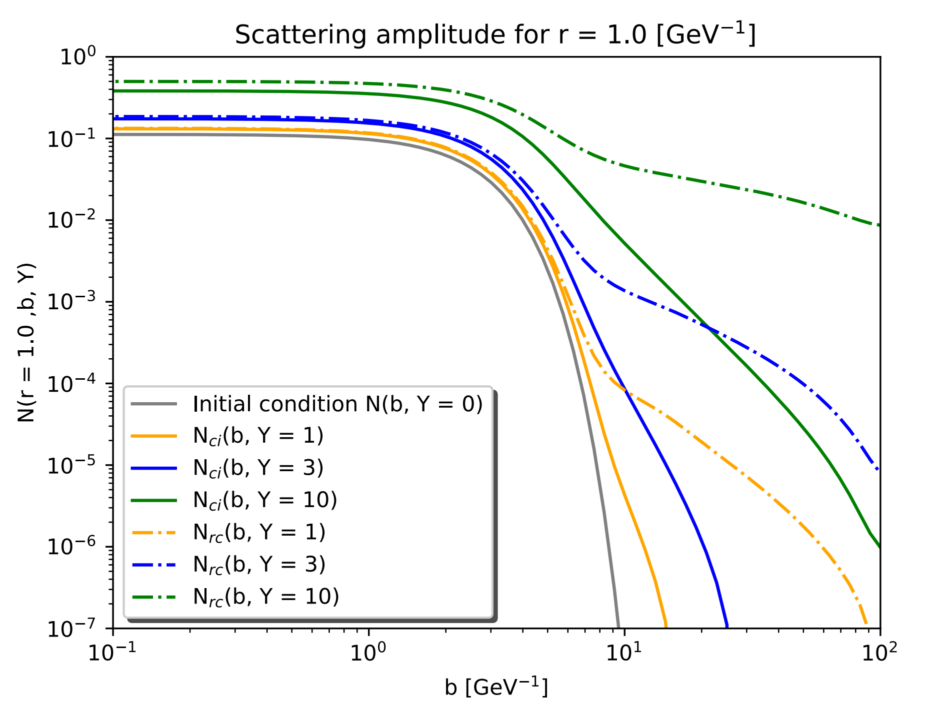
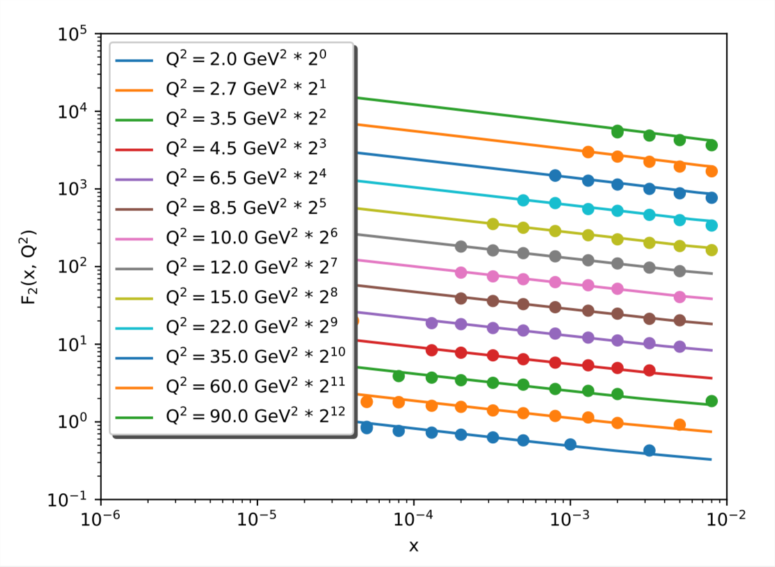




J. Cepila, J. G. Contreras, M. Matas; Phys. Rev. D 99, 051502

COMPARISON TO DATA

RESULTS



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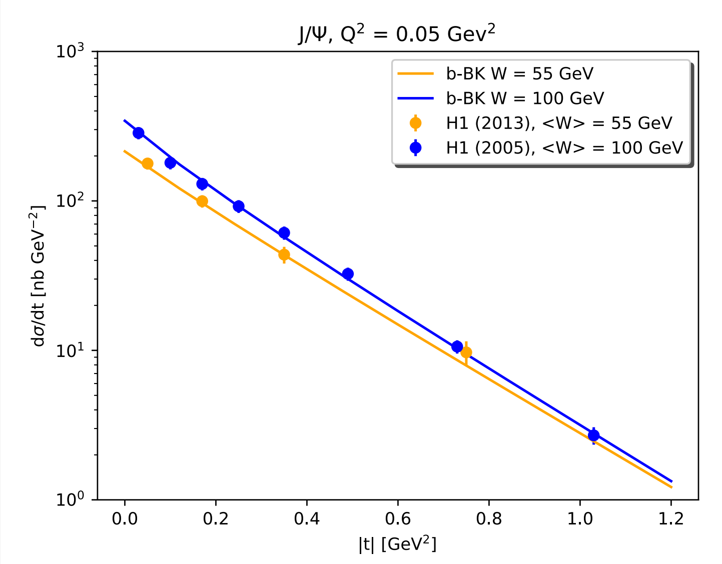

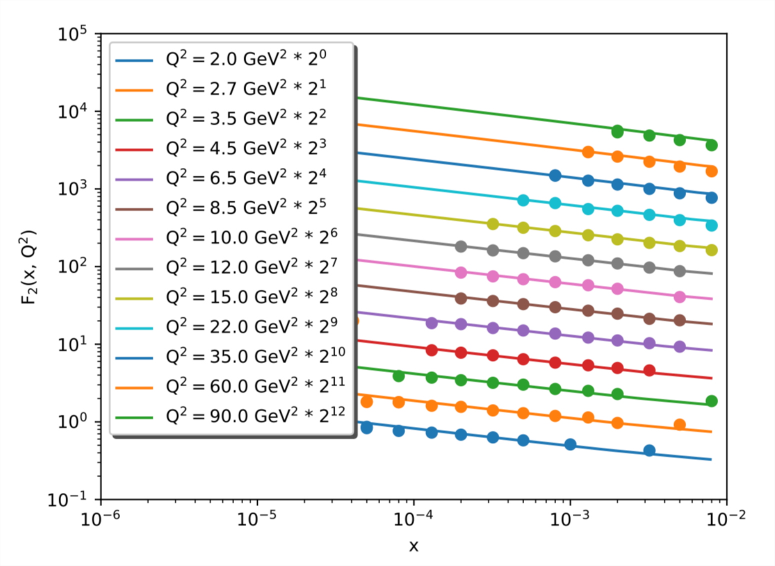

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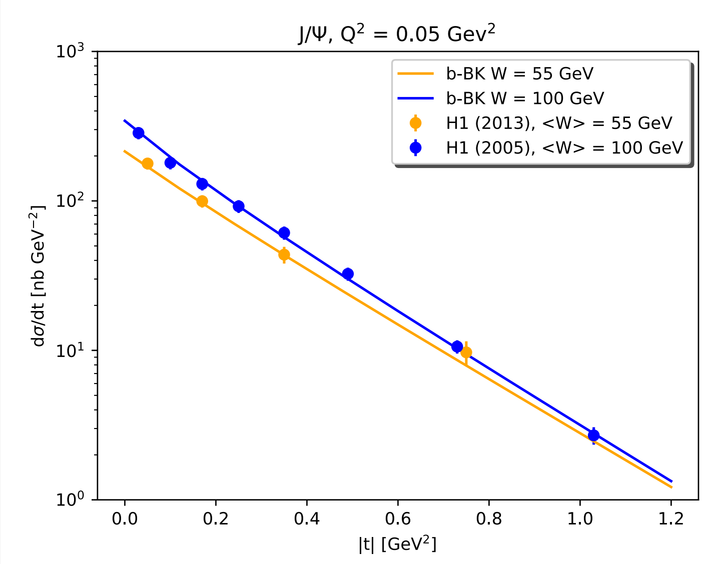
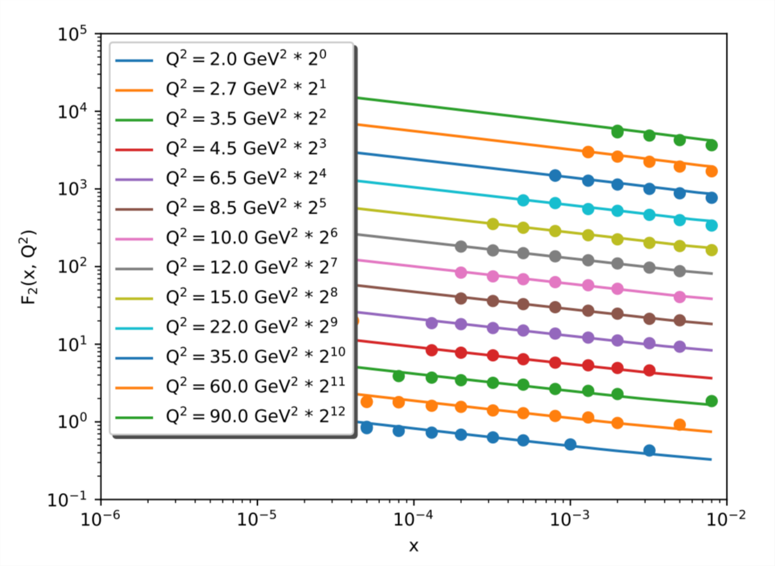
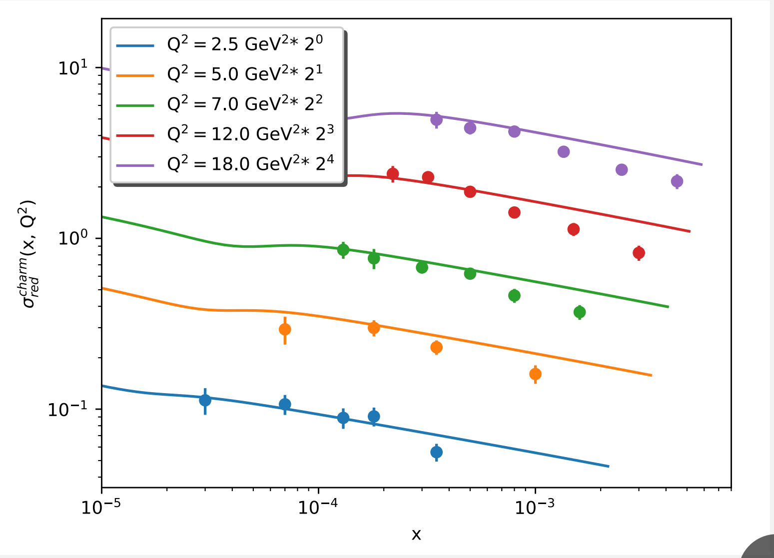
RESULTS







J. Cepila, J. G. Contreras, M. Matas; Phys. Rev. D 99, 051502

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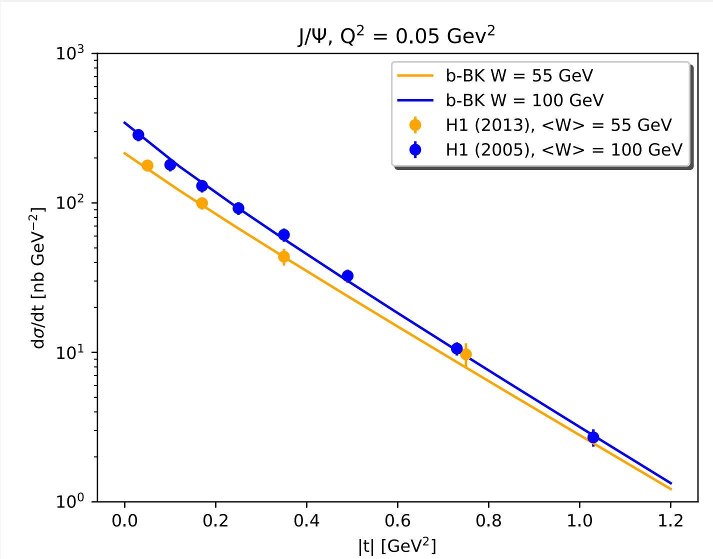

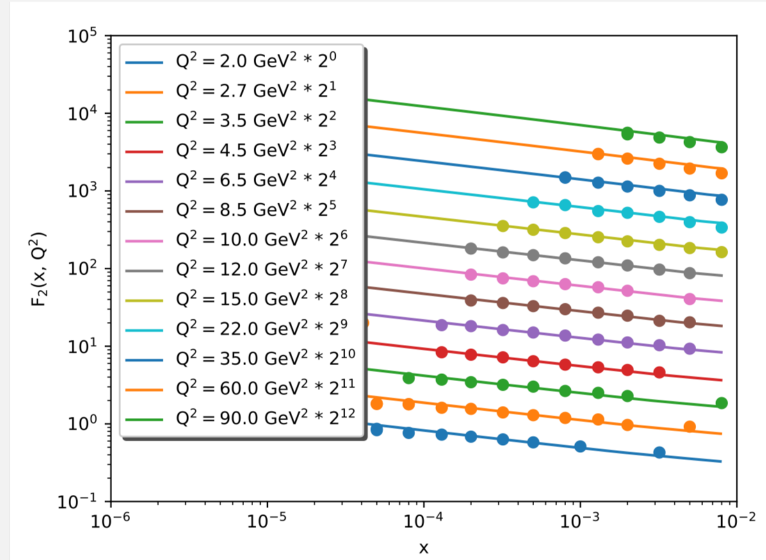

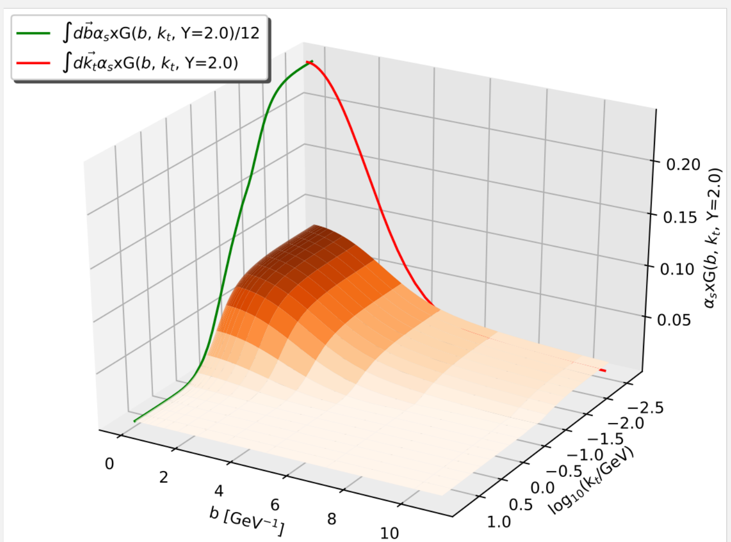

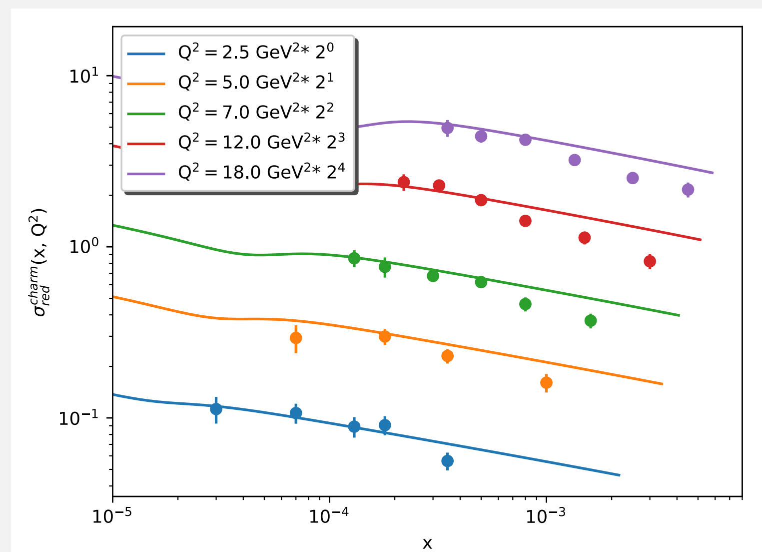

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CONCLUSIONS

- The BK equation is a crucial tool in our understanding of QCD and saturation physics
- The predictive power of the the impact-parameter dependent BK equation can be spoiled by the unphysical growth of the so-called Coulomb tails.
- These can be suppressed by suppressing the evolution for large daughter dipoles r_1 and r_2.
- The collinearly improved kernel suppresses the Coulomb tails so that the b-dependent BK equation describes data over a large phase-space and various processes.
- We have currently published a paper with all details

  Phys. Rev. D 100, 054015.

THANK YOU FOR YOUR ATTENTION

J. Cepila, J. G. Contreras, M. Matas; Phys. Rev. D 99, 051502

D. Bendova, J. Cepila, J. G. Contreras, M. Matas; arXiv:1907.12123

BACKUP

KERNEL CUTOFF

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$$K^{col}(r, r_1, r_2) = \frac{\overline{\alpha}_s}{2\pi} \frac{r^2}{r_1^2 r_2^2} \left[\frac{r^2}{\min(r_1^2, r_2^2)} \right]^{\pm \overline{\alpha}_s A_1} K_{DLA}(\sqrt{L_{r_1 r} L_{r_2 r}})$$

where $K_{DLA}(\rho) = \frac{J_1(2\sqrt{\overline{\alpha}_s \rho^2})}{\sqrt{\overline{\alpha}_s}\rho}$ with $L_{r_i r} = \ln\left(\frac{r_i^2}{r^2}\right)$

$\pm\overline{\alpha}_s A_1$ is positive when r is smaller than the daughter dipoles and negative otherwise and $A_1 = 11/12$

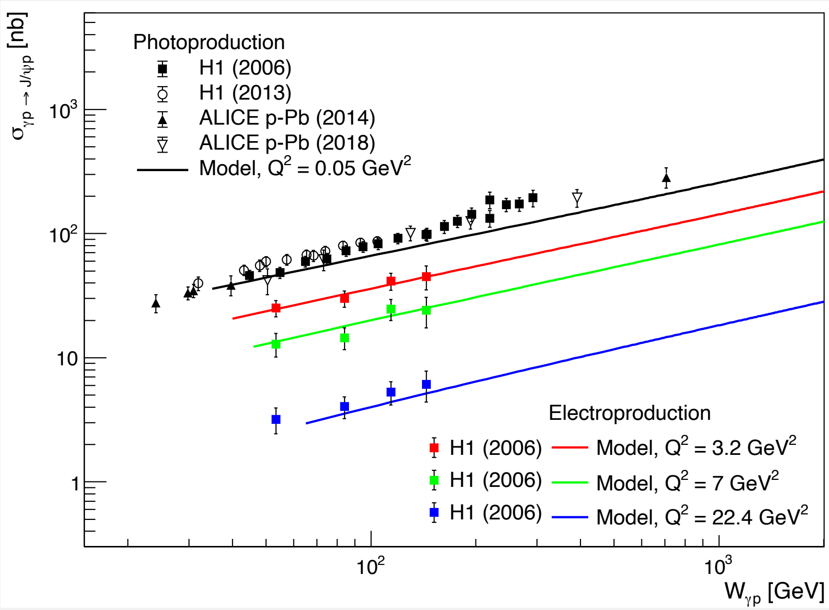
Running coupling is of the usual scheme for the BK computations as in [J. L. Albacete at al, Eur.Phys.J. C71 (2011) 1705] at the minimal scale given by

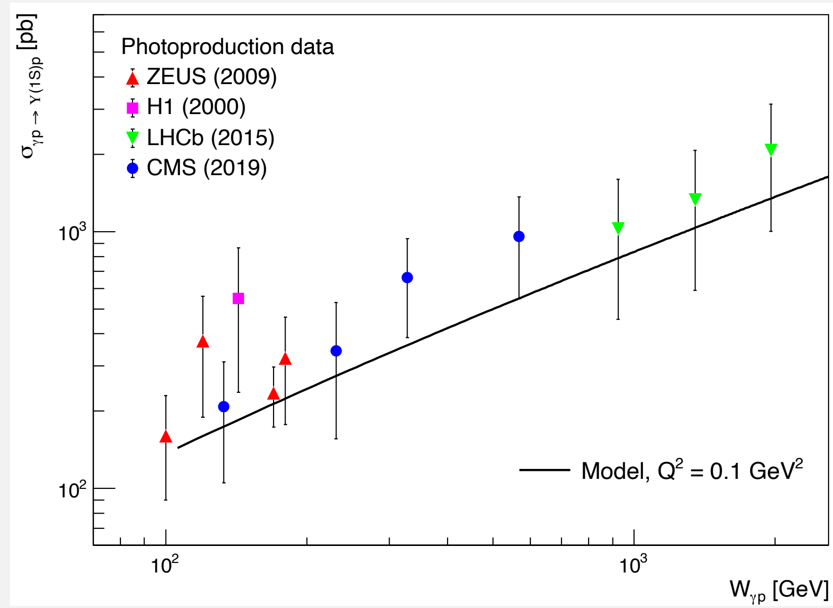
$\overline{\alpha}_s = \alpha_s \frac{N_c}{\pi}$ $\alpha_s = \alpha_s(r_{\min})$ $r_{\min} = \min(r_1, r_2, r)$ with C = 9.

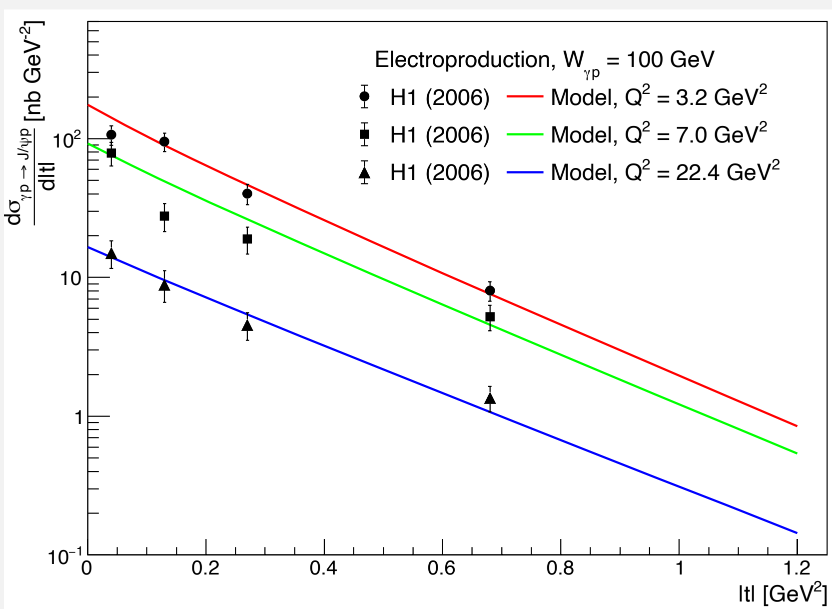
The factor in square brackets represents the contribution of single collinear logarithms and DLA term resums double collinear logarithms to all orders.

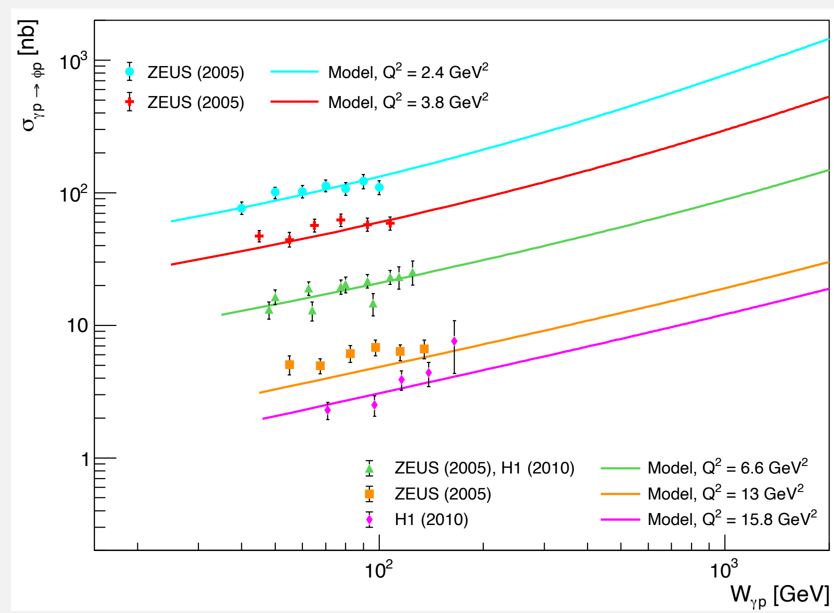
E. Iancu at al, Phys. Let. B. (2015) 1507.03651

RESULTS









D. Bendova, J. Cepila, J. G. Contreras, M. Matas; Phys. Rev. D 100, 054015