Jets and strong coupling parameter

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- QCD Lagrangian
- (running) coupling constant
- jets vs. partons
- equation of re-normalization group
- α_s measurements
  - one scale measurements
  - inclusive measurements
- summary

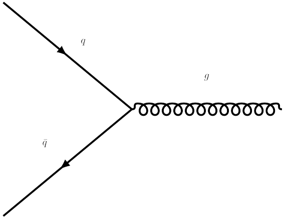
QCD Lagrangian

$$\mathcal{L}_{QCD} = \sum_q \left[i\bar{\psi}_q \gamma^\mu \left(\partial_\mu - ig_s \frac{\lambda^a}{2} A^a_\mu(x) \right) \psi_q - m_q \bar{\psi}_q \psi_q \right] + \mathcal{L}_{gauge}$$

- summing over quarks q
- quark - dirac field ψ_q
- gluon - vector field A^a_μ, $a \in \hat{8}$
- non-abelian color group $SU(3)$ - Gellman matrices λ^a

two kind of parameters:

- quark mass m_q
- color charge g_s



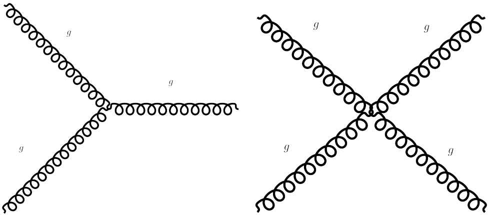
QCD Lagrangian continues

$$\mathcal{L}_{gauge} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

$$= -\frac{1}{4} A^a_{\mu\nu} A^{a\mu\nu} - \frac{1}{2} g_s f^{abc} (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) A^{b\mu} A^{c\nu} - \frac{1}{4} g_s^2 f^{abc} f^{ajk} A^b_\mu A^c_\nu A^{j\mu} A^{k\nu} \tag{1}$$

with abelian tensor field straight $A^a_{\mu\nu}$:

$$A^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$$

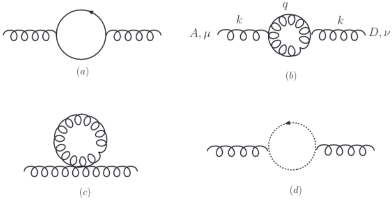


Coupling constant

- coupling constant relates closely with color charge g_s

$$\alpha_s = \frac{g_s^2}{4\pi}$$

- re-normalization procedure & Re-normatization Group Equation



rung coupling constant

$$\alpha(Q^2) = \frac{\alpha(Q_0^2)}{1 - B \cdot \alpha(Q_0^2) \ln \frac{Q^2}{Q_0^2}}$$

$$B_{QED} = \frac{2}{3\pi}, \qquad B_{QCD} = -\frac{11N_c - 2N_f}{6\pi}.$$

- α_s is small enough at high energy momentum transfers Q
  $\rightarrow$ asymptotic freedom
  $\rightarrow$ perturbation theory is applicable also above at TeV scale
- world average value is currently

$$\alpha_s(m_Z) = 0.1181 \pm 0.0011$$

Jets

- however quarks are not observed in detector directly
- we observe secondary produced hadrons which form jets
  - carry information about quarks and gluons
  - but be careful - jet is not quark or gluon
    $\rightarrow$ Jet physic
    $\rightarrow$ parton vs. jet

from experimental point of view:

- how find jet $\rightarrow$ jet algorithms and input parameters
- jet simulation, calibration ...
- jet tagging

from phenomenology point of view:

- parton distribution functions
- hadronization models
- etc.

How to evaluate α_s at e^+e^- colliders vol. 1

- ratio cross sections etc. reduces amount of systematic unc.
  - e^-e^+ colliders at $\sqrt{s} = M_X$:
    - α_s measured at the scale $Q = M_X$, $X \in \{Z^0, \tau, ...\}$

$$R_\tau = \frac{\Gamma(\tau \to hadrons + \nu_\tau)}{\Gamma(\tau \to e^- + \nu_e + \nu_\tau)}$$

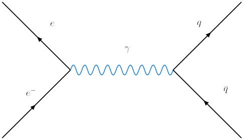
$$R_Z = \frac{\Gamma(Z^0 \to hadrons)}{\Gamma(Z^0 \to e^- + e^+)}$$

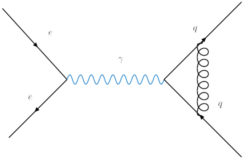
How to evaluate α_s at e^+e^- colliders vol. 2

- e^+e^- colliders at any energy:
  - inclusive measurements of α_s at various scale
  - mostly jets ration cross section measurements

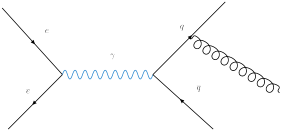
$$R_{32} = \frac{\sigma(3jets)}{\sigma(2jets)} = \frac{\alpha_s}{\pi}$$

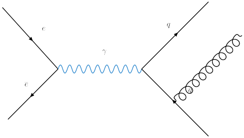
$$\sigma_0 \ldots e^- e^+ \rightarrow 2jets$$





$$\sigma \ldots e^- e^+ \rightarrow 3jets$$





Event shape variables

motivation for event shape variables:

- strong coupling constant α_s
- test asymptotic freedom
- constrain color factors for quark and gluon couplings

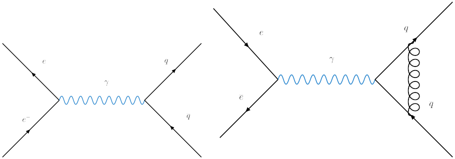
an example of event shape variable: thrust T na sphericity S

$$T = \max_{\vec{n}_T} \frac{\sum_i |\vec{p}_i \vec{n}_T|}{\sum_i |\vec{p}_i|}$$

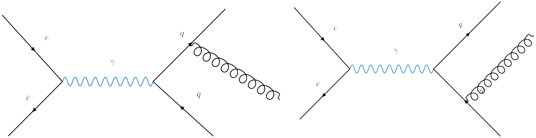
- $\vec{p}_i$ ... momentum of i-th jet
- $\vec{n}_T$ ... thrust axis ... unit vector in a direction of the most energetic jet
- $T = 1$ for exclusive dijet event
- $T = \frac{1}{2}$ for spherical event

Event shape - Thrust - calculation $e^- e^+ \rightarrow jets$

$\sigma_0 \ldots e^- e^+ \rightarrow 2jets$



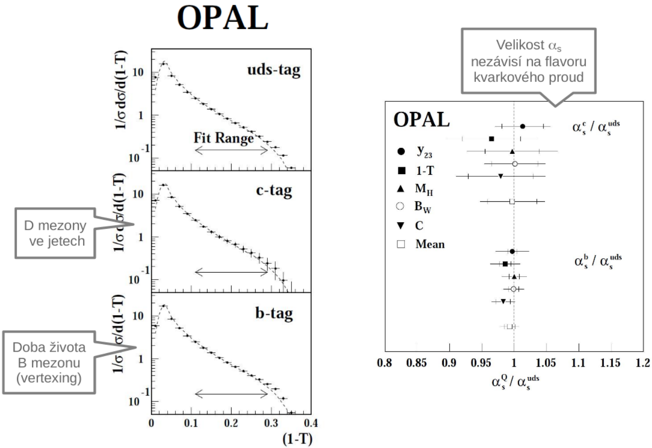
$\sigma \ldots e^- e^+ \rightarrow 3jets$



$$\frac{1}{\sigma_0}\frac{d\sigma}{dT} = C_F \frac{\alpha_s}{2\pi}\left[\frac{2(3T^2 - 3T + 2)}{T(1-T)} \ln \frac{2T-1}{1-T} - \frac{3(3T-2)(2-T)}{1-T}\right]$$

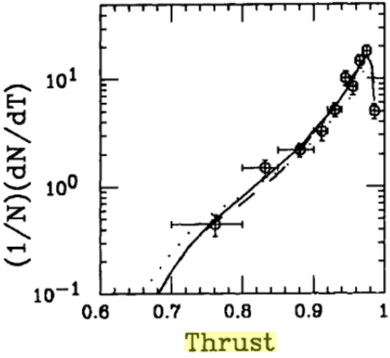
Event shape - Thrust and α_s at OPAL experiment

- α_s does not depend on flavor



slide from Martin Spousta MFF
lecture of *Strong interactions at high energies*

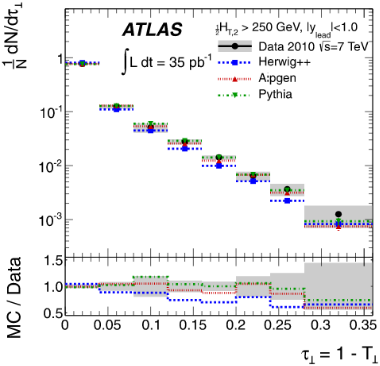
Event shape - Thrust at ALEPH and ATLAS



ALEPH experiment at LEP, e^+e^- collisions
Proceedings of The Rice Meeting vol.1
1990 Meeting of the Division of Particles and Fields of American Physical Society

Properties of Hadronic Events in e^+e^- Annihilation at

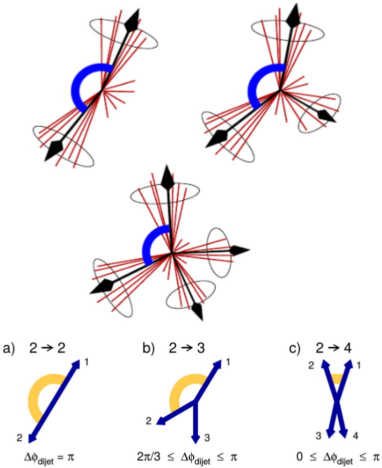
$\sqrt{s} = 91$ GeV

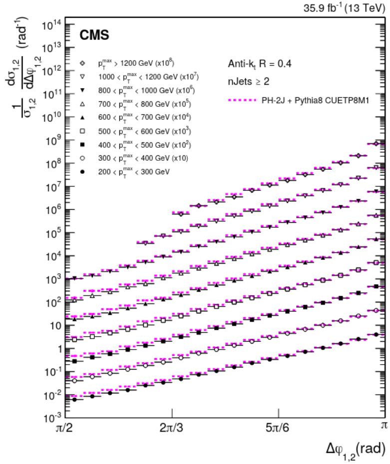


ATLAS experiment at LHC, *pp* collisions

Eur. Phys. J. C (2012) 72:2211

Azimuth angle measurement





inclusive dijet cross section differential in $\Delta\phi_{12}$ at CMS INSPIRE doi:10.22323/1.297.0167

Transverse energy-energy correlations TEEC and its asymmetry ATEEC

$$\frac{1}{\sigma}\frac{d\Sigma}{d(\cos\phi)} = \frac{1}{\sigma}\sum_{ij}\int \frac{d\sigma}{dx_{Ti}dx_{Tj}d(\cos\phi)} x_{Ti}x_{Tj}dx_{Ti}dx_{Tj}$$

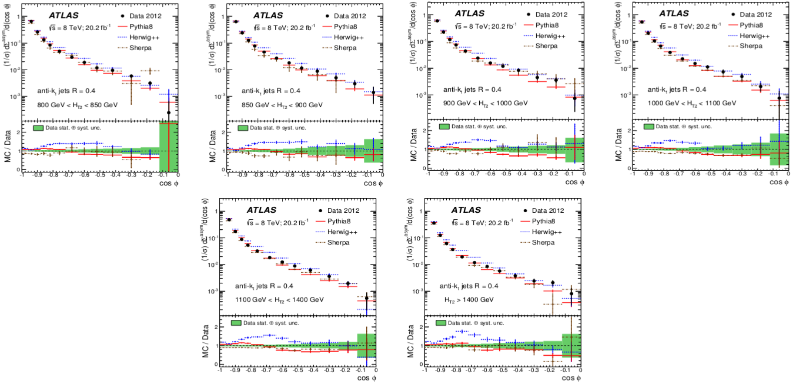
$$= \frac{1}{N}\sum_{A=1}^{N}\sum_{ij}\frac{E_{Ti}^{A}E_{Tj}^{A}}{(\sum_k E_{Tk}^{A})^2}\delta(\cos\phi - \cos\phi_{ij})$$

- A running over all N hard-scattering multi-jet events
- i and j run over all jets in event
- $x_{Ti} = \frac{E_{Ti}}{E_T}$ ...transverse energy of i-th jet
- $E_T = \sum_i E_{Ti}$ ...total transverse energy
- ϕ_{ij} ...azimuth angle between jets i and j

Asymmetry TEEC

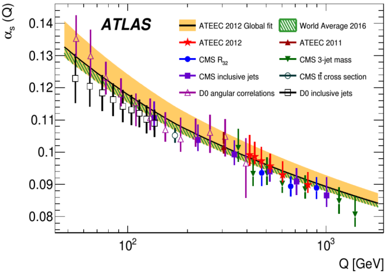
In order to cancel uncertainties which are constant over $\cos\phi \in (-1, 1)$, it is useful to define the azimuth asymmetry of the TEEC (ATEEC):

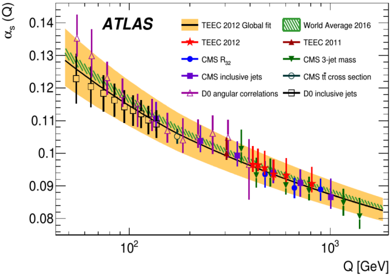
$$\frac{1}{\sigma}\frac{d\Sigma^{asym}}{d(\cos\phi)} = \frac{1}{\sigma}\frac{d\Sigma}{d(\cos\phi)}\bigg|_{\phi} - \frac{1}{\sigma}\frac{d\Sigma}{d(\cos\phi)}\bigg|_{\pi-\phi}$$



arXiv:1707.02562 [hep-ex]

α_s from TEEC and ATEEC





- χ^2 global fit 2016
- scale avaluated as $\frac{H_{T2}}{2}$

$$\alpha_s^{TEEC}(M_Z) = 0.1162 \pm 0.0011(exp.)^{+0.0076}_{-0.0061}(scale) \pm 0.0018(PDF) \pm 0.0003(NP)$$
$$\alpha_s^{ATEEC}(M_Z) = 0.1196 \pm 0.0013(exp.)^{+0.0061}_{-0.0013}(scale) \pm 0.0017(PDF) \pm 0.0004(NP)$$

arXiv:1707.02562 [hep-ex]

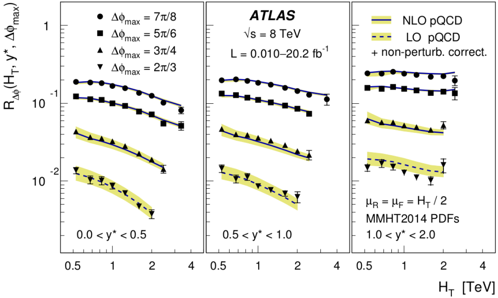
α_s form Azimuth decorrelations

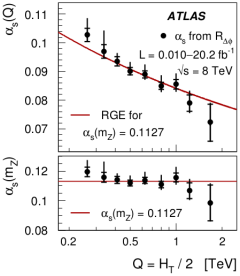
$$H_T = \sum_i p_{Ti}$$

$$R_{\Delta\phi(H_T, y^*, \Delta\phi_{max})} = \frac{\frac{d^2\sigma_{dijet}(\Delta\phi_{dijet} < \Delta\phi_{max})}{dH_T dy^*}}{\frac{d^2\sigma_{dijet}(inclusive)}{dH_T dy^*}}$$

$$y^* = \frac{1}{2}|y_1 - y_2|$$

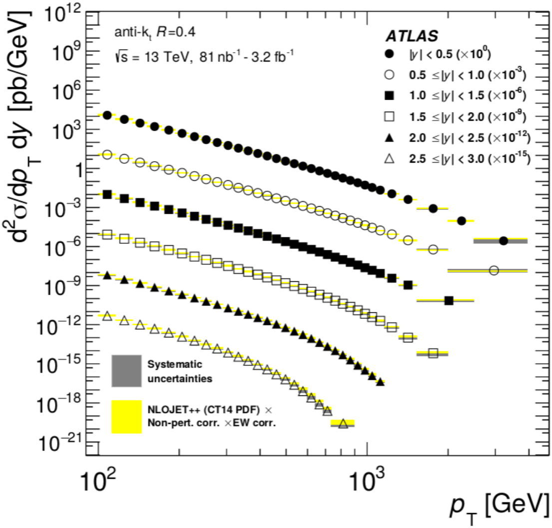
$$y_{boost} = \frac{1}{2}|y_1 + y_2|$$





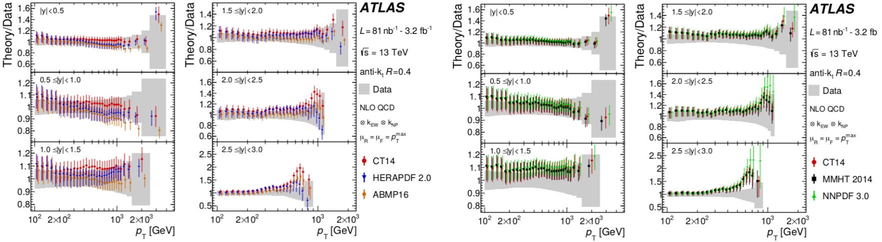
arXiv:1805.04691 [hep-ph].

inclusive jets measurement



arXiv:1711.02692 [hep-ex]

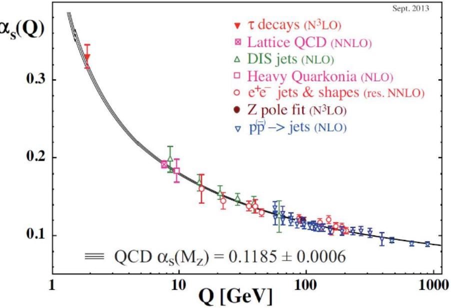
inclusive jets measurement

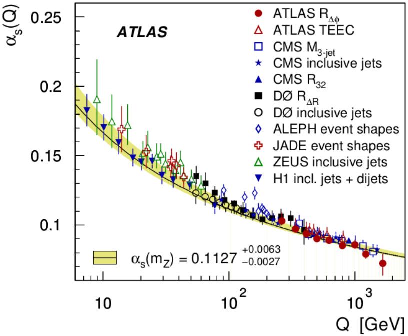


arXiv:1711.02692 [hep-ex]

- NLO pQCD predictions ratio predictions / data cross-sections
- different PDF sets:
  CT14, HERAPDF 2.0, ABMP16, MMHT 2014, NNPDF 3.0
- systematic (JES, JER, unfolding, jet cleaning, luminosity)

α_s measurements summary plots





dependence of α_s on the scale from arXiv:1805.04691 [hep-ph].

summary

- not each constant is thrully constant
- considering higher order of perturbative theory $\alpha \to \alpha(Q^2)$
- summary measurements of α_s
  - one Q^2 scale point
    - ratio of Γ
  - various Q^2 scale points - jet measurements
    - ratio of (differential) cross section
    - event shape variables, TEEC, ATEEC, Azimuth decorrelations ...
    - global fit based on χ^2

$$\alpha_s(m_Z) = 0.1181 \pm 0.0011$$

Thank you for your attention!

Infrared & collinear safety

- properties of ideal jet algorithm-suiteble theoretical properties, universality, reasonably fast, independence experiment, effectivity

Theoretical safety

For an observable distribution to be calculable in [fixed-order] perturbation theory, the observable should be *infra-red safe*, i.e. Insensitive to the emission of soft or collinear gluons. In particular if $\vec{p}_i$ is any momentum occurring in its definition, it must be invariant under the branching

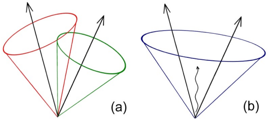
$$\vec{p}_i \longrightarrow \vec{p}_j + \vec{p}_k ,$$

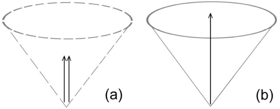
whenever $\vec{p}_j$ and $\vec{p}_k$ are parallel (collinear) or one of them is small (infrared).

gluon radiation cross section:

$$d\sigma_{q\to qg} = \frac{\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta}{sin\theta} \frac{d\phi}{2\pi}$$

- It diverges for $E \to 0$ - infrared (or soft) divergence
- It diverges for $\theta \to 0$ and $\theta \to \pi$ - collinear divergence





General formula for clustering algorithms

p_t	transverse momentum
y	rapidity
ϕ	azimuth angle
d_{ij}	distance between object i and j
d_{iB}	distance between i and beam
R	radius
p	parameter identification

- jets are defined by jet algorithm and set of input parameters
- cone alg.- surround significant flows of particles by cone with radius R
- clustering alg.- cluster particles retrospectively during the QCD branching

$$d_{min} = min(d_{ij}, d_{iB})$$

$$d_{iB} = p_{t_i}^{2p}$$

$$d_{ij} = min(p_{t_i}^{2p}, p_{t_j}^{2p}) \cdot \frac{\Delta R_{ij}}{R}$$

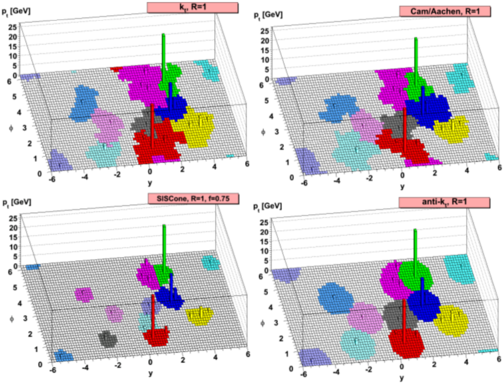
$$\Delta R_{ij} = \sqrt{(y_i - y_j)^2 + (\phi_i - \phi_j)^2}$$

1. find the smallest of d_{ij} and d_{iB}
2. if $d_{ij} < d_{iB} \rightarrow$ recombine them
3. if $d_{iB} < d_{ij} \rightarrow$ call i as a jet and remove it from the list of particles
4. repete it from step 1 until there are no particles

p	algorithm
1	k_t
0	Cambridge/Aachen (C./A.)
-1	anti-k_t

- k_t, anti-k_t: strong depends on p_t
  - anti-k_t: heavy and energetic particles are clustered first
  - k_t: soft particles are clustered first
- C./A.: $d_{ij} = \frac{\Delta R_{ij}}{R}$ & $d_{iB} = 1$
  - clustered until all $\Delta R_{ij} > R$

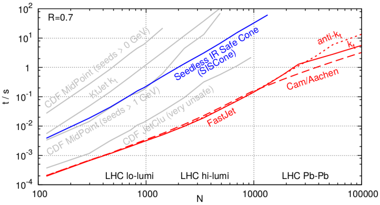
Properties of jet algorithms 1/2



anti-kt alg:

- $p = -1 \Rightarrow$ heavy particles are clustered first
- gives regular circular shapes $\Rightarrow$ convenient for jet energy calibration
- proved to be infra-red and collinear safe (IRC safety)
- reasonably fast
- currently the most widely use and safest jet algorithm

Properties of jet algorithms 2/2



- basic IR safety cone algorithms implemented in NLOJet provides "slozitost" $O(N2N)$, what is reasonable for $N < 4$, but e.q. for $N \sim 100$ means $10^{1}7$ year (Pb collisons)
- jet algorithms in FastJet package are based on Delaunay triangulatio of graph theory (mathematical CGAL package) to optimalize "nejlepsÃ slozitost"

název algoritmu	Typ algoritmu typ	p	IR safety	colinear safety	symetrický výstup	časová náročnost
k_t	klastrovací	1	✓	✓	✗	$N \ln N$
C./A.	klastrovací	0	✓	✓	✗	$N \ln N$
anti-k_t	klastrovací	-1	✓	✓	✓	$N^{3/2}$
SIS Cone	kuželový		✓	✗	✓	$N^2 \ln N$

RGE

$$\mu_R^2 \frac{d\alpha_s}{d\mu_R^2} = \beta(\alpha_s)$$

$$\beta(\alpha_s) = -(b_0 \alpha_s^2 + b_1 \alpha_s^3 + \ldots)$$