Attachment

Authorship of the article

Credits for the submitted work, titled "*Vortex solutions of Liouville equation and quasi spherical surfaces*", are shared by two authors: Bc. Pavel Kůs, student at the Faculty of Mathematics and Physics, Charles University, and doc. Alfredo Iorio, associated professor of theoretical physics at the same faculty. The article is based on the results of the bachelor thesis, titled "*Conformal symmetry and vortices in graphene*", defended by P. Kůs in June 2019 and supervised by doc. A. Iorio. The complete thesis is enclosed as an attachment, where the main points are described in chapter 4. Results of the article come from this chapter, pages 27-37. Graphical illustrations were created using Wolfram Mathematica by P. Kůs. Both authors contributed to the text.

This paper in a broader context

The article fits into the broader context of the so called *Analog gravity*. The idea is to replace a system of interest by another system, which shares key features with the original one and then *analog phenomena* can be predicted and *directly tested*.

Typical object the description of which requires both GR and QFT is a black hole, where the famous phenomenon *Hawking evaporation* is supposed to take place. However, if this effect were there, it would be completely overshadowed by the cosmic microwave background, since its temperature of 2.7 K is greater than the Hawking radiation temperature of astrophysically relevant black holes. This problem encouraged many physicists to search for *analog systems*, that could mimic environment of the black hole. In such environment a phenomenon analogous to the Hawking radiation may take place and be directly observable in lab facilities. (1)

Let us now turn to a specific important example of an analog: graphene. The dispersion relation $E(p)$ of low energy conductivity electrons is of the relativistic form:

$$E = v_F p, \tag{1}$$

where v_F is the Fermi velocity. Electrons behave like relativistic (massless) particles, which are described by a Dirac spinor ψ in 2+1 dimensional spacetime. 'Classical physics' is described by an action, which in the simplest case (flat spacetime, $\eta_{\mu\nu}$) is of the form:

$$A(\eta_{\mu\nu}, \psi) = i \int d^3x \bar{\psi} \gamma^a \partial_a \psi. \tag{2}$$

However, if the spacetime has an inner curvature, the action becomes:

$$A(g_{\mu\nu}, \psi_\Sigma) = i \int d^3x \sqrt{g} \bar{\psi}_\Sigma \gamma^a E_a^\mu D_\mu \psi_\Sigma, \tag{3}$$

where ψ_Σ denotes a classical field, μ is the Einstein (curved) index and a is the Minkowski (flat) index, $g_{\mu\nu}^{(3)}$ is a metric of the spacetime associated to a graphene membrane $\sqrt{g} \equiv \sqrt{|\det g_{\mu\nu}^{(3)}|}$, γ^a is a standard γ-matrix satisfing the Clifford algebra,

E_a^μ is an inverse Dreibein (mapping between flat and curved space), D_μ is a covariant derivative. Spinors ψ_Σ and ψ are related to each other in a specific way, which we will show in what follows. (2)

But what does it actually mean that a graphene membrane has an associated ”spacetime”, and what is its structure? The ”spacetime” is considered to be static and we assume the relevant metric to be flat in time. Its spatial part is assumed to be a surface's metric, described by laboratory coordinates (u, v). The spacetime is then understood as the product ”surface” $\times \mathbb{R}$:

$$g_{\mu\nu}^{(3)}(t,u,v) = \begin{pmatrix} 1 & 0 \\ 0 & -g_{ij}^{(2)}(u,v) \end{pmatrix}.$$

It turns out that the action (3) possess the Weyl symmetry. Particularly, for (2+1)-dim spacetimes Weyl related to the Minkowski spacetime, we understand the symmetry as $A(\eta_{\mu\nu}, \psi) = A(g_{\mu\nu}, \psi_\Sigma)$, where

$$g_{\mu\nu}(x) = e^{2\Sigma(x)}\eta_{\mu\nu}(x), \quad \psi_\Sigma(x) = e^{-\Sigma(x)}\psi(x). \tag{4}$$

That makes 'classical physics' of all Weyl related systems (graphene membranes with their Dirac fields) indistinguishable from a flat spacetime. In such case, measurable differences should be of quantum nature. (3)

This explains why conformally flat spacetimes are of particular importance in this scenario. In covariant form, the spacetime of 2+1 dim is conformally flat if and only if its Cotton tensor vanishes. Parametrizing the surface by so called *isothermal coordinates* $(\tilde{x}, \tilde{y})$, the line element takes the form:

$$dl^2 = e^{2\sigma(\tilde{x},\tilde{y})}(d\tilde{x}^2 + d\tilde{y}^2). \tag{5}$$

In these coordinates it is easier to see that the vanishing of the Cotton tensor gives a constrain on the spatial part of the metric $g_{\mu\nu}^{(3)}$, i.e. the surface:

$$\Delta_{(\tilde{x},\tilde{y})}\sigma(\tilde{x},\tilde{y}) = -Ke^{2\sigma(\tilde{x},\tilde{y})}, \tag{6}$$

where $\Delta_{(\tilde{x},\tilde{y})} = \partial_{\tilde{x}}^2 + \partial_{\tilde{y}}^2$ and K is a *constant Gaussian curvature*. This can be recognised to be the famous *Liouville equation*, important equation of mathematical physics, and its validity suggests that relevant surfaces, or graphene membranes, are those with constant Gaussian curvature. (2)

In the submitted paper we present the solution of a problem encountered in (2). Namely, we found here the coordinate transformations from the isothermal abstract coordinates to the laboratory cartesian coordinates for a specific and potentially interesting set of solutions of the Liouville equation called ”non-topological vortices”.

Besides being a challenging mathematical problem (solution of a system of partial differential equations), it is also relevant for future experimental set-ups. Indeed, knowing the cartesian coordinates, we know the actual shape of the membrane those vortices correspond to. Hence, their laboratory realization is, in principle, doable.

Bibliography

[1] Iorio, Alfredo Curved Spacetimes and Curved Graphene: a status report of the Weyl-symmetry approach. Int. J. Mod. Phys. D 4 (2015)

[2] Iorio, Alfredo. Weyl-Gauge Symmetry of Graphene. Annals Phys.326:1334-1353,2011.

[3] Iorio, Alfredo. Pais, Pablo. Revisiting the gauge fields of strained graphene Phys. Rev. D 92, 125005 (2015)