Real Options Valuation: A Dynamic Programming Approach

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    - Optimal strategy, maximizing the reward function

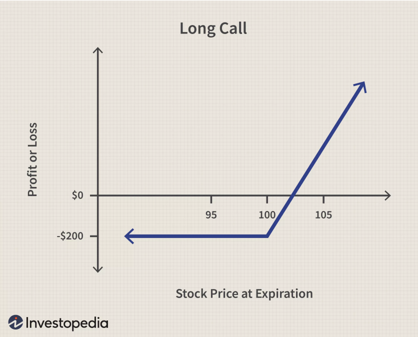

Figure: Call Option [1]

Introduction
Detailed

- Three classes of ROA authors (BSM model analogy)
  - Full analogy - limited to very simple cases
  - Partial analogy - CAPM
  - No analogy
- SDT framework > frameworks used in economy
- Formulation of ROA problem in SDT framework

Table 7.1. Analysis of the Option to Invest in a Project when Construction Takes One Period

$X(i,n)$	0	1	2	3	4	5
0	100.00	125.00	156.25	195.31	244.14	305.18
1		80.00	100.00	125.00	156.25	195.31
2			64.00	80.00	100.00	125.00
3				51.20	64.00	80.00
4					40.96	51.20
5						32.77

Step 1: Construct binomial tree for the state variable

$V_b(i,n)$	0	1	2	3	4	5
0	1.88	7.82	31.93	65.41	107.26	0.00
1		0.30	1.43	6.76	31.93	0.00
2			0.00	0.00	0.00	0.00
3				0.00	0.00	0.00
4					0.00	0.00
5						0.00

Step 2: Fill in final column using equation (7.1)

Step 3: Fill in remaining columns using equation (7.2)

Figure: Economical framework example [2]

ROA inspiration

Graeme Guthrie

- Replicating portfolio (no arbitrage principle)
- Risk neutral probabilities
- Backward induction
- Probability of up and down move [2]:

$$\pi_u = \frac{ZR_f - X_d}{X_u - X_d}, \pi_d = \frac{X_u - ZR_f}{X_u - X_d}, \tag{1}$$

where

$$Z = \frac{E[\tilde{X}] - (E[\tilde{R_m}] - R_f)(\frac{Cov[\tilde{X}, \tilde{R_m}]}{Var[\tilde{R_m}]})}{R_f} \tag{2}$$

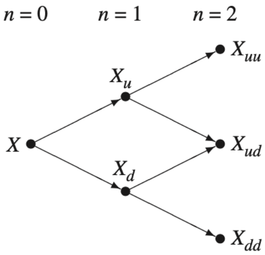

Figure: Binomial model [2]

ROA limitations

General

- Limited number of uncertainty sources - one (no arbitrage principle)
- Simple distributions - binomial model
- Limited by computational complexity for higher-dimensional problems
- Complicated scaling in types and number of real options

- Allows for multiple sources of uncertainty - seamless integration
- Allows continuous distributions, theoretically of any type
- Computational complexity tools - ADP
- Real options (actions) easily scaled with action set

Furthermore:

- Allows for simple integration of Bayesian learning
- Allows for complex creation of prior probability densities

- Time value of money - discounting factors
- Risk aversion of investors - utility theory
- Risk-neutral probabilities - Bayesian priors

SDT Improvements
ADP

- Value iteration, value function approximation
- Value function modeled as linear model with interactions up to second degree
- Solves the problem of uncountable state space: i.e. for Gas power plant example:

$$|\mathbb{R}| \times |\mathbb{R}| \times |\mathbb{R}| \times 5 \times 3 \times 3 \times 241 \times 241 \quad (3)$$

Valuation of gas power plant

General Idea

- Power generating company
- In the next 5 years, possibility to build 200MW or 400MW gas unit for 65/130M EUR
- Prices of gas, power, CO2 allowances are 24EUR, 9EUR, 40EUR per MWh
- Government policy favoring renewables can rise → higher volatility of prices
- Lifespan of gas power plant is set to 25 years, loan possible with 3% interest rate
- Power plant is selling its power as monthly contracts

Valuation of gas power plant

Details

Considered real options:

- Wait with construction for better prices - Timing option
- Build 200MW/400MW gas power plant - Scaling option
- Sell the power plant - Abandonment option
- Ability to not run the power plant - Switching option
- Mothball the plant. No production, lower fixed costs

Sources of uncertainty:

- Price of gas, power and CO2 - lognormal processes
- Government policy for renewables (1-5) - discrete with positive mean

Results

- To be computed

Conclusion

The contributions of my thesis are

- Summary of ROA state of the art
- Identification of the core ROA knowledge
- Formulation of ROA problem in SDT framework
- Identification of a useful ADP algorithm
- More robust tools allowing: more sources of uncertainty, more possible and complicated actions, complicated distributions, high-dimensional problems and Bayesian learning.

Possible directions of future research:

- Real data application
- Deeper study of replicating portfolios
- Use case in IT development
- More complicated model of free cash flow
- Continuous time

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