LO Balitsky-Kovchegov equation

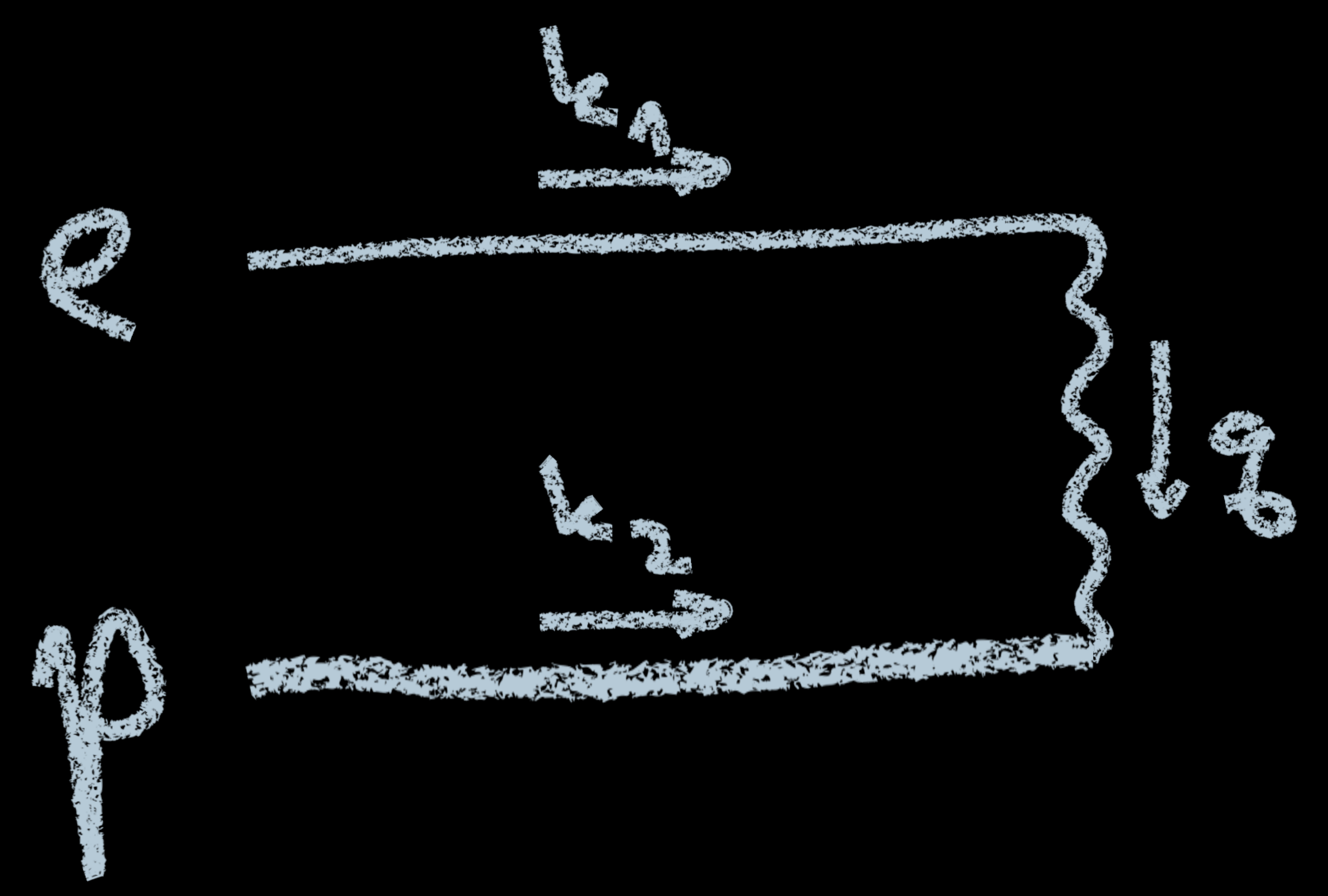
CGC
CDM

high-E QCD

DIS observables

Matěj Vaculčiak, 14.9.2021

Electron-proton scattering

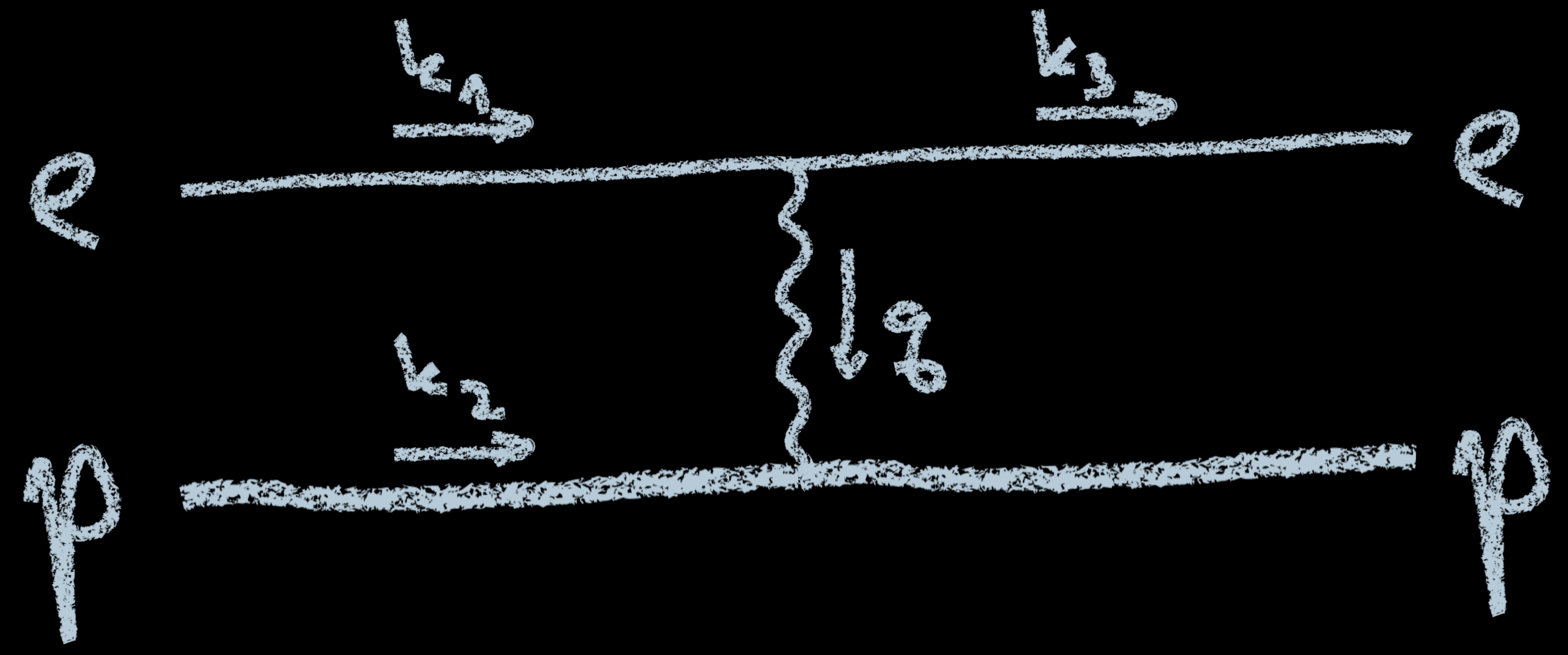


Electron-proton scattering
elastic scattering

$y := 1 - \frac{E_3}{E_1}$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{m_p^2 y^2}{Q^2}\right) f_2(Q^2) + \frac{1}{2} y^2 f_1(Q^2)\right]$$

$Q^2 := -q^2$

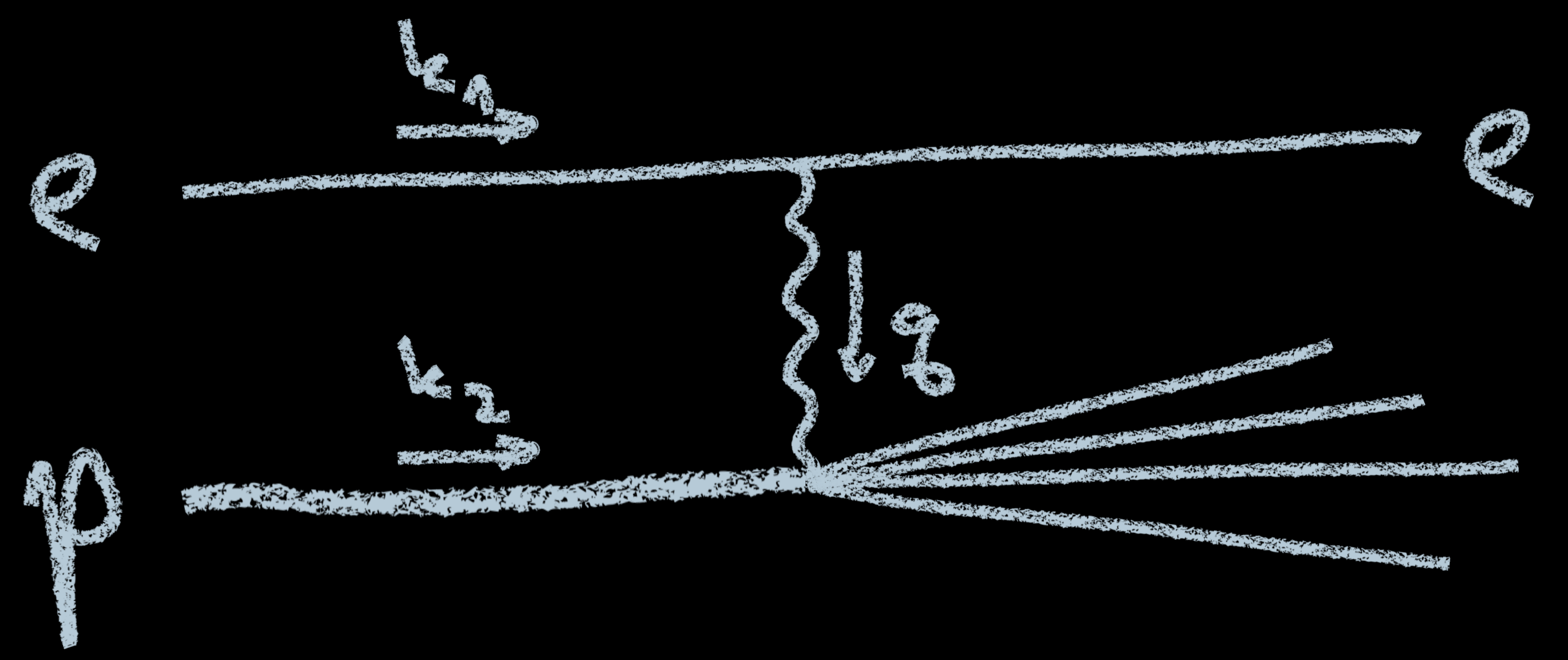


Electron-proton scattering

inelastic scattering

$\frac{d^2\sigma}{dQ^2 dx}$ $\qquad$ $\frac{F_2(x,Q^2)}{x}$ $\qquad$ $2F_1(x,Q^2)$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{m_p^2 y^2}{Q^2}\right) f_2(Q^2) + \frac{1}{2} y^2 f_1(Q^2)\right]$$

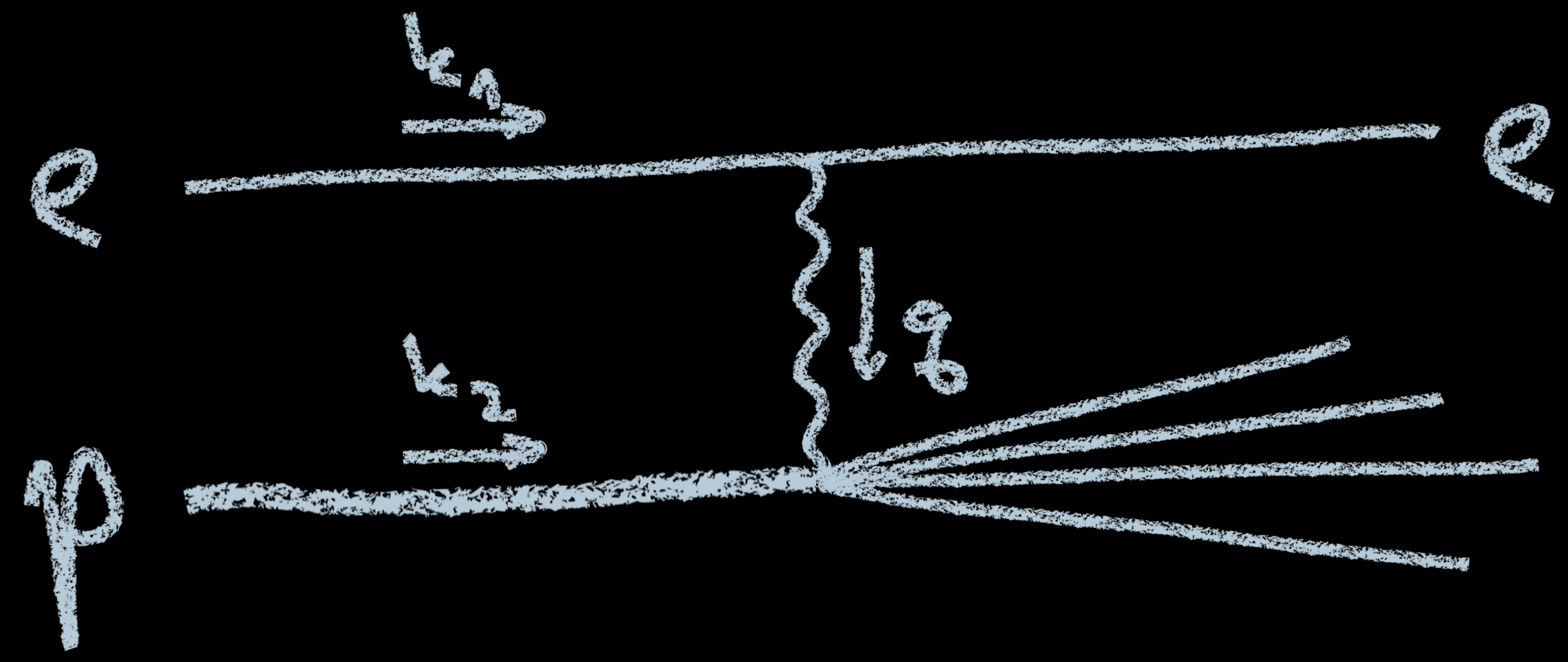


Electron-proton scattering

inelastic scattering

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}Q^2\mathrm{d}x} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{m_p^2 y^2}{Q^2}\right)\frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2)\right]$$

Bjorken x



$$\Downarrow$$

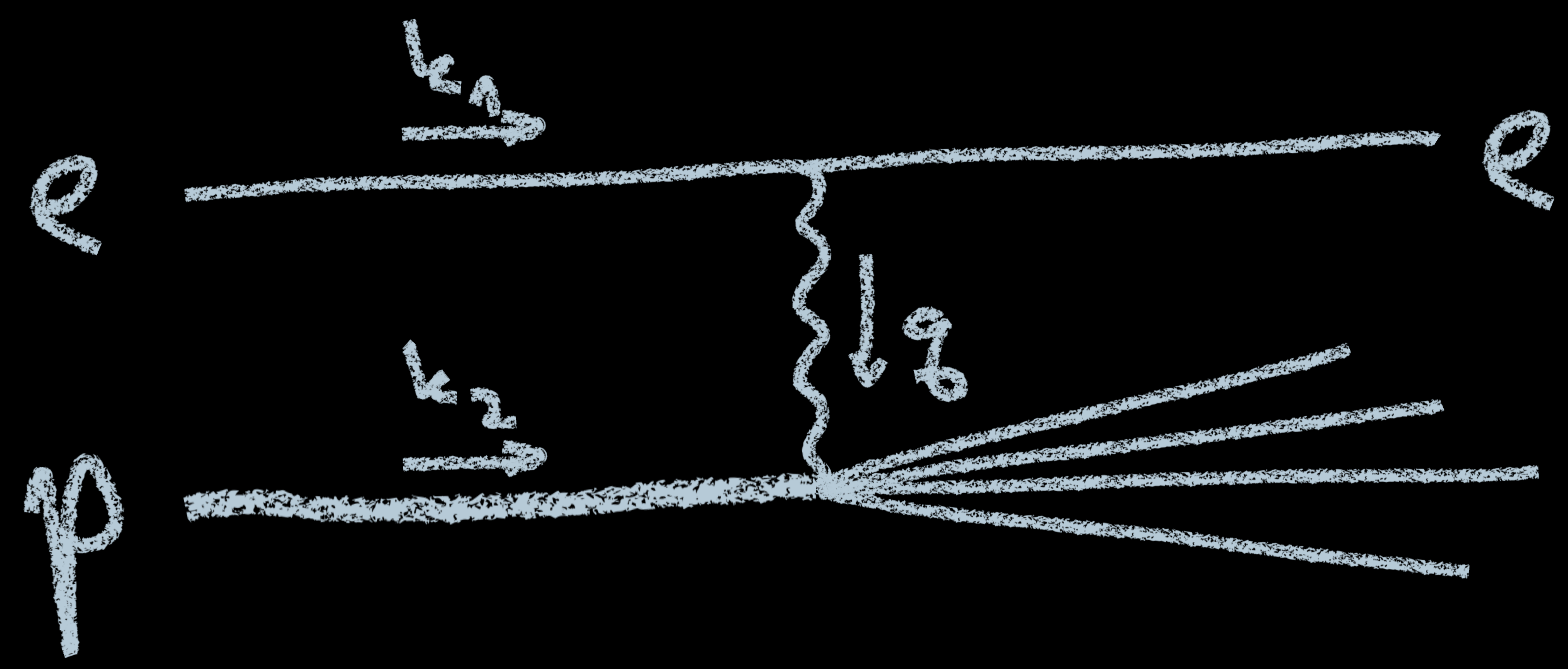
$$k^\mu_d = x\, k^\mu_p$$

Electron-proton scattering

deep inelastic scattering

$\rightarrow 0$

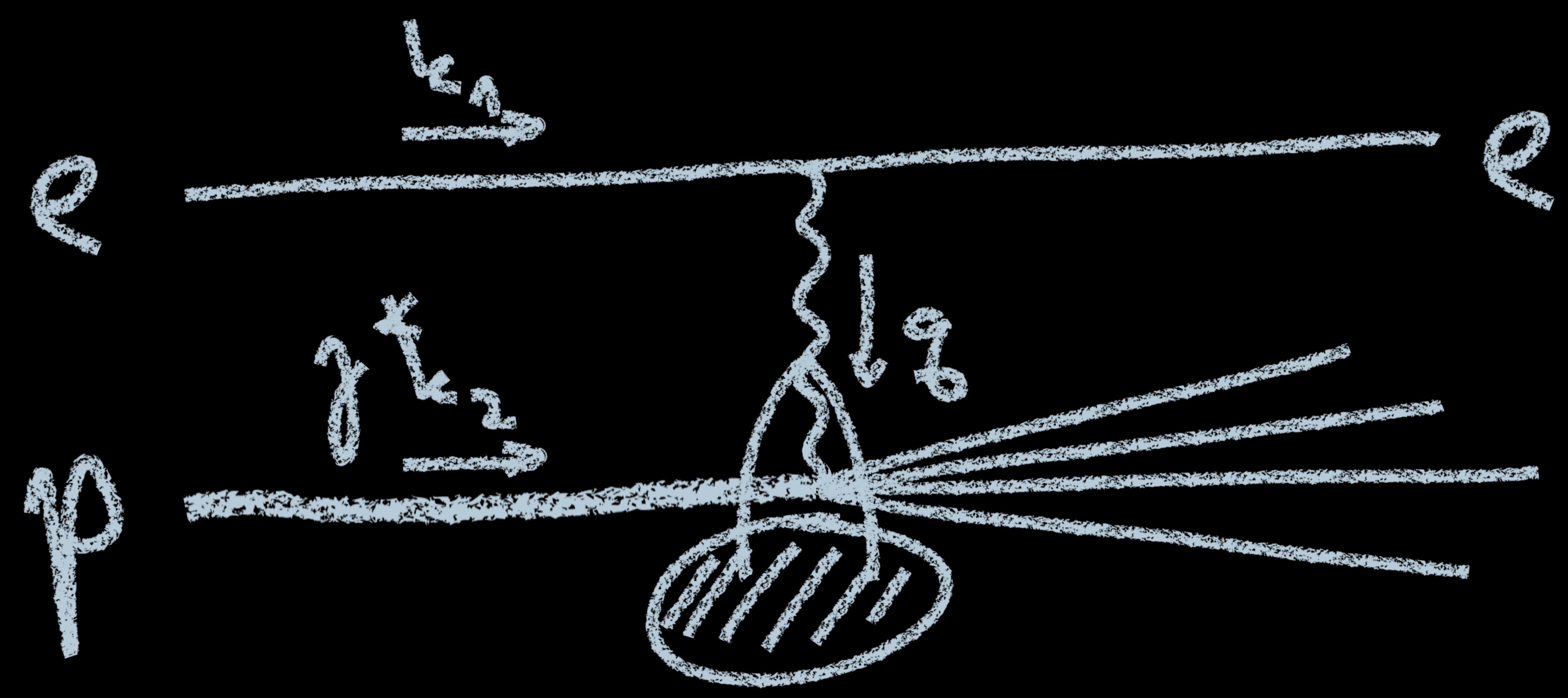
$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}Q^2\mathrm{d}x} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{m_p^2 y^2}{Q^2}\right)\frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right]$$



Electron-proton scattering

deep inelastic scattering

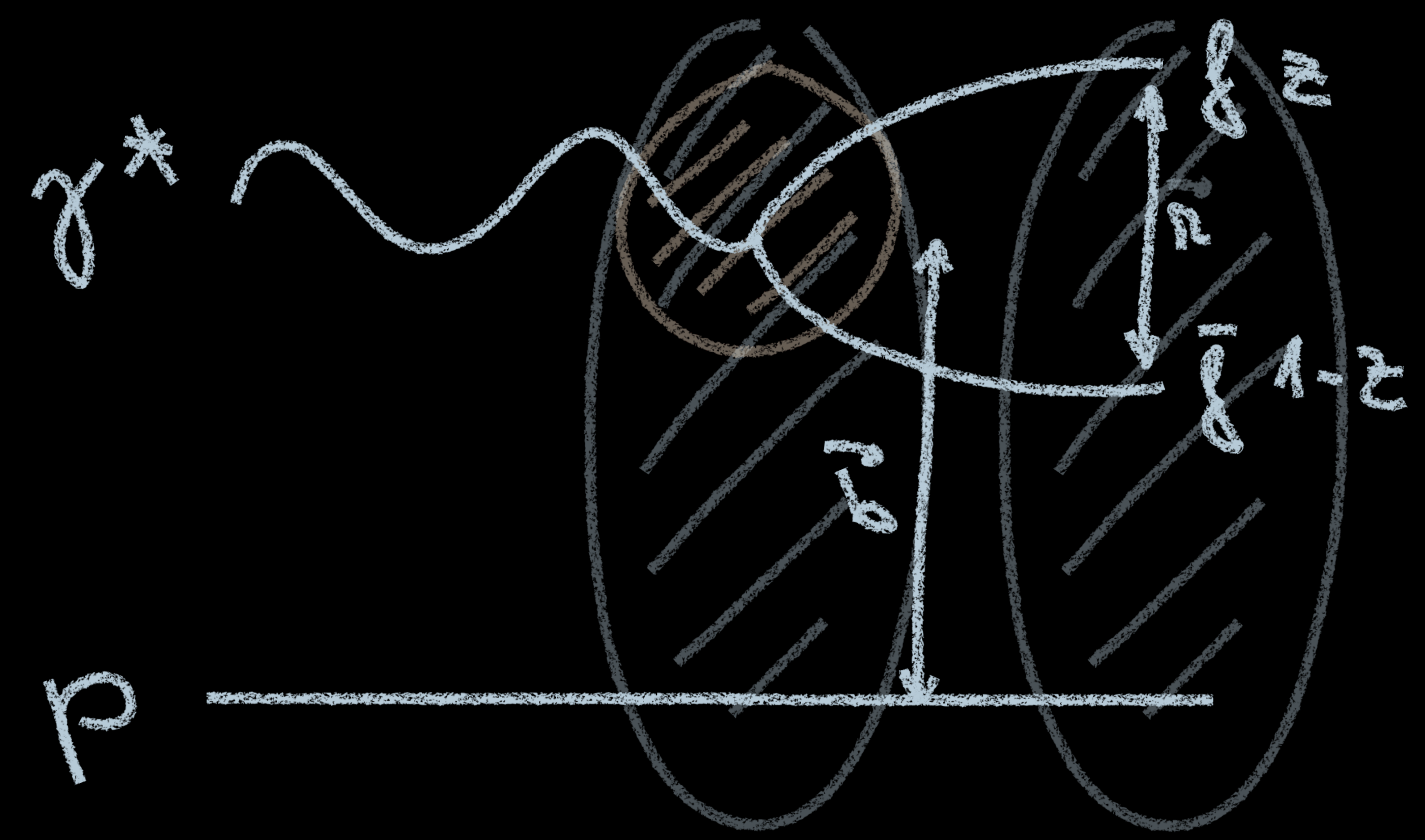
$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}Q^2\mathrm{d}x} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1-y\right)\frac{F_2(x,Q^2)}{x} + y^2F_1(x,Q^2)\right]$$



Dipole scattering amplitude

$$F_2(x, Q^2) = \frac{Q^2}{4\pi\alpha_{em}}\left(\sigma_L^{\gamma^* p}\left(x, Q^2\right) + \sigma_T^{\gamma^* p}\left(x, Q^2\right)\right)$$

$$F_L(x, Q^2) = \frac{Q^2}{4\pi\alpha_{em}}\sigma_L^{\gamma^* p}\left(x, Q^2\right)$$



- the photon-proton cross-section

$$\sigma_{L,T}^{\gamma^* p}(x, Q^2) = \sum_f \int \mathrm{d}^2\vec{r} \int_0^1 \mathrm{d}z \,|\, \psi_{T,L}^{(f)}(\vec{r}, Q^2, z)\,|^2\, 2\int \mathrm{d}^2\overrightarrow{b}\, N(\vec{r}, \overrightarrow{b}, \tilde{x}_f(x))$$

$\tilde{x}_f = x\left(1 + \frac{4m_f^2}{Q^2}\right)$

LO Balitsky-Kovchegov equation

dipole scattering amplitude

$$\frac{\partial N(r,b,Y)}{\partial Y} = \int \mathrm{d}\vec{r}_1 K(r,r_1,r_2) \left[N(r_1,b_1,Y) + N(r_2,b_2,Y) - N(r,b,Y) - N(r_1,b_1,Y)N(r_2,b_2,Y) \right]$$

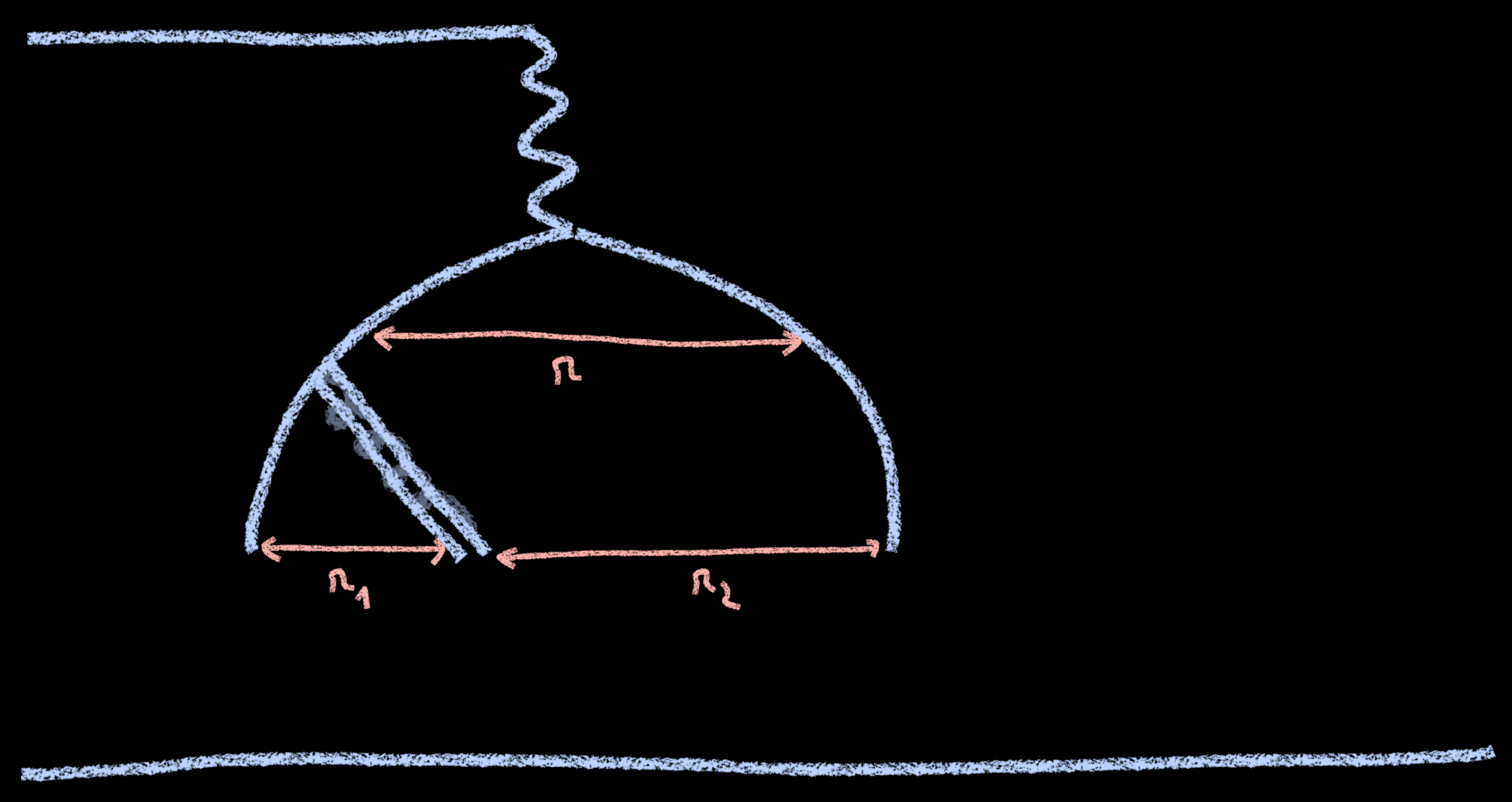
$Y = \ln \frac{x_0}{x}$

kernel

$$K = \frac{\alpha_s(r) N_c}{2\pi^2}\left[\frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_1^2}\left(\frac{\alpha_s(r_1)}{\alpha_s(r_2)} - 1\right) + \frac{1}{r_2^2}\left(\frac{\alpha_s(r_2)}{\alpha_s(r_1)} - 1\right)\right]$$

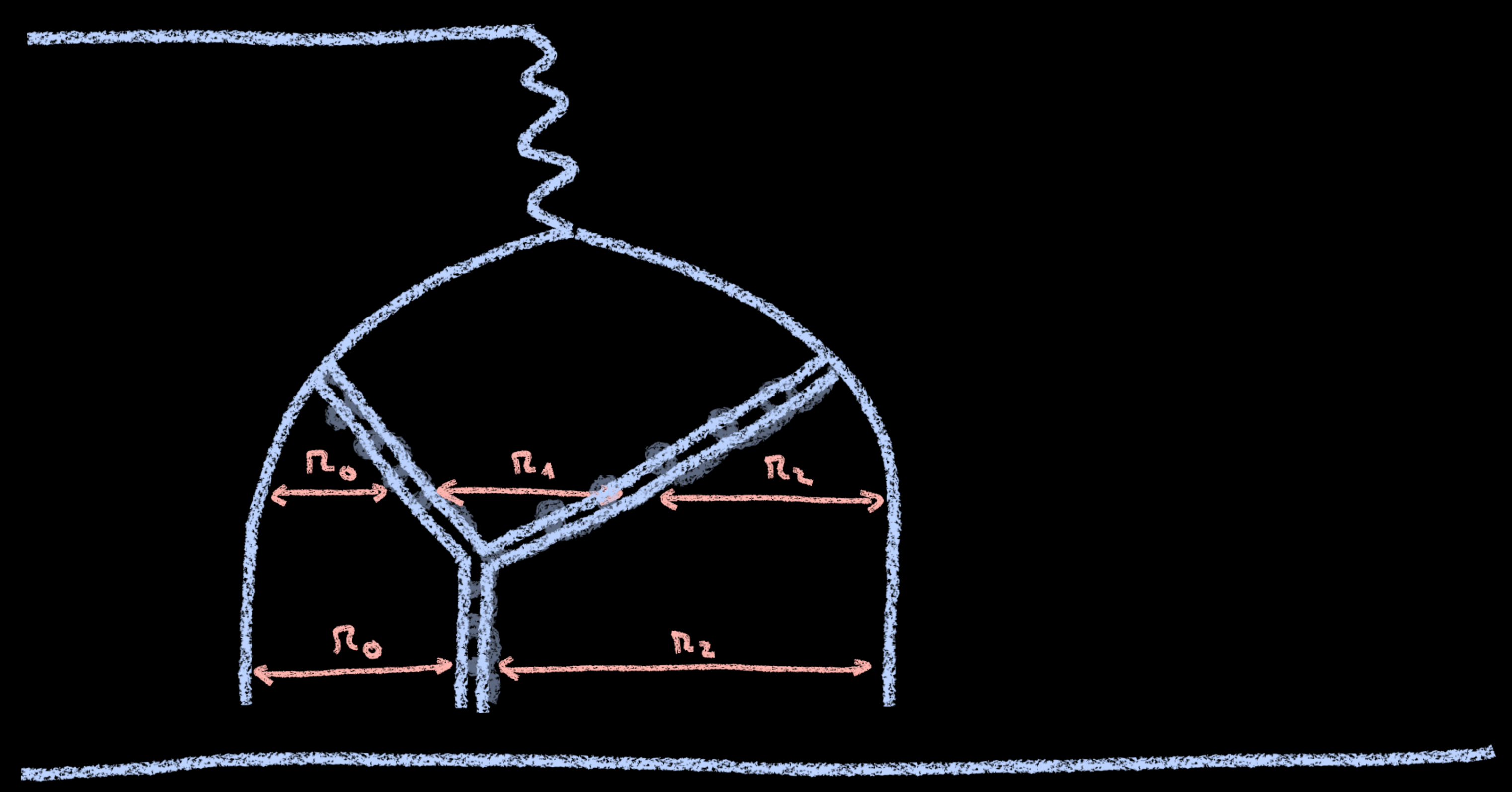
LO Balitsky-Kovchegov equation

$$\frac{\partial N(r,b,Y)}{\partial Y} = \int \mathrm{d}\vec{r}_1 K(r,r_1,r_2) \; \left[N(r_1,b_1,Y) + N(r_2,b_2,Y) - N(r,b,Y) - N(r_1,b_1,Y)N(r_2,b_2,Y)\right]$$



LO Balitsky-Kovchegov equation

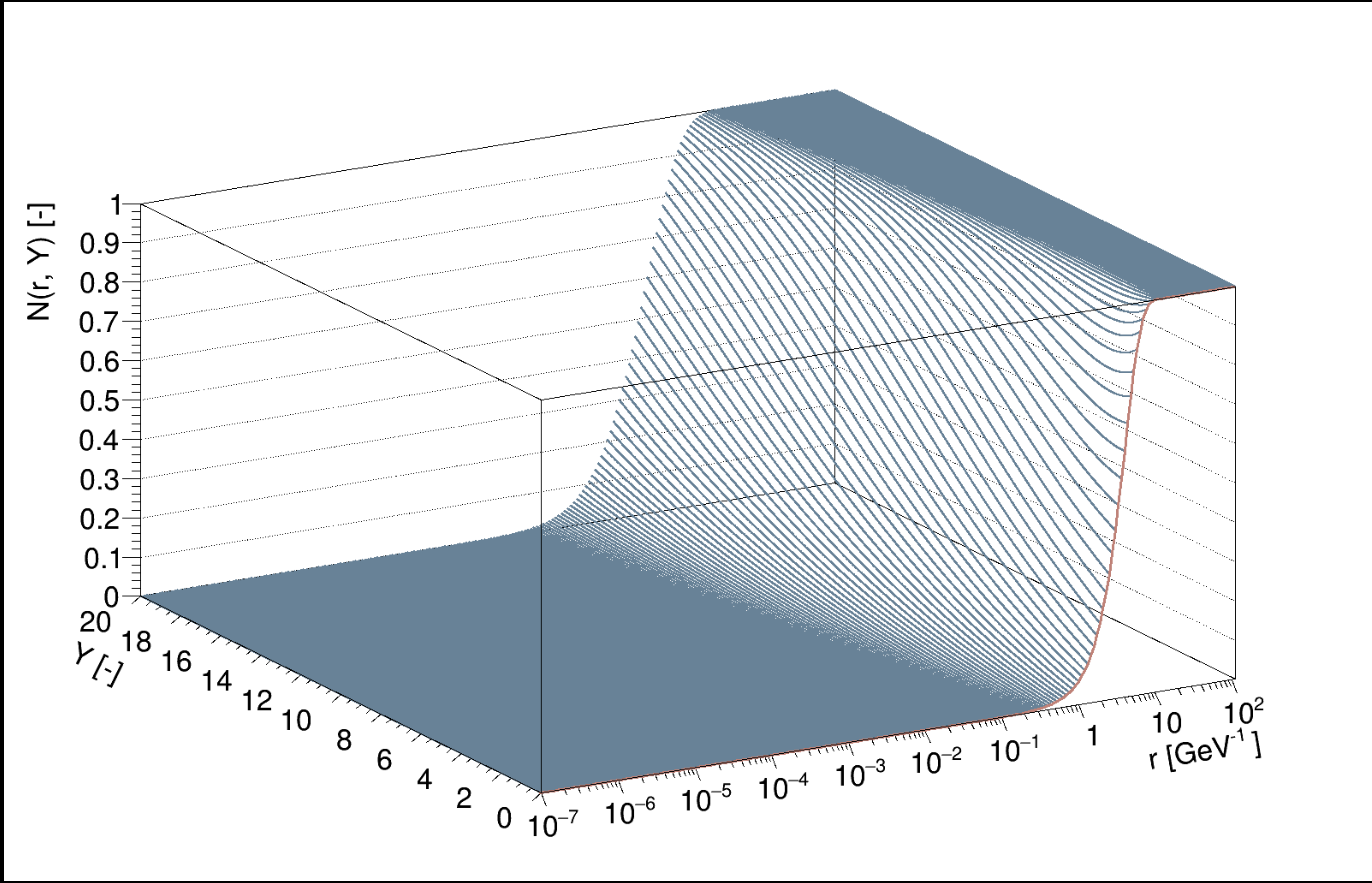
$$\frac{\partial N(r,b,Y)}{\partial Y} = \int \mathrm{d}\vec{r}_1 K(r,r_1,r_2) \;\; \left[N(r_1,b_1,Y) + N(r_2,b_2,Y) - N(r,b,Y) - N(r_1,b_1,Y)N(r_2,b_2,Y)\right]$$



1D solution

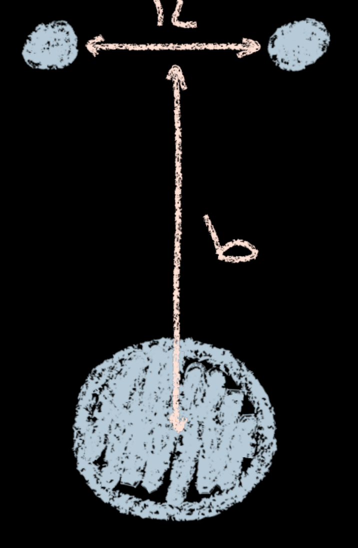
$$2\int \mathrm{d}\overrightarrow{b}\, N(\underbrace{\vec{r}, \overrightarrow{b}}_{\text{4 dim}}, Y) \approx \sigma_0 N(r, Y)$$

1D solution

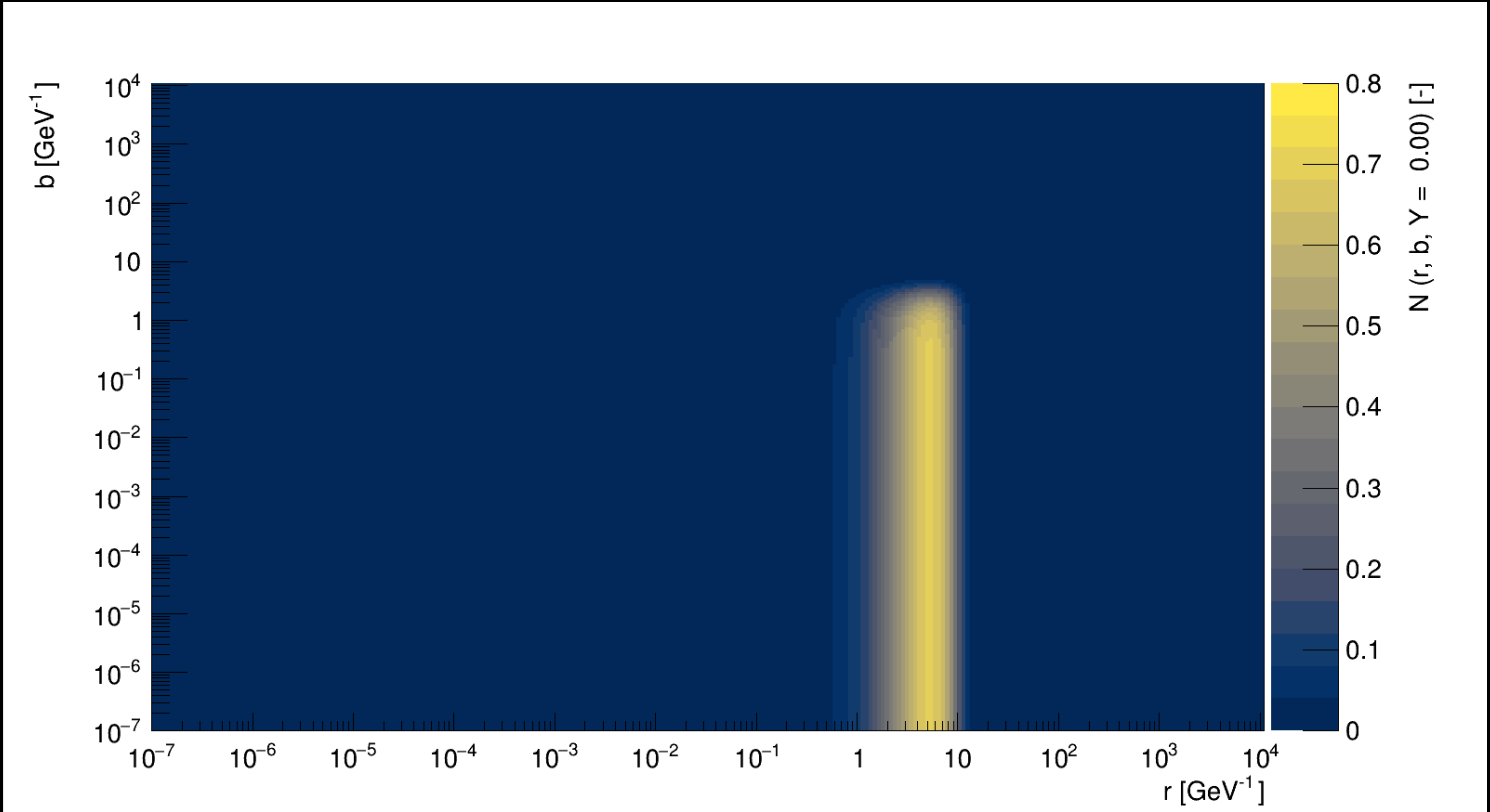


2D solution

$$2\int \mathrm{d}\overrightarrow{b}\,N(\vec{r},\overrightarrow{b},Y) \approx 4\pi\int \mathrm{d}b N(r,b,Y)$$



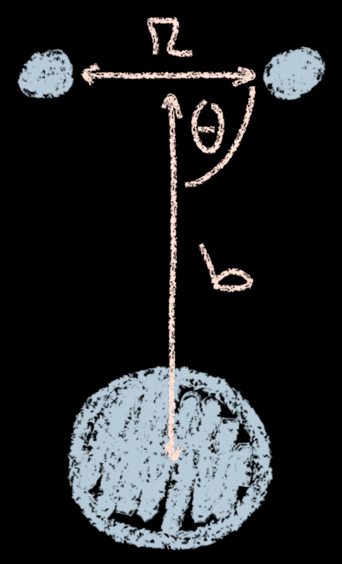
2D solution



b [GeV^{-1}]
10^4
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 10 10^2 10^3 10^4
r [GeV^{-1}]
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
N (r, b, Y = 0.00) [-]

Further steps

- calculating observables from the 2D solution
  - fitting necessary parameters
  - vector meson observables
- calculating the 3D solution
- calculating the 4D solution
- implementing higher order corrections



Thank you for your attention