Real Options Valuation: A Dynamic Programming Approach

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- Project valuation problem
- Project - alternation or creation of a process
- Real option analysis (ROA)
  - Economic framework
  - Highly uncertain environments
  - Inspired by financial option valuation theory (BSM)
  - Financial options

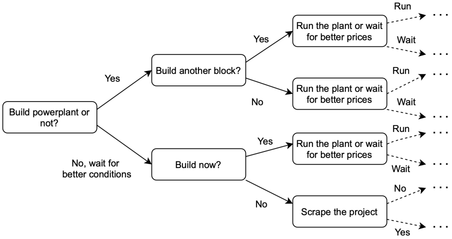

Figure: Illustration of DM problem - gas powerplant valuation.

Introduction
SDT

- Stochastic decision theory (SDT)
  - 3 sets - **T**, **S**, **A**, 2 functions - p and r
  - Optimal strategy, maximizing the cumulative reward
- SDT framework for DM > frameworks used in economy
- Formulation of ROA problem in SDT framework

Table 7.1. Analysis of the Option to Invest in a Project when Construction Takes One Period

$X(i,n)$	0	1	2	3	4	5
0	100.00	125.00	156.25	195.31	244.14	305.18
1		80.00	100.00	125.00	156.25	195.31
2			64.00	80.00	100.00	125.00
3				51.20	64.00	80.00
4					40.96	51.20
5						32.77

Step 1: Construct binomial tree for the state variable

$V_b(i,n)$	0	1	2	3	4	5
0	1.88	7.82	31.93	65.41	107.26	0.00
1		0.30	1.43	6.76	31.93	0.00
2			0.00	0.00	0.00	0.00
3				0.00	0.00	0.00
4					0.00	0.00
5						0.00

Step 2: Fill in final column using equation (7.1)

Step 3: Fill in remaining columns using equation (7.2)

Figure: Economical framework example [1]

ROA idea

Inspiration -Graeme Guthrie

- Partial BSM analogy
- Replicating portfolio (no arbitrage principle)
- Risk neutral probabilities
- Backward induction
- Probability of up and down move [1]:

$$\pi_u = \frac{ZR_f - X_d}{X_u - X_d}, \pi_d = \frac{X_u - ZR_f}{X_u - X_d}, \quad (1)$$

where

$$Z = \frac{E[\tilde{X}] - (E[\tilde{R_m}] - R_f)(\frac{Cov[\tilde{X},\tilde{R_m}]}{Var[\tilde{R_m}]})}{R_f} \quad (2)$$

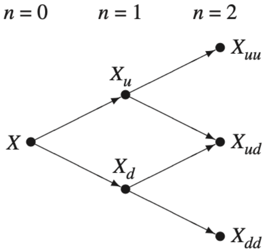

Figure: Binomial model [1]

ROA limitations

General

- Limited number of uncertainty sources - one (no arbitrage principle)
- Simple distributions - binomial model
- Limited by computational complexity for higher-dimensional problems
- Complicated scaling in types and number of real options

SDT Improvements

General

- Allows for multiple sources of uncertainty - seamless integration
- Allows continuous distributions, theoretically of any type
- Computational complexity tools - ADP
- Real options (actions) easily scaled with action set

Furthermore:

- Allows for simple integration of Bayesian learning
- Allows for complex creation of prior probability densities

SDT Improvements

Preserving domain-specific concepts

- Time value of money - discounting factors
- Risk aversion of investors - utility theory
- Risk-neutral probabilities - Bayesian priors

- Value iteration, value function approximation
- Value function modeled as three piecewise linear functions (for gas power plant example)

$$v_t(s) = \sum_{i=0}^{2} I_i(s^4) pw(x, k^1_{i,t}, k^2_{i,t}, x_{i,t}, y_{i,t}) \tag{3}$$

- Solves the problem of uncountable state space: i.e. for Gas power plant example:

$$|\mathbb{R}| \times |\mathbb{R}| \times |\mathbb{R}| \times 3 \times |\mathbb{R}| \tag{4}$$

Valuation of gas power plant

General Idea

- Utility company has possibility to build 1 or 2 200MW gas powerplant blocks for 65M each in the following 25 years.
- Power plant sell its power and buying the needed gas and CO2 allowances as monthly contracts.
- Ability to use FLL at 6% r_b and invest with risk-free interest rate 2%
- Initial prices of gas, CO2 allowances and power are 24EUR, 9EUR, 40EUR per MWh

Optimized actions:

- Build a new block or don't
- Run the existing installed capacity or don't

Sources of uncertainty:

- Price of gas, power and CO2 - lognormal processes, 2 sets of volatilites

Results

Optimal management + its value Comparison to three baseline strategies

- Build both blocks and run all the time
- Build both blocks and run only if spark price "in the money"
- Build both blocks, when the spark price is 40 EUR lower than the price of power

Results

Initial setup

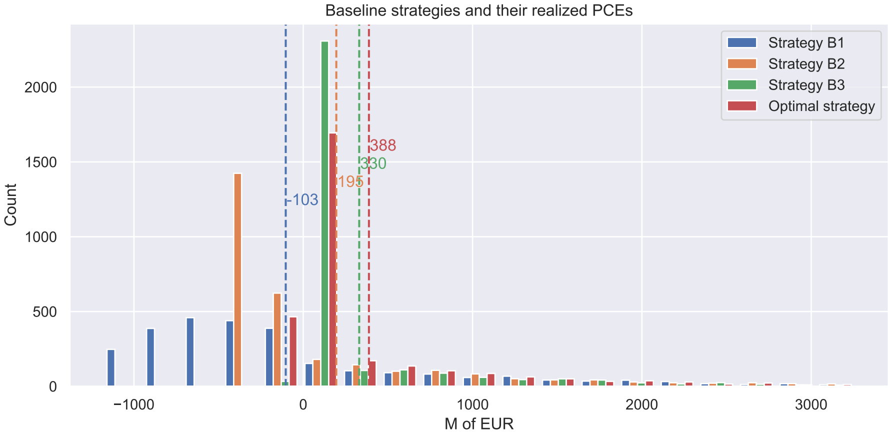

Figure: PCE comparison between 3 baseline strategies and the optimal one - initial setup.

Results

Increased volatility

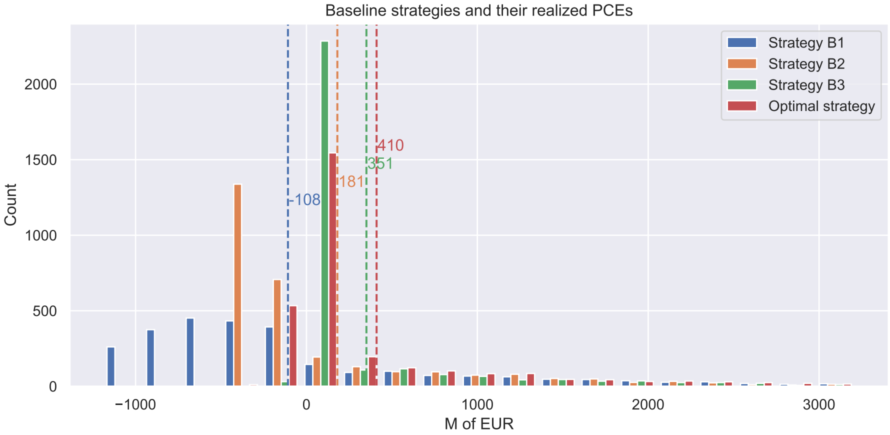

Figure: PCE comparison between 3 baseline strategies and the optimal one - increased price volatility.

Conclusion

The contributions of my thesis are

- Summary of ROA state of the art
- Formulation of ROA project valuation problem in SDT framework
- Identification of a fitting ADP algorithm
- Demonstration of the applicability on a gas powerplant project

Possible directions of future research:

- real data application;
- sensitivity analysis of time granularity;
- so-called option games (SDT+Game theory) competition vs collaboration;
- deeper study of utility in the micro and macro optimization.

Bibliography

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Real Options in Theory and Practice.
Financial Management Association Survey and Synthesis. Oxford University Press, 2009.