Jet Energy Loss in Relativistic Heavy-Ion Collisions with Realistic Medium Modeling

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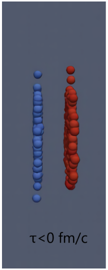
Heavy-Ion Collision



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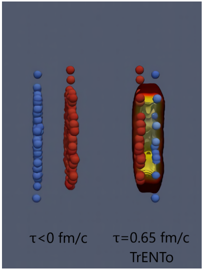

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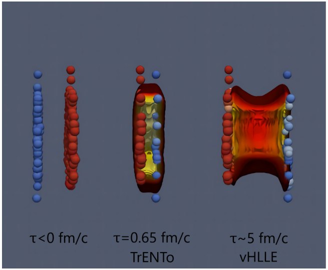

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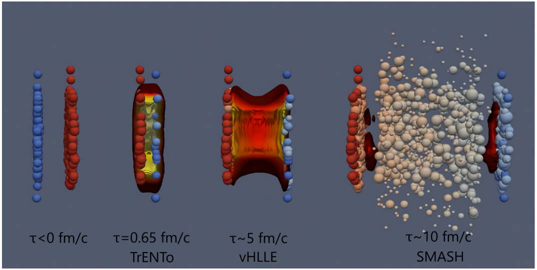

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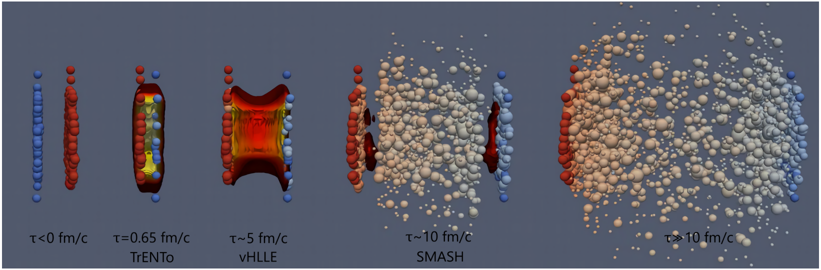

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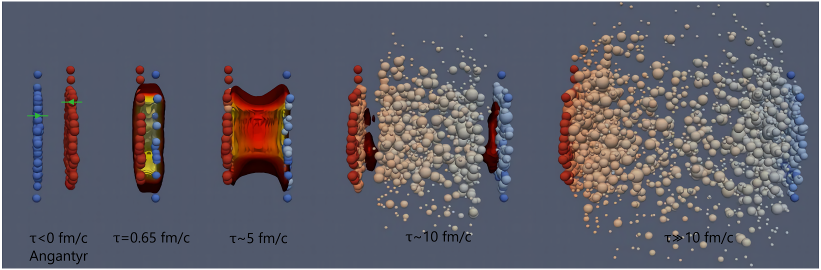

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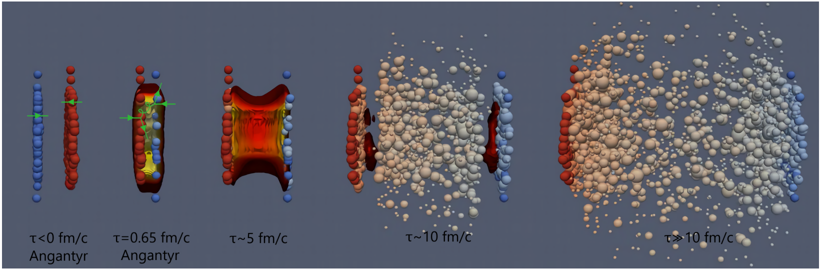

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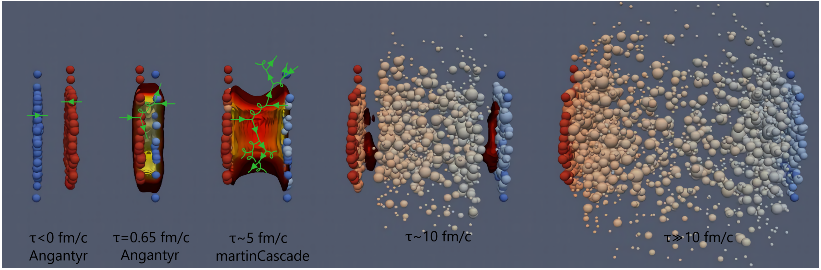

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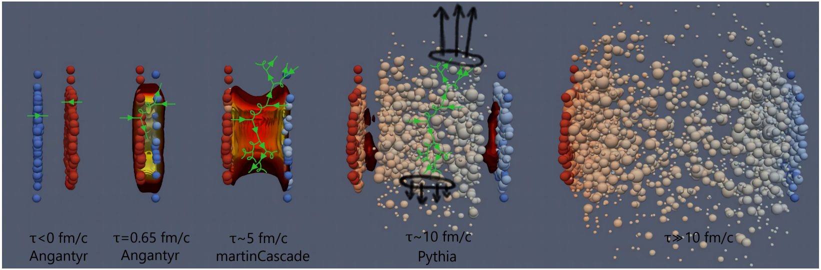

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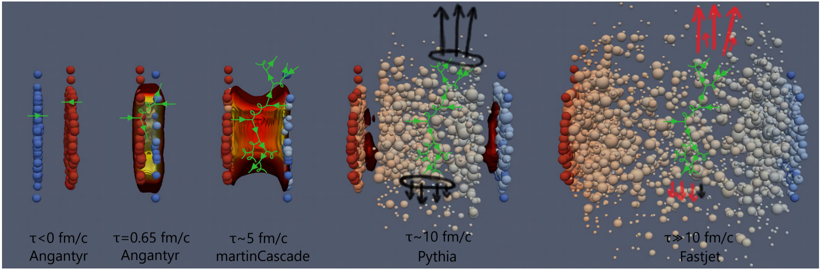

Medium Modeling

Medium Modeling

- Initial state model (non-dynamical)
  - Describe the initial conditions at $\tau = \tau_0$ (equilibrium proper time)
  - Nuclear density, σ_{NN}, Impact parameter $\longrightarrow \begin{cases} \varepsilon(\tau = \tau_0) \\ \text{or} \\ s(\tau = \tau_0) \end{cases}$
  - i.e., Glauber model or **TRENTo**

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- Hydrodynamic simulation
  - Expansion and cooling of the hot and dense matter
  - Hydrodynamic equations, equation of state, and transport coefficients
- Freeze-out of the QGP
  - Transition from fluid to hadrons (at $T_c \approx 150$ MeV)
  - Cooper-Frye formula

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- Reconstruction
  - Sequential recombination algorithm reconstruction (anti-k_T)
  - Background subtraction (k_T+ghost particle)

TRENTo Initial State

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Participant thickness function:

$$T_A(x, y) \equiv \int \rho_A^{\mathrm{part}}(x, y, z)\mathrm{d}z$$

Reduced thickness function:

$$T_R\,(p;\, T_A,\, T_B) \equiv \left(\frac{T_A^p + T_B^p}{2}\right)^{1/p} \propto \left.\frac{\mathrm{d}s}{\mathrm{d}\eta_s}\right|_{\substack{\tau=\tau_0 \\ \eta_s=0}}$$

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The average multiplicity of charged particles:

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+ Effective inelastic parton-parton cross section

TRENTo Initial State

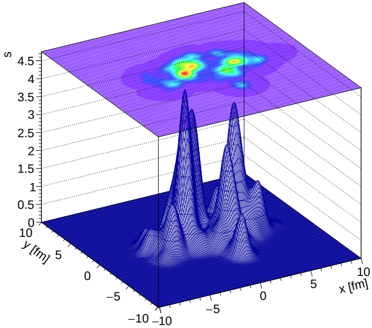


Figure: Normalized entropy distribution in transverse plane of Pb+Pb collision at $\sqrt{s_{NN}} = 2.76$ TeV and $20 - 30\%$ centrality. On the left side is a randomly picked single event and on the right side is the average of 2000 events.

Second-Order Hydrodynamics

Israel-Stewart equations in the 14-momentum approximation:

$$u^\mu \partial_\mu \Pi = \frac{-\zeta \partial_\mu u^\mu - \Pi}{\tau_\Pi} - \frac{\delta_{\Pi\Pi}}{\tau_\Pi} \Pi \partial_\mu u^\mu + \frac{\lambda_{\Pi\pi}}{\tau_\Pi} \pi^{\mu\nu} \sigma_{\mu\nu}$$

$$u^\alpha \partial_\alpha \pi^{\langle\mu\nu\rangle} = \frac{2\eta\sigma^{\mu\nu} - \pi^{\mu\nu}}{\tau_\pi} - \frac{\delta_{\pi\pi}}{\tau_\pi} \pi^{\mu\nu} \partial_\mu u^\mu + \frac{\phi_7}{\tau_\pi} \pi_\alpha{}^{\langle\mu} \pi^{\nu\rangle\alpha} - - \frac{\tau_{\pi\pi}}{\tau_\pi} \pi_\alpha{}^{\langle\mu} \sigma^{\nu\rangle\alpha} + \frac{\lambda_{\pi\Pi}}{\tau_\pi} \Pi \sigma^{\mu\nu}, \quad \text{where}$$

$$\sigma^{\mu\nu} \equiv \eta \partial^{\langle\mu} u^{\nu\rangle} = \eta \left[\frac{1}{2} (\Delta^{\alpha\mu} \Delta^{\beta\nu} + \Delta^{\beta\mu} \Delta^{\alpha\nu}) - \frac{1}{3} \Delta^{\mu\nu} \Delta^{\alpha\beta} \right] \partial_\alpha u_\beta$$

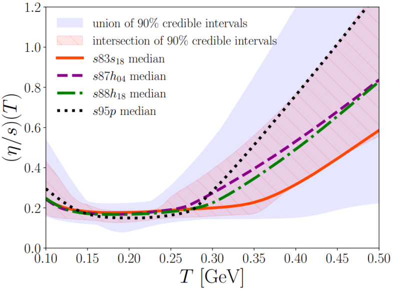
Transport coefficients: η, ζ,

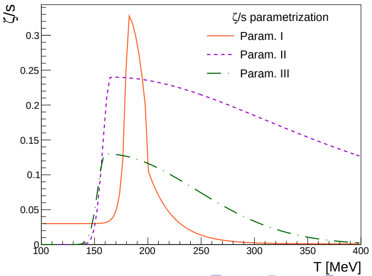
$$\frac{\delta_{\Pi\Pi}}{\tau_\Pi} = \frac{2}{3}, \quad \frac{\lambda_{\Pi\pi}}{\tau_\Pi} = \frac{8}{5}\left(\frac{1}{3} - c_s^2\right), \quad \frac{\delta_{\pi\pi}}{\tau_\pi} = \frac{4}{3}, \quad \phi_7 = \frac{9}{70p}, \quad \frac{\tau_{\pi\pi}}{\tau_\pi} = \frac{10}{7}, \quad \frac{\lambda_{\pi\Pi}}{\tau_\pi} = \frac{6}{5}.$$

Relaxation times: $\tau_\pi = \frac{5\eta}{sT}$, $\tau_\Pi = \frac{\zeta}{15(\frac{1}{3} - c_s^2)^2 sT}$

Bulk and Shear Viscosity

- η/s can only be computed for simplified scenarios
  - pQCD (leading log): $\frac{\eta}{s} \sim \frac{1}{\alpha_s^2 \ln(\alpha_s^{-1})}$ (small α_s)
  - AdS/CFT limit: $\frac{\eta}{s} \geq \frac{1}{4\pi} \approx 0.08$ (large α_s)
- η/s cannot be computed for realistic QGP
  - Comparing different η/s to the data (i.e., Bayesian analysis)
- ζ/s cannot be computed even for simplyfied scenarios
  - Must be carefully tested for cavitation stability





Freeze-Out of the Fluid

- Cooper-Frye formula

$$E\frac{\mathrm{d}N_i}{\mathrm{d}^3p} = \int_\Sigma \mathrm{d}\Sigma_\mu p^\mu f_i\left(T, p_\mu u^\mu, \pi^{\mu\nu}\right)$$

  - Σ_μ - Cooper-Frye freeze-out hypersurface
  - $f_i\left(T, p_\mu u^\mu, \pi^{\mu\nu}\right)$ - particle distribution functions
- Hadronic rescattering
- Resonance decays

- Source of the background event

Parton Initial State

Parton Initial State

- Standard Woods-Saxon ‘a la GLISSANDO

$$\rho(r) = \frac{\rho_0}{1 + \exp\left(r - R\left(1 + \beta_2 Y_{20} + \beta_4 Y_{40}\right)\right)/a}$$

- Glauber–Gribov

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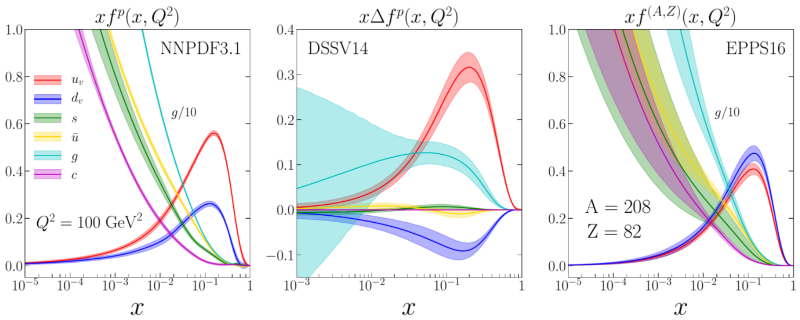
- Glauber–Gribov
- Parton distribution function $f_{a/p}(x)$
- $\sum_{ab} f_{a/p} \star f_{b/p} \star \widehat{\sigma}_{ab}$
- $\frac{d\sigma_{2\to 2}}{dp_\perp^2} \propto \frac{\alpha_s^2(p_\perp^2)}{p_\perp^4} \to \frac{\alpha_s^2(p_\perp^2 + p_{\perp 0}^2)}{\left(p_\perp^2 + p_{\perp 0}^2\right)^2}$

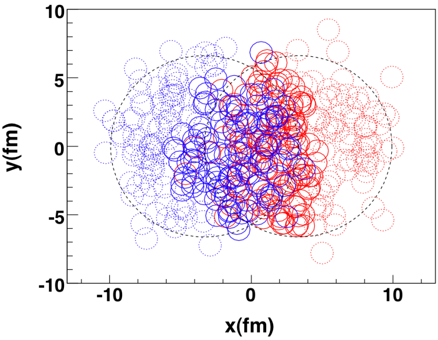
Parton Distribution Functions



Parton Initial State

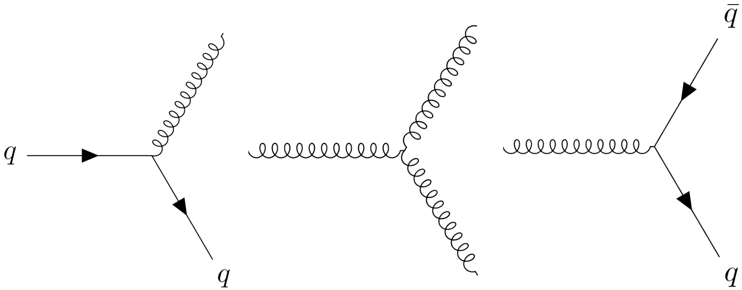
- Participant plane determination (Anganthyr does not give reaction plane angle)

$$\tan(2\phi^*) = 2\frac{\langle xy\rangle - \langle x\rangle\langle y\rangle}{\mathrm{var}(y) - \mathrm{var}(x)}$$



Parton Splitting in Vacuum

- Possible splitting processes:
  - $q \to q + g$
  - $g \to g + g$
  - $g \to q + \overline{q}$



Parton Splitting in Vacuum

- LO parton splitting functions

$$\widehat{P}_{qq}(z) = \frac{4}{3}\frac{1+z^2}{1-z}$$
$$\widehat{P}_{gg}(z) = 3\left[\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)\right]$$
$$\widehat{P}_{qg}(z) = \frac{1}{2}\left[z^2 + (1-z)^2\right]$$

- Parton branching is given in terms of the virtuality of the parent parton Q
- Monte Carlo formulation of the parton shower is based on the Sudakov form factor

$$S_a\left(Q_\mathrm{i}^2, Q_\mathrm{f}^2\right) = \exp\left[-\int_{Q_\mathrm{f}^2}^{Q_\mathrm{i}^2}\frac{\mathrm{d}Q^2}{Q^2}\int_{z_-}^{z_+}\mathrm{d}z\,\frac{\alpha_\mathrm{s}}{2\pi}\sum_b \widehat{P}_{ba}(z)\right]$$

Parton Splitting in Vacuum

- Requirements to fix integral divergence

$$k_\perp \geq f \cdot \Lambda_{\mathrm{QCD}} \text{ and } Q_b, Q_c \geq Q_0/2$$

- Allowed range of momentum fraction

$$z_\pm\left(Q^2, E_a\right) = \frac{1}{2} \pm \frac{1}{2}\sqrt{\left(1 - \frac{Q_0^2 + 4\left(f \cdot \Lambda_{\mathrm{QCD}}\right)^2}{Q^2}\right)\left(1 - \frac{Q^2}{E_a^2}\right)}$$

- Parton can split only if

$$Q_a > Q_{\mathrm{min}} = \sqrt{Q_0^2 + 4\left(f \cdot \Lambda_{QCD}\right)^2}$$

Parton Energy Loss

Collisional energy loss

- Perturbative calculation+regularisation $\rightarrow \sigma^{\mathrm{el}} = \dots$ (too long)
- Probability that parton will not experience elastic scattering during time period tau

$$P_{\text{no scatt}}(\tau) = \exp\left(-\int_{t_{\mathrm{p}}}^{t_{\mathrm{p}}+\tau} \mathrm{d}t' \sigma^{\text{elas}} \left(\vec{r}\left(t'\right), t'\right) n\left(\vec{r}\left(t'\right), t'\right)\right)$$

Radiative energy loss

- Medium induced gluon radiation ($2 \rightarrow 3$)
    - Inelastic $2 \rightarrow 3$ scattering same footing as elastic
    or
    - $\hat{P}_{ba}(z) \rightarrow (1 + f_{\text{med}})\, \hat{P}_{ba}(z)$ inside the medium ($1 \rightarrow 2$)

Space-Time Trajectories of jet Partons

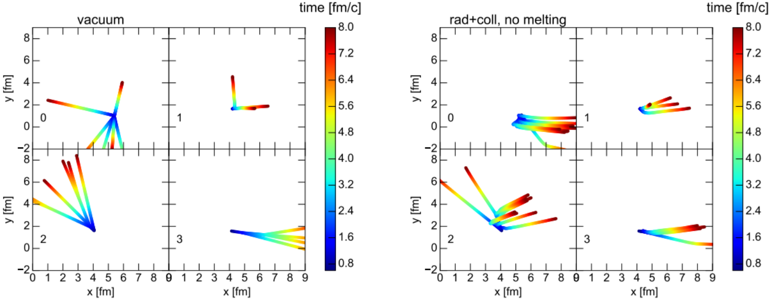


Figure: Space-time trajectories of jet partons from central at 2.76 TeV PbPb event. The left panel shows the evolution without medium effects. Right panel shows the evolution with medium effects.

Lund String Model Hadronization

- QCD potential

$$V_{\mathrm{QCD}} = -\frac{4}{3}\frac{\alpha_s}{r} + \kappa r + \ldots$$

+ κ is the string tension, which is around 1 GeV/fm

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- Production probability using the WKB approximation

$$\frac{1}{\kappa}\frac{\mathrm{d}\mathcal{P}_q}{\mathrm{d}^2 p_\perp} \propto \exp\left(-\pi m_{\perp q}^2/\kappa\right) = \exp\left(-\pi p_\perp^2/\kappa\right)\exp\left(-\pi m_q^2/\kappa\right)$$

Reconstruction

- Fastjet anti-k_T:

$$d_{ij} = \min\left(\frac{1}{p_{\mathrm{T},i}^2}, \frac{1}{p_{\mathrm{T},j}^2}\right)\frac{\Delta R_{ij}^2}{R}$$

$$d_{iB} = \frac{1}{p_{\mathrm{T},i}^2}$$

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- Substracting large, fluctuating background

$$p_{T,jet} = p_{T,jet}^{\mathrm{raw}} - \rho A$$

Reconstruction

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$$d_{iB} = \frac{1}{p^2_{\mathrm{T},i}}$$

- Substracting large, fluctuating background

$$p_{T,jet} = p^{\mathrm{raw}}_{T,jet} - \rho A$$

- A - Area of the jet (Fastjet ghost particle algorithm)

Reconstruction

- Fastjet anti-k_T:

$$d_{ij} = \min\left(\frac{1}{p_{\mathrm{T},i}^2}, \frac{1}{p_{\mathrm{T},j}^2}\right)\frac{\Delta R_{ij}^2}{R}$$

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- Substracting large, fluctuating background

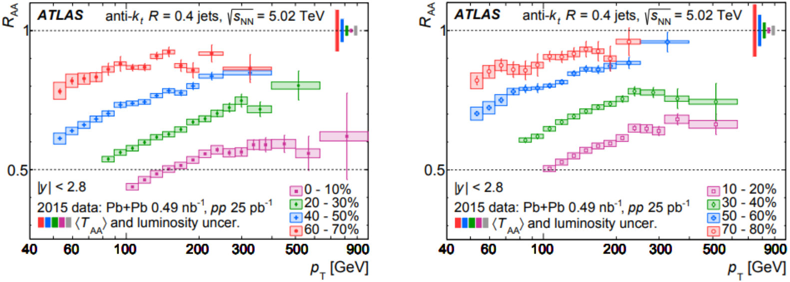
$$p_{T,jet} = p_{T,jet}^{\mathrm{raw}} - \rho A$$

- A - Area of the jet (Fastjet ghost particle algorithm)
- $\rho = \text{median}\left\{\frac{p_{T,\,\text{jet}}^{\mathrm{raw},j}}{A_{\mathrm{jet}}^j}\right\}$ - transverse momentum density of background in the event (hardest jets are excluded) (Fastjet k_T algorithm)

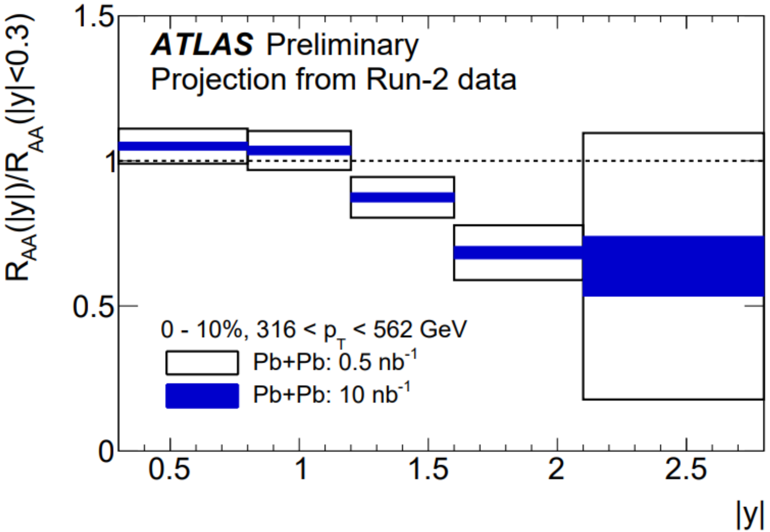
Jet observables?

Nuclear Modification Factor R_{AA}

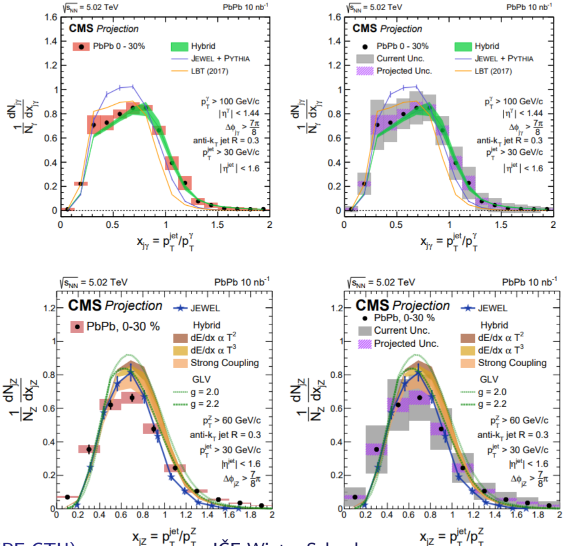
$$R_{AA} = \frac{\frac{\mathrm{d}^2 N^{AA}_{jet}}{\mathrm{d}p_T \mathrm{d}y}}{\langle T_{AA} \rangle \frac{\mathrm{d}^2 \sigma^{pp}_{jet}}{\mathrm{d}p_T \mathrm{d}y}}$$



Nuclear Modification Factor Rapidity Ratio

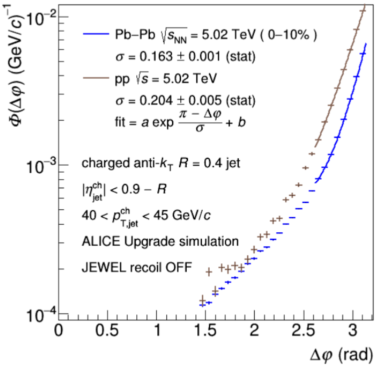


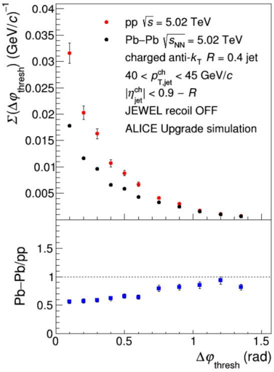
photon-jet distribution of momentum fraction "$x_{j\gamma}$" and Z-jet distribution of momentum fraction "x_{jZ}"



Acoplanarity and Cumulative Large-Angle Yield

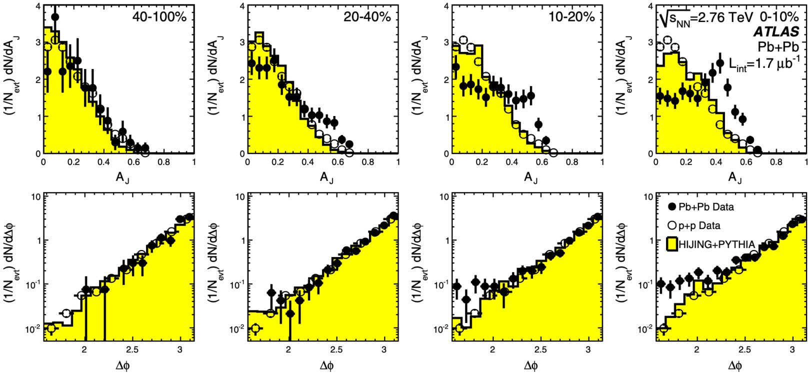
$$\Sigma\left(\Delta\varphi_{\text{thresh}}\right) = \int_{\pi/2}^{\pi-\Delta\varphi_{\text{thresh}}} \Phi(\Delta\varphi)\mathrm{d}\Delta\varphi$$





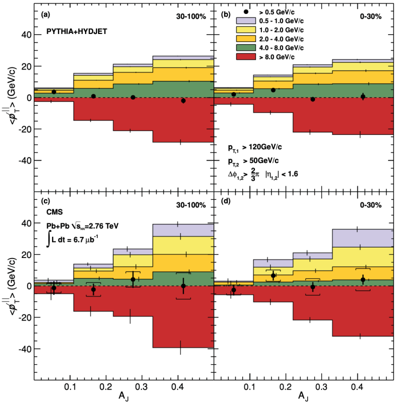
Imbalance and acoplanarity of dijet events

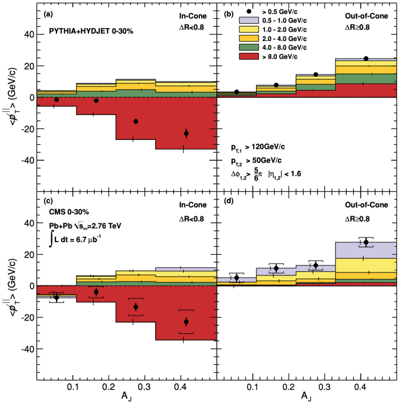
$$A_J = \frac{p_{T,1} - p_{T,2}}{p_{T,1} + p_{T,2}}, \qquad A_J = \frac{E_{T,1} - E_{T,2}}{E_{T,1} + E_{T,2}}$$



Average Missing Transverse Momentum $\langle \not{p}_{\mathrm{T}}^{\parallel} \rangle$

$$\not{p}_{\mathrm{T}}^{\parallel} = \sum_i -p_{\mathrm{T}}^{i} \cos\left(\phi_i - \phi_{\mathrm{LJ}}\right)$$





Work plan

- Simulate a simple scenario (brick of QGP)
- Simulate pp and PbPb collisions at LHC energy
- Compare procedure with JETSCAPE
- Find optimal reconstruction procedure
- Get obsarvables comparable to experimental results
- Hydrodynamic initial state based on participating nucleons from Angantyr (?)
- Detector effects (smearing, response matrix, unfolding) (PhD?)

Thank you for your attention!

This work was supported by the project Centre of Advanced Applied Sciences with the number: CZ.02.1.01/0.0/0.0/16-019/0000778. Project Centre of Advanced Applied Sciences is cofinanced by the European Union.

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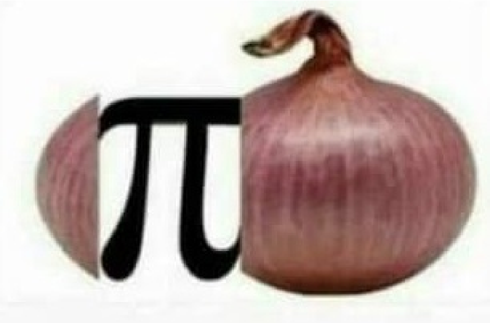
Questions
and



Backup: Ideal hydrodynamics

Ideal fluid dynamics:

$$T_0^{\mu\nu} = \varepsilon u^\mu u^\nu - p\Delta^{\mu\nu}, \quad \text{were} \quad \Delta^{\mu\nu} = \eta^{\mu\nu} - u^\mu u^\nu$$

$$N_{0,i}^{\mu} = n_i u^\mu$$

Equation of motion with orthogonal ($u^\mu \Delta_{\mu\nu} = 0$) projection:

$$u_\mu \partial_\nu T_0^{\mu\nu} = 0 \longrightarrow u^\mu \partial_\mu \varepsilon + (\varepsilon + p)\partial_\nu u^\nu = 0 \qquad \text{(Continuity eq.)}$$

$$\Delta_{\sigma\mu} \partial_\nu T_0^{\mu\nu} = 0 \longrightarrow (\varepsilon + p) u^\mu \partial_\mu u_\sigma - \Delta_\sigma^{\ \nu} \partial_\nu p = 0 \qquad \text{(Euler eq.)}$$

Backup: Navier-Stokes formalism (first order)

$$\begin{aligned} T^{\mu\nu} &= T_0^{\mu\nu} + \Pi^{\mu\nu} \\ &= T_0^{\mu\nu} + \pi^{\mu\nu} - \Pi\Delta^{\mu\nu} \\ &= \varepsilon u^\mu u^\nu - (p + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu} \end{aligned}$$

$$\pi^{\mu\nu} = \eta\partial^{<\mu}u^{\nu>} = \eta\left[\frac{1}{2}(\Delta^{\alpha\mu}\Delta^{\beta\nu} + \Delta^{\beta\mu}\Delta^{\alpha\nu}) - \frac{1}{3}\Delta^{\mu\nu}\Delta^{\alpha\beta}\right]\partial_\alpha u_\beta$$

$$\Pi = -\zeta\partial_\mu u^\mu$$

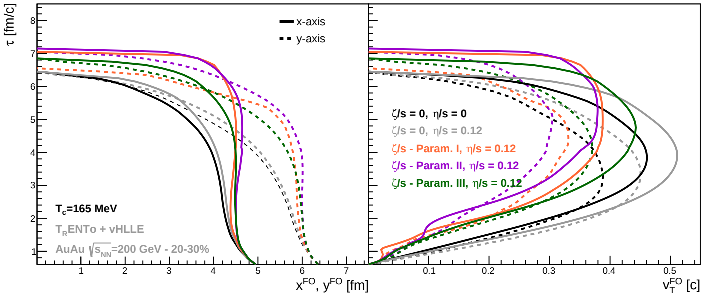
Equation of motion with orthogonal projection:

$$u^\mu\partial_\mu\varepsilon + (\varepsilon + p + \Pi)\partial_\nu u^\nu + \pi_{\mu\nu}\partial^{<\mu}u^{\nu>} = 0 \qquad \text{(Continuity eq.)}$$

$$(\varepsilon+p+\Pi)u^\mu\partial_\mu u^\sigma - \partial^\sigma(p+\Pi) + \Delta^{\sigma\mu}\partial^\nu\pi_{\mu\nu} - \pi^{\sigma\nu}u^\mu\partial_\mu u_\nu = 0 \qquad \text{(N-S eq.)}$$

Backup: Freeze-out evolution with bulk viscosity

- Transverse size of a freeze-out hyper-surface at $\eta_s = 0$ (left)
- Transverse velocity of a freeze-out hyper-surface at $\eta_s = 0$ (right)



- Bulk viscosity suppresses transverse flow and delays the break-up of the fireball

Backup: parton kinematics in vacuum

$$p_{a\mu} = \left(E_a, \overrightarrow{0}, p_a\right) \quad \text{with} \quad p_a^2 = E_a^2 - Q_a^2$$
$$p_{b\mu} = \left(zE_a, \vec{k}_\perp, p_b\right)$$
$$p_{c\mu} = \left((1-z)E_a, -\vec{k}_\perp, p_a - p_b\right).$$

$$p_b = \frac{2zE_a^2 - Q_a^2 - Q_b^2 + Q_c^2}{2p_a}$$

$$k_\perp^2 = -z^2 \frac{Q_a^2 E_a^2}{p_a^2} - z\frac{E_a^2}{p_a^2}\left(Q_c^2 - Q_a^2 - Q_b^2\right) - \frac{\left(Q_c^2 - Q_a^2 - Q_b^2\right)^2}{4p_a^2} - Q_b^2$$

$$k_\perp^2 \approx z(1-z)Q_a^2$$

$$\theta_a \approx \frac{Q_a}{\sqrt{z(1-z)}E_a}$$