Predictions for future EICs with the BK equation

What can it tell us about parton saturation?

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(E)JČF workshop in Bílý potok

13th June, 2022

Q1: What is the structure of hadrons and how does it evolve with increasing energy?

Q2: How does the structure of a free proton changes when it's bound inside a nucleus?

Parton model

- Partons = constituent particles inside hadrons (Feynman)
- From our current perspective – quarks and gluons
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- Partons = constituent particles inside hadrons (Feynman)
- From our current perspective – quarks and gluons
- Valence quarks determine the quantum numbers of hadrons
- However certain sum rules are not fulfilled with only (valence) quarks
- Fraction of the proton momentum carried by all q and $\bar{q}$ together
  - Naive expectation

$$\int_0^1 x\Sigma(x)\mathrm{d}x = 1$$

  - Experimental value

$$\int_0^1 x\Sigma(x)\mathrm{d}x \approx 0.5$$

→ sea quarks and gluons are also part of hadrons!

Deep Inelastic Scattering (DIS)

- Probes inner structure of hadrons.
- DIS cross section

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[y^2 F_1(x,Q^2) + (1-y)\frac{F_2(x,Q^2)}{x}\right] \tag{1}$$

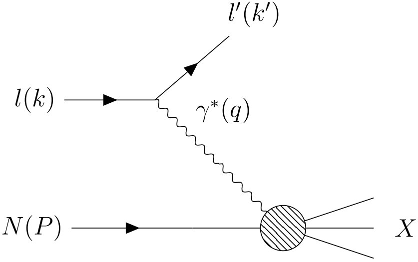
- $F_1,\ F_2$ – structure functions, include photon-proton interaction.

$$W^2_{\gamma^* p} = (P+q)^2$$

$$Q^2 = -q^2 = -(k-k')^2$$

$$x = \frac{Q^2}{2P\cdot q}$$

$$y = \frac{P\cdot Q}{P\cdot k}$$



DIS in Feynmann's parton model picture

- Scattering of γ^* on one of the constituent partons.
- Infinite momentum frame - incoming particles carry only longitudinal momentum.
- Partons carry fraction x of the proton momentum P.
- Electron-parton scattering cross section

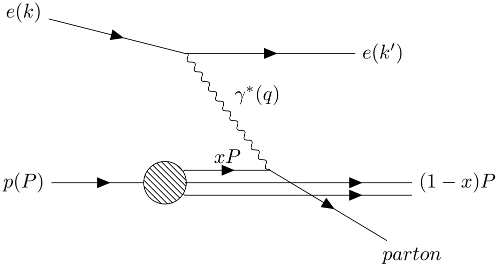
$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right]\sum_i e_i^2 f_i(x), \tag{2}$$

- If partons are ferminons:

$$F_1(x) = \frac{1}{2}\sum_i e_i^2 f_i(x)$$

$$F_2(x) = \sum_i e_i^2 x f_i(x)$$

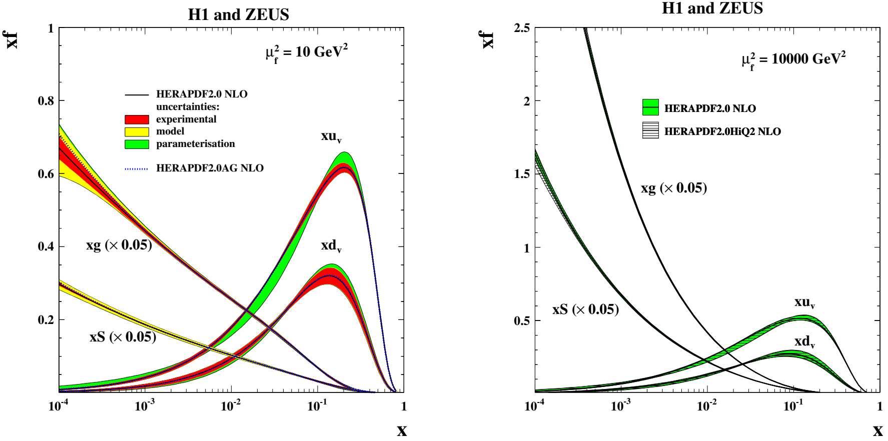
$$F_2(x) = 2xF_1(x).$$



- Independence on Q^2 (Bjorken scaling) → violated at low-x due to gluon branching

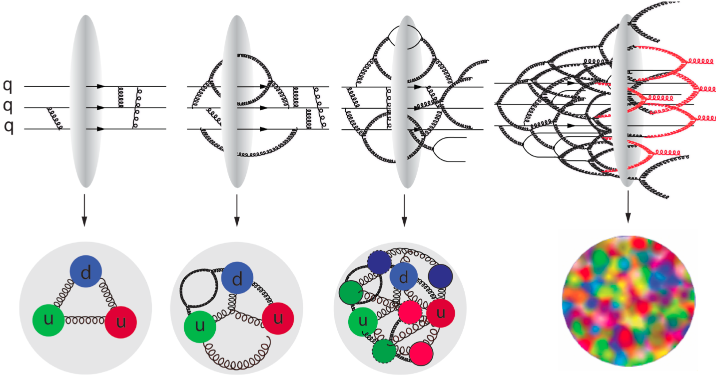
Evolution of proton structure

- Composition of the proton changes with x and Q^2.
- At low energies proton dominated by valence quarks.
- At large energies gluons dominate.



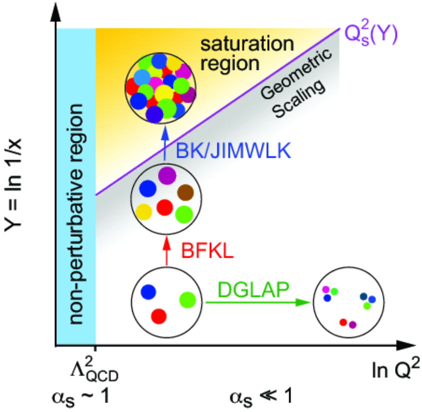
Are gluons growing to infinity?

- Untamed grow of gluon densities towards low-x
  - The gluon distribution function would diverge
  - Unphysically large values of the cross section (unitarity violation)
- Gluon recombination slows down the evolution of gluon densities.
- So far, there has been no direct observation of saturation effects!



M. Matas

Evolution of parton densities and saturation



C. Marquet, Nucl.Phys. A904-905 (2013) 294c-301c.

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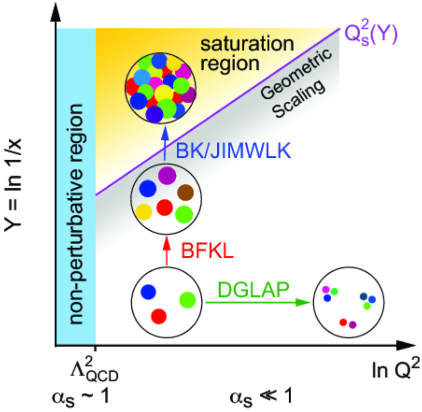

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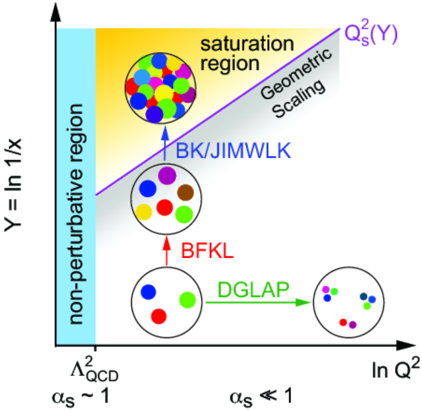

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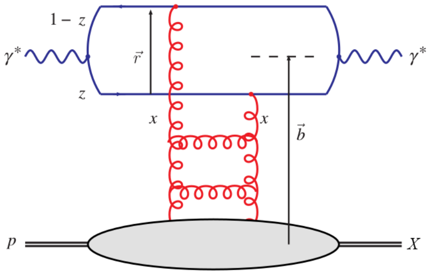
    - **Above** $Q_S^2(x) \rightarrow$ dense regime, non-linear evolution of the gluon density (JIMWLK, BK).

QCD phenomenology at high energies

Color dipole model (of DIS)

- Electron (or a hadron with large EM field) emits a virtual photon
- Photon interacts via its $q\bar{q}$ Fock state with the hadron in the target rest frame.

  $\rightarrow$ both γ^* and $q\bar{q}$ are colorless
- The scattering amplitude is proportional to:
  - $\gamma^* \rightarrow q\bar{q}$ wave function (QED)
  - Dipole-hadron cross section (QCD is here!)



- r - transverse size of the $q\bar{q}$ dipole
- z - fraction of photon momentum carried by quark
- b - impact parameter
- x - light-front momentum fraction of the proton carried by gluons attached to the dipole

Balitsky–Kovchegov equation in QCD phenomenology

- The dipole-hadron cross section

$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}\vec{b}} = 2N(x, \vec{r}, \vec{b})$$

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- We are interested in the next step of the full $N\left(x, \vec{r}, \vec{b}\right)$ evolution

$$N\left(x, \vec{r}, \vec{b}\right) \to N(x, r, b)$$

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- Evolution of the scattering amplitude N of a $q\bar{q}$ dipole off a hadronic target.

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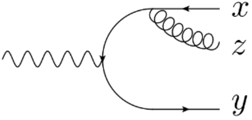

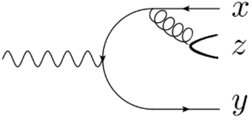

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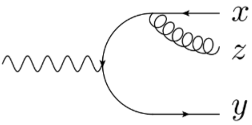

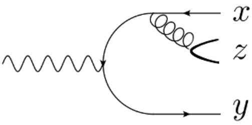

- Dipole and the target hadron are interconnected

  → evolution of the dipole structure gives information about target structure

Evolution kernel

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$$K^{rc}(r_{xy}, r_{xz}, r_{zy}) = \frac{\alpha_S(r_{xy}^2) N_C}{2\pi^2} \left[\frac{r_{xy}^2}{r_{xz}^2 r_{zy}^2} + \frac{1}{r_{xz}^2} \left(\frac{\alpha_S(r_{xz}^2)}{\alpha_S(r_{zy}^2)} - 1 \right) + \frac{1}{r_{zy}^2} \left(\frac{\alpha_S(r_{zy}^2)}{\alpha_S(r_{xz}^2)} - 1 \right) \right]$$

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- **Collinearly improved kernel**

$$K^{ci}(r_{xy}, r_{xz}, r_{zy}) = \frac{\alpha_S(r^2) N_C}{2\pi^2} \frac{r_{xy}^2}{r_{xz}^2 r_{zy}^2} \left[\frac{r_{xy}^2}{\min(r_{xz}^2, r_{zy}^2)} \right]^{\pm \bar{\alpha}_S A_1} K_{DLA}\left(\sqrt{L_{r_{xz} r_{xy}} L_{r_{zy} r_{xy}}} \right)$$

Implementation of impact-parameter dependence

- The equation is solved using a new initial condition that takes into account the location of the end-points of the dipole

$$N(\vec{r}, \vec{b}, Y = 0) = 1 - \exp\left[-\frac{1}{2}\frac{Q_S^2}{4} r^2 T(\vec{b}_{q_1}, \vec{b}_{q_2})\right]$$

$$T(\vec{b}_{q_1}, \vec{b}_{q_2}) = \exp\left(-\frac{b_{q_1}^2}{2B}\right) + \exp\left(-\frac{b_{q_2}^2}{2B}\right)$$

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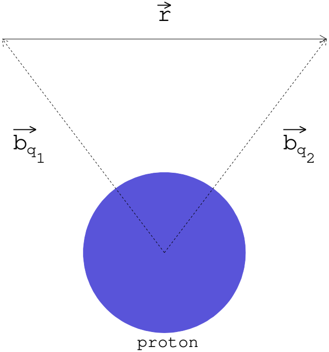
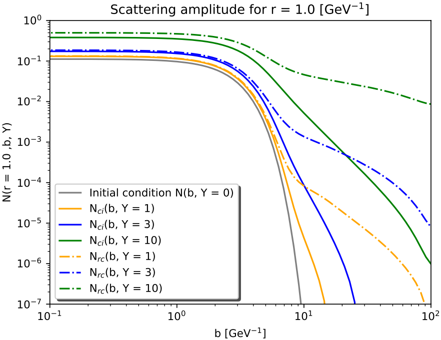


- The *r* behavior mimics the models for the *b*-independent dipole amplitude.
- The *b* behavior contains an exponential fall-off for dipole-ends far away from the target.

The problem of Coulomb tails

- Evolution with an exponentially falling initial condition and K^{rc} (*dashed lines*) increases the contribution at large impact parameters into a power-like growth.
  - Complications for phenomenological applications.
- Evolution at high-b can be suppressed by suppressing large daughter dipoles.



J. Cepila, J. G. Contreras, M. Matas; Phys. Rev. D 99, 051502

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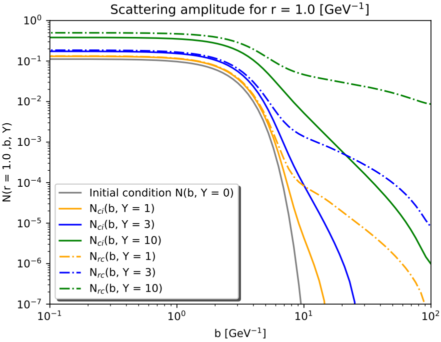

J. Cepila, J. G. Contreras, M. Matas; Phys. Rev. D 99, 051502

- Using the collinearly improved kernel K^{ci}(*full lines*)
  - Large daughter dipoles are suppressed as a result of time-ordering of the emissions.
  - Suppression by several orders of magnitude at large b.

Modification of b-BK to the nuclear case

- **Glauber–Gribov approach (b-BK-GG)**
  - Solution of the b-BK for proton target coupled to Glauber–Gribov prescription

$$N^{A}_{\mathrm{GG}}(r_{xy}, b_{xy}, Y) = \left[1 - \exp\left(-\frac{1}{2} T_A(b_{xy}) \sigma_{q\bar{q}}(Y, r_{xy})\right)\right]$$

$$\sigma_{q\bar{q}}(Y, r_{xy}) = \int \mathrm{d}^2 b \, 2N^{p}(r_{xy}, b_{xy}, Y)$$

  - Nuclear thickness function $T_A(b_{xy})$ obtained from a Woods–Saxon distribution
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- **Nuclear BK evolution (b-BK-A)**
  - The initial condition represents a specific nucleus

$$N^A(r_{xy}, b_{xy}, Y = 0) = 1 - \exp\left[-\frac{1}{2} \frac{Q^2_{s_0}(A)}{4} \, r^2_{xy} \, T_A\left(b_{q_1}, b_{q_2}\right)\right]$$

$$T_A\left(b_{q_1}, b_{q_2}\right) = k\left[T_A(b_{q_1}) + T_A(b_{q_2})\right]$$

→ $T_A(b_{q_i})$ uses the Woods–Saxon distribution

Balitsky–Kovchegov equation applied to QCD observables

Deep inelastic scattering

- Structure function F_2

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2\alpha}\left(\sigma_T^{\gamma^* p} + \sigma_L^{\gamma^* p}\right)$$

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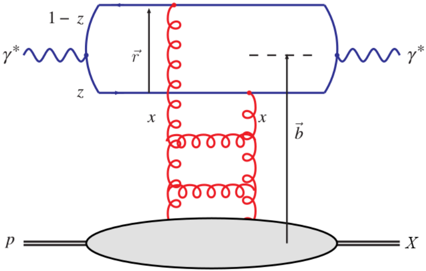
$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2\alpha}\left(\sigma_T^{\gamma^* p} + \sigma_L^{\gamma^* p}\right)$$

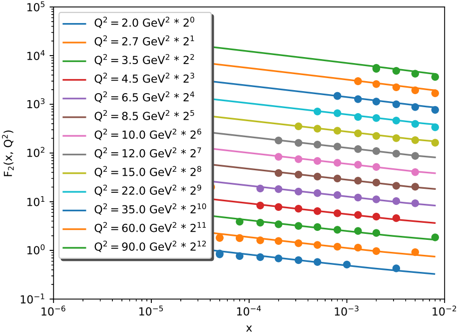
- The γ^*p cross section is calculated using
  - $\gamma^* \to q\bar{q}$ wave function
  - Dipole-hadron cross section

$$\sigma_{q\bar{q}}(x, \vec{r}) = 2\int \mathrm{d}^2 b\, N\left(x, \vec{r}, \vec{b}\right)$$



DIS: Proton structure function $F_2(x, Q^2)$

- Good agreement with the data measured at HERA[1].
- Results obtained without any additional ad-hoc terms (or parameters) needed to describe the data correctly.



DB, J. Cepila, J. G. Contreras, M. Matas; Phys. Rev. D 100, 054015

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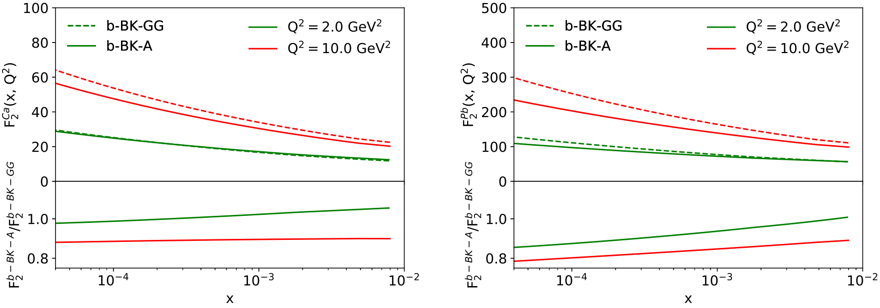
[1] H1, ZEUS: JHEP 01 (2010) 109.

DIS: Nuclear structure function $F_2(x, Q^2)$

- A sizeable difference between the b-BK-GG and b-BK-A approaches

  → the difference clearly depends on x, Q^2, and also A



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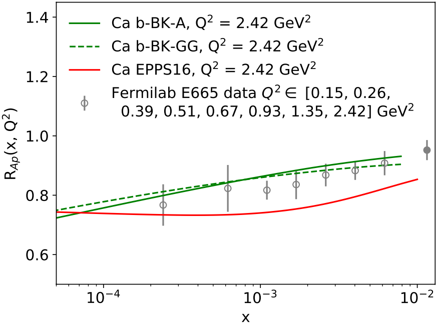
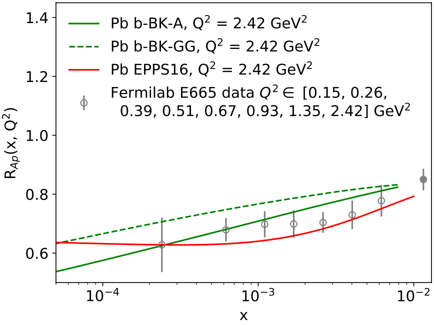
DIS: Nuclear suppression factors

- Ca: b-BK-GG and b-BK-A describe data.
- For Pb, a *x*-dependent difference is clearly observed between the b-BK-GG and b-BK-A approaches.





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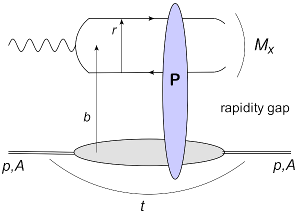
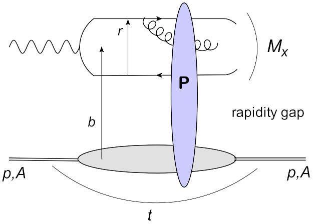
Diffractive DIS

- Diffractive structure function $F_2^{D(3)}$

$$F_2^{D(3)}(Q^2,\beta,x_{\mathbb{P}}) = F^D_{q\bar{q},L} + F^D_{q\bar{q},T} + F^D_{q\bar{q}g,T}$$

$$x_{\mathbb{P}} = \frac{Q^2+M_X^2}{Q^2+W^2}; \qquad \beta = \frac{Q^2}{Q^2+M_X^2}; \qquad x = \beta x_{\mathbb{P}}$$





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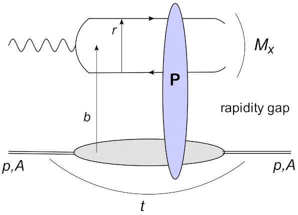
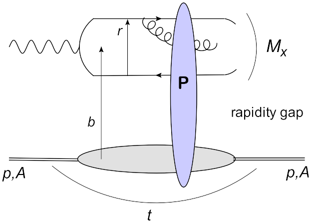
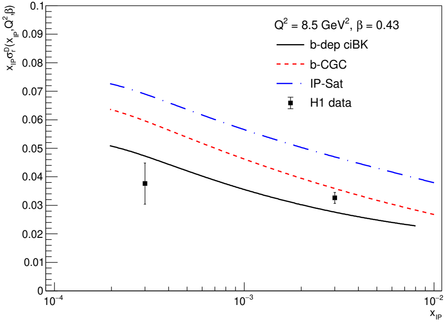


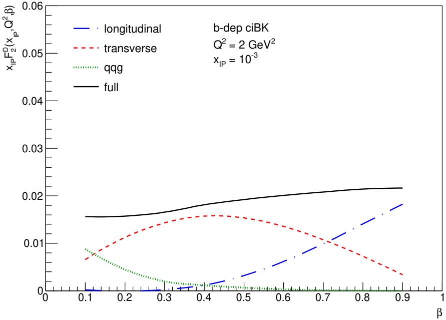


- $F_2^{D(3)}(Q^2, \beta, x_{\mathbb{P}})$ has a different sensitivity to the dipole-target amplitude (especially in the gluon part) than the inclusive F_2

Diffractive DIS: proton structure functions and the reduced cross section

- A reasonable agreement with the HERA data on the diffractive reduced cross section.
- Results obtained with b-BK amplitudes predict smaller proton $F_2^{D(3)}$ compared to other models.

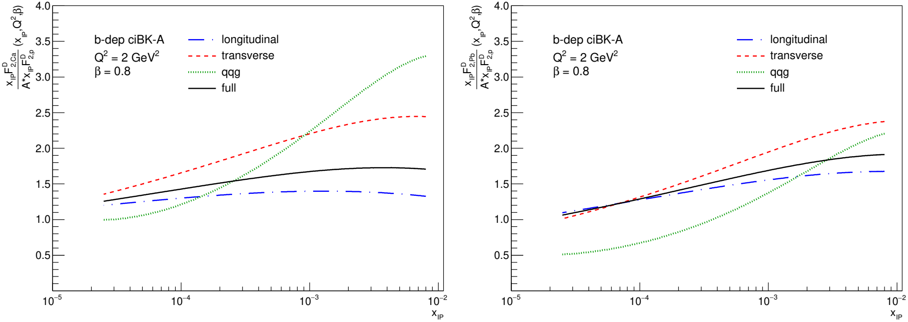




DB, J. Cepila, J. G. Contreras, V.P. Goncalves, M. Matas; Eur. Phys. J. C81 (2021), 211.

Diffractive DIS: nuclear structure functions

- Results of the b-BK-A approach are shown for Ca and Pb at low Q^2 and large β
- F_2^D ratio slightly larger for heavier nuclei
- $q\bar{q}g$ component (*green line*) strongly suppressed at larger A.

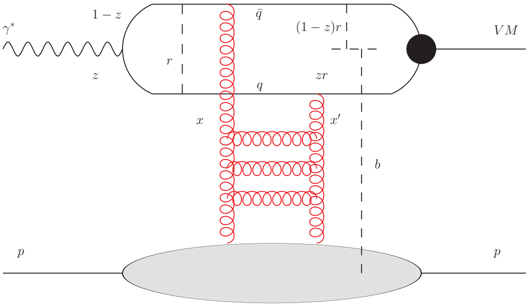


DB, J. Cepila, J. G. Contreras, V.P. Goncalves, M. Matas; Eur. Phys. J. C81 (2021), 211.

Production of vector mesons

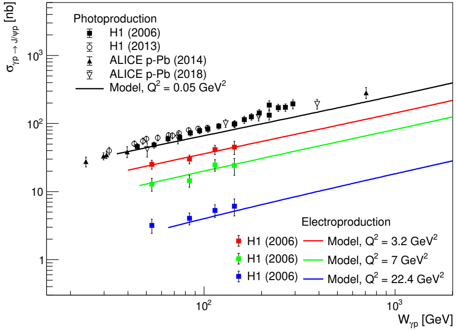
- The dipole interacts with the target hadron

  → VM is formed as a result of the interaction
- The scattering amplitude is given as a convolution of
  - The photon-meson wave function
  - The dipole-target cross section
- A schematic picture:

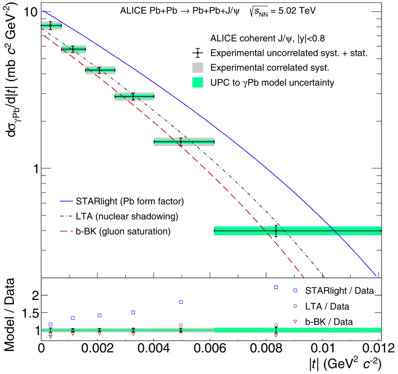


Production of vector mesons in γp and γPb collisions

- Predictions compared to HERA and LHC data
- Good agreement of the predictions with data for wide range of VM types
- Comparison with the first J/ψ $|t|$-distribution measurement at ALICE (right)



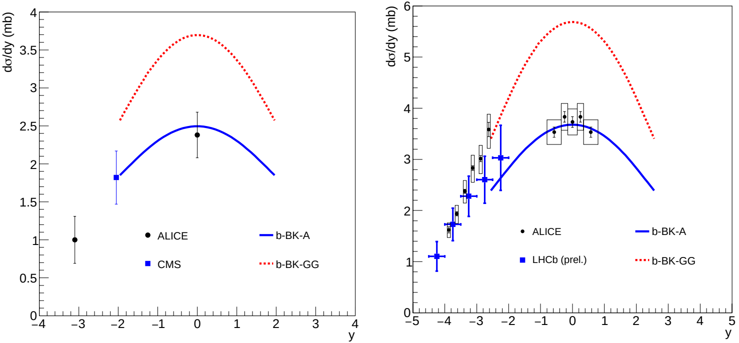
DB, J. Cepila, J. G. Contreras, M. Matas; Phys. Rev. D 100, 054015.



ALICE Collaboration; Phys. Lett. B 817 (2021) 136280.

Vector mesons in γPb collisions

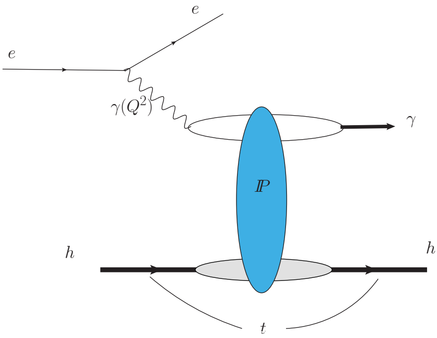
- Coherent J/ψ photoproduction
- Comparison to the LHC data from UPC
  - LHC Run 1 data (left) prefer the b-BK-A.
  - LHC Run 2 data don't provide a clear message.



DB, J. Cepila, J. G. Contreras, M. Matas; Phys. Lett. B 817 (2021) 136306

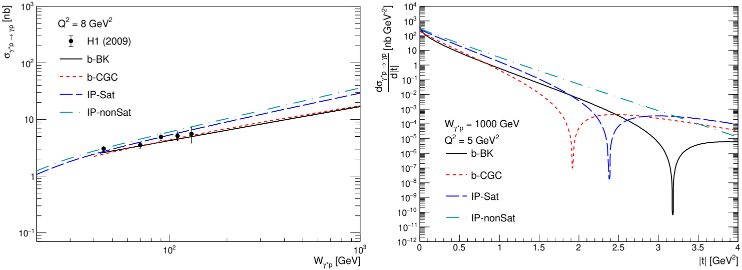
Deeply virtual Compton scattering

- Process similar to DIS, however a real γ is formed.
- The target hadron remains intact.
- A rapidity gap separates the two final states.
- DVCS process probes directly the QCD dynamics of the dipole-target interaction.
- Gluons dominate the interaction at large energies – direct probe of gluon GPD.



DVCS predictions

- Comparison to HERA data in a good agreement
  - b-BK model shows a milder *W*-dependence.
- Predictions for EIC and LHeC energies
  - Positions of the diffractive minima for proton clearly displaced among different models.



DB, J. Cepila, J. G. Contreras, M. Matas; Eur. Phys. Journal C 82 (2022) 99

Conclusions

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  - Measurements of nuclear structure functions and nuclear suppression
  - Saturation – it's expected to onset sooner in nuclei than for protons
  - The tomography picture of hadrons from exclusive processes
    → measurements of TMDs and GPDs

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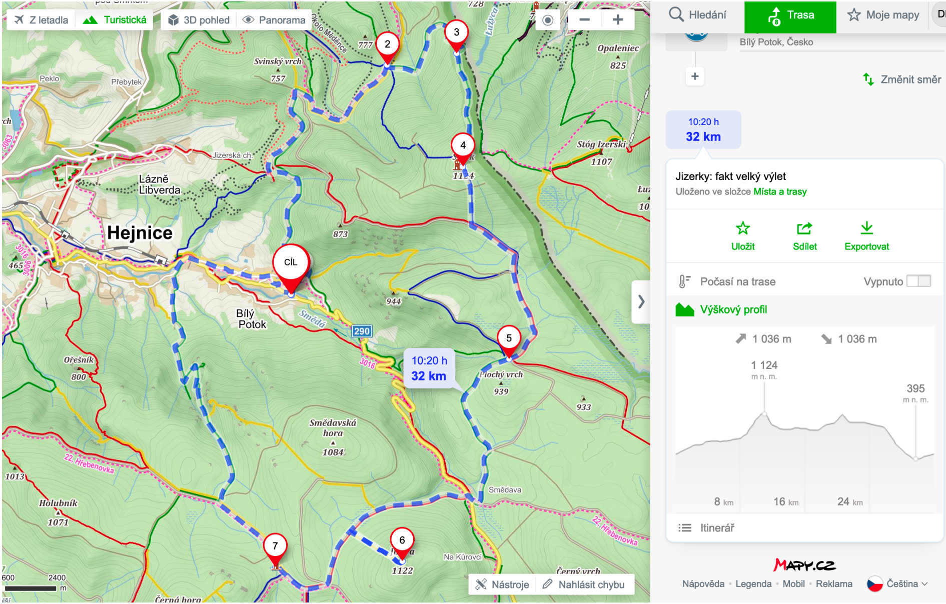
- New measurements can improve our models and their input
- Most important – better understanding of QCD at high energies ahead of us!

WEDNESDAY HIKE!!!

Ultimate hike via all the hills around Bílý potok



Bílý Potok, Česko

10:20 h
32 km

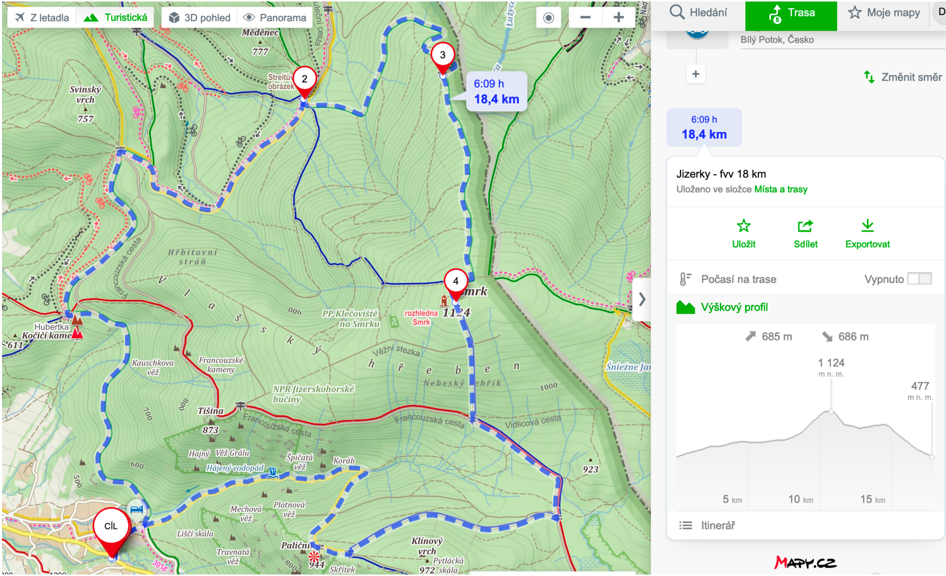
Jizerky: fakt velký výlet
Uloženo ve složce Místa a trasy

Počasí na trase Vypnuto

Výškový profil

1 036 m ↗ 1 036 m ↘

A shorter variant



Bílý Potok, Česko

6:09 h
18,4 km

Jizerky - fvv 18 km
Uloženo ve složce Místa a trasy

Počasí na trase Vypnuto

685 m 686 m




Beyond the (SM) limits!