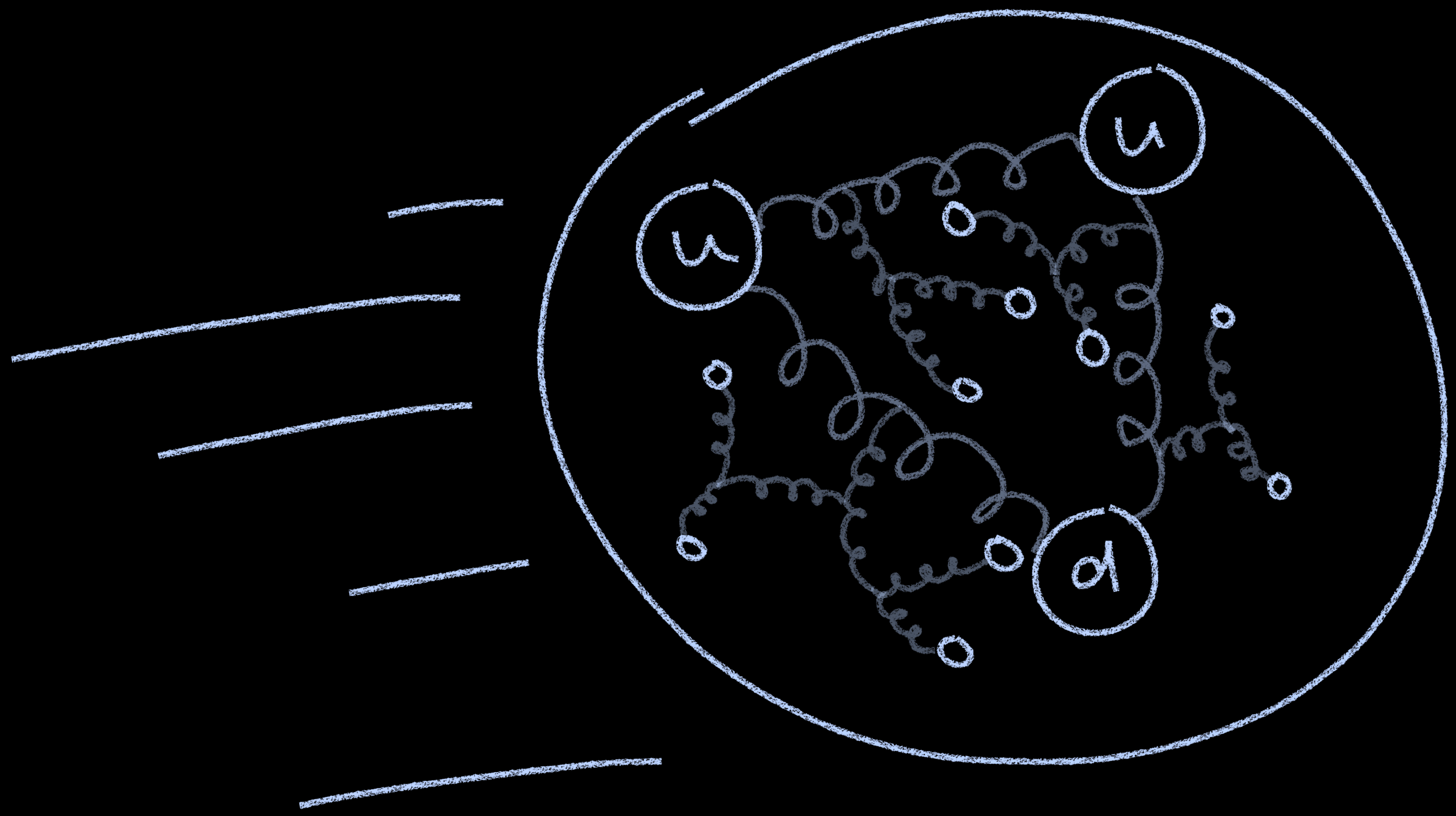
Study of non-linear evolution of the hadron structure within quantum chromodynamics

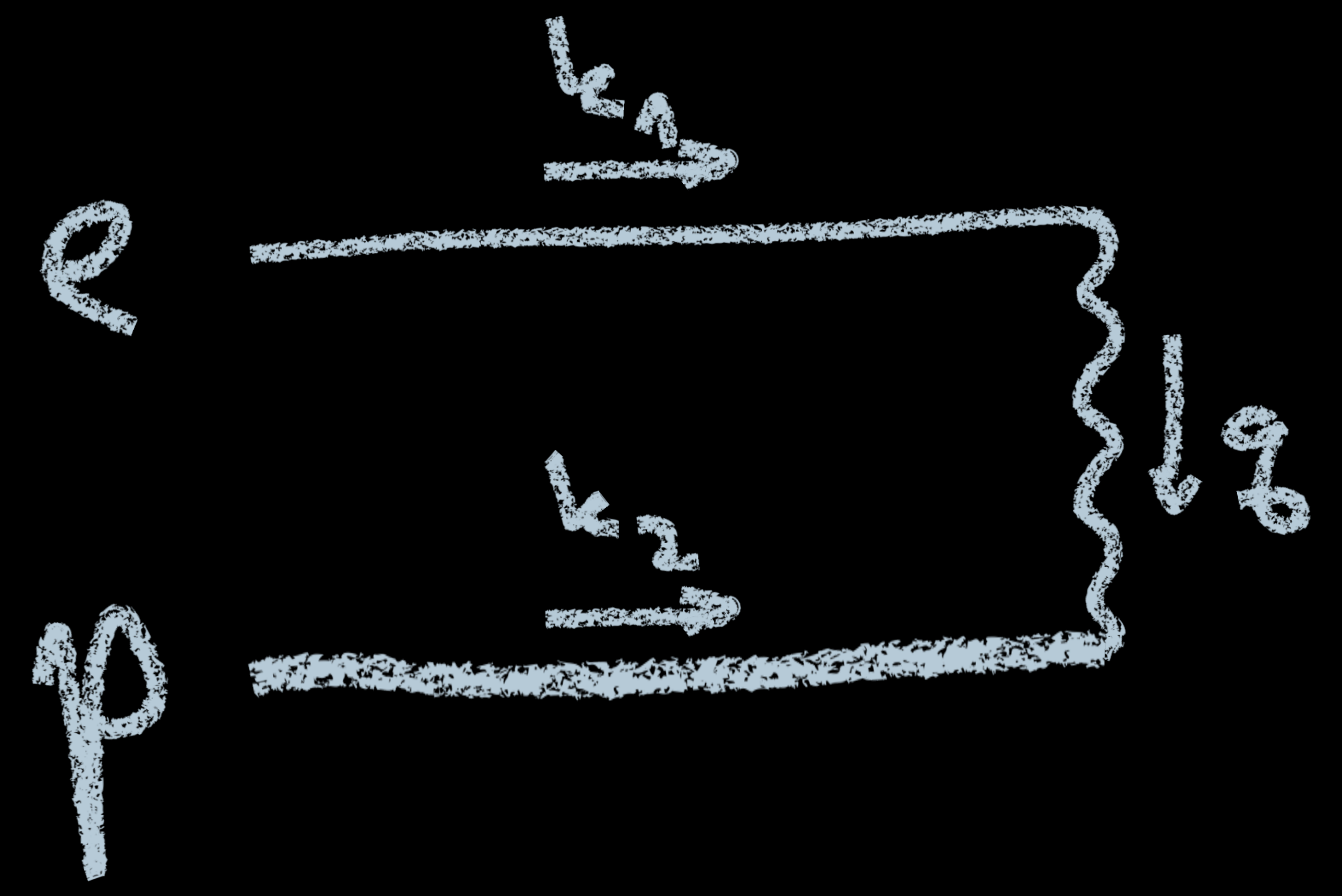
Matěj Vaculčiak, 13 June 2022

doc. Jan Čepila

Study of non-linear evolution of the **hadron structure** within quantum chromodynamics



Electron-proton scattering



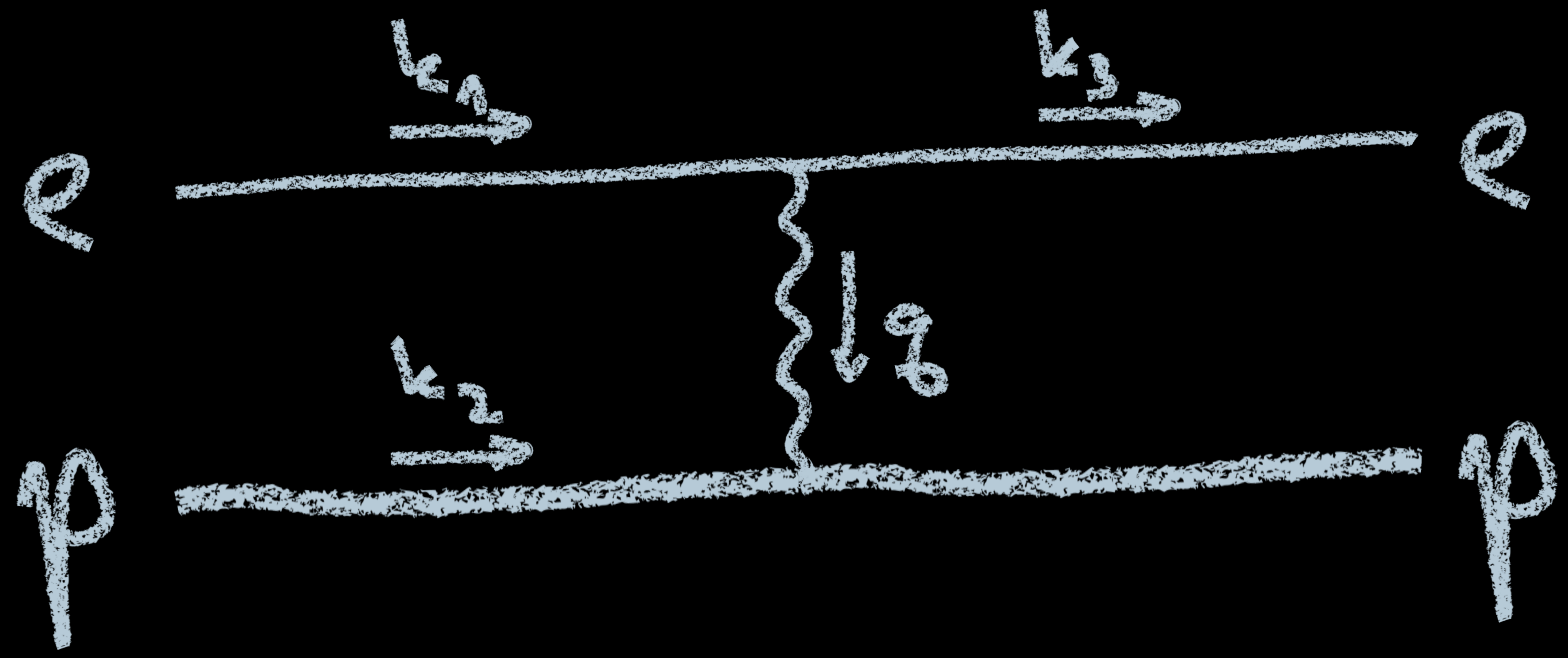
Electron-proton scattering

elastic scattering

$y := 1 - \frac{E_3}{E_1}$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{m_p^2 y^2}{Q^2}\right) f_2(Q^2) + \frac{1}{2} y^2 f_1(Q^2)\right]$$

$Q^2 := -q^2$



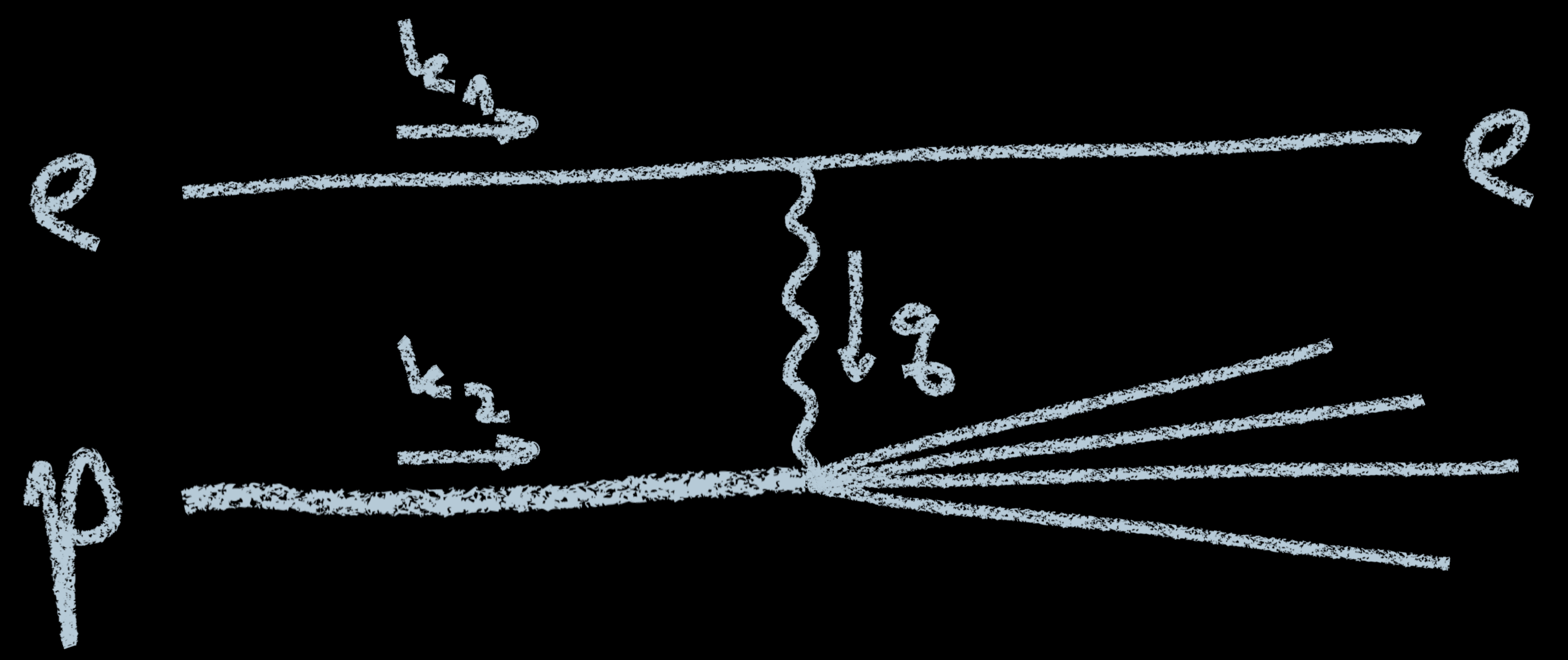
Electron-proton scattering

inelastic scattering

$$\frac{d^2\sigma}{dQ^2 dx}$$

$$\frac{F_2(x,Q^2)}{x} \qquad 2F_1(x,Q^2)$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{m_p^2 y^2}{Q^2}\right) f_2(Q^2) + \frac{1}{2} y^2 f_1(Q^2)\right]$$

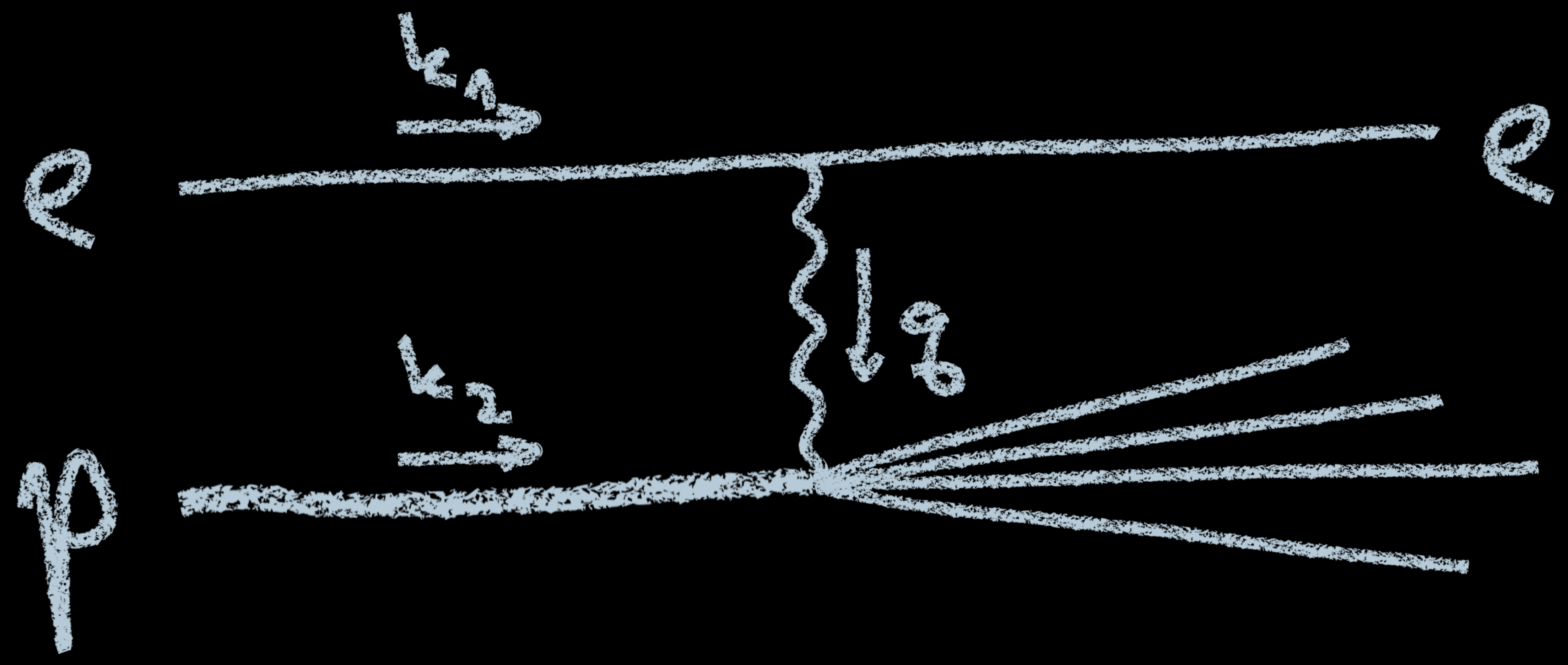


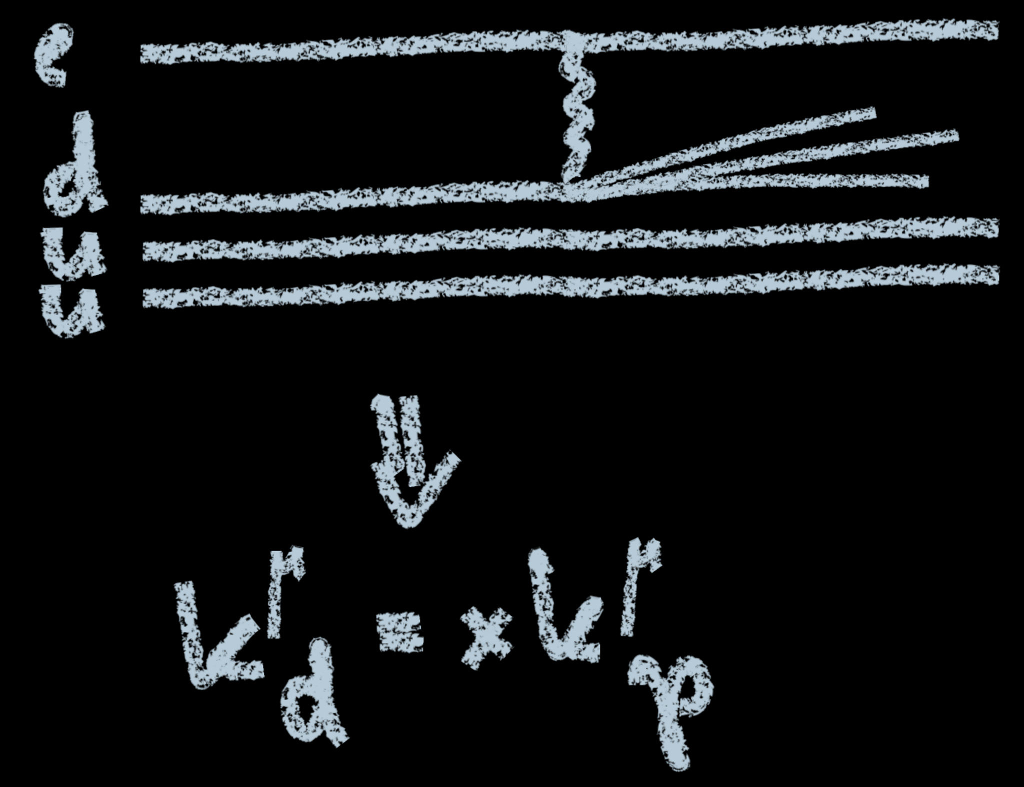
Electron-proton scattering

inelastic scattering

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}Q^2\mathrm{d}x} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{m_p^2 y^2}{Q^2}\right)\frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2)\right]$$

Bjorken x

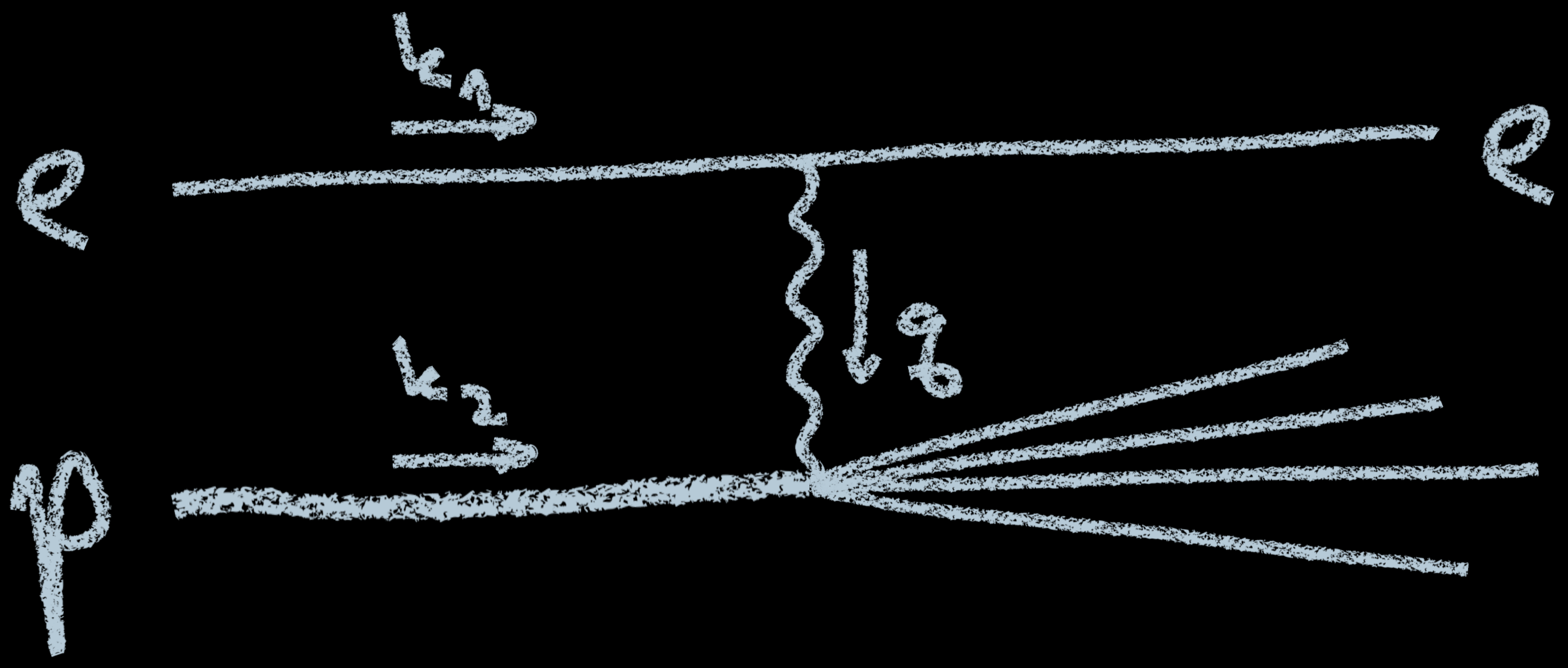




Electron-proton scattering

deep inelastic scattering

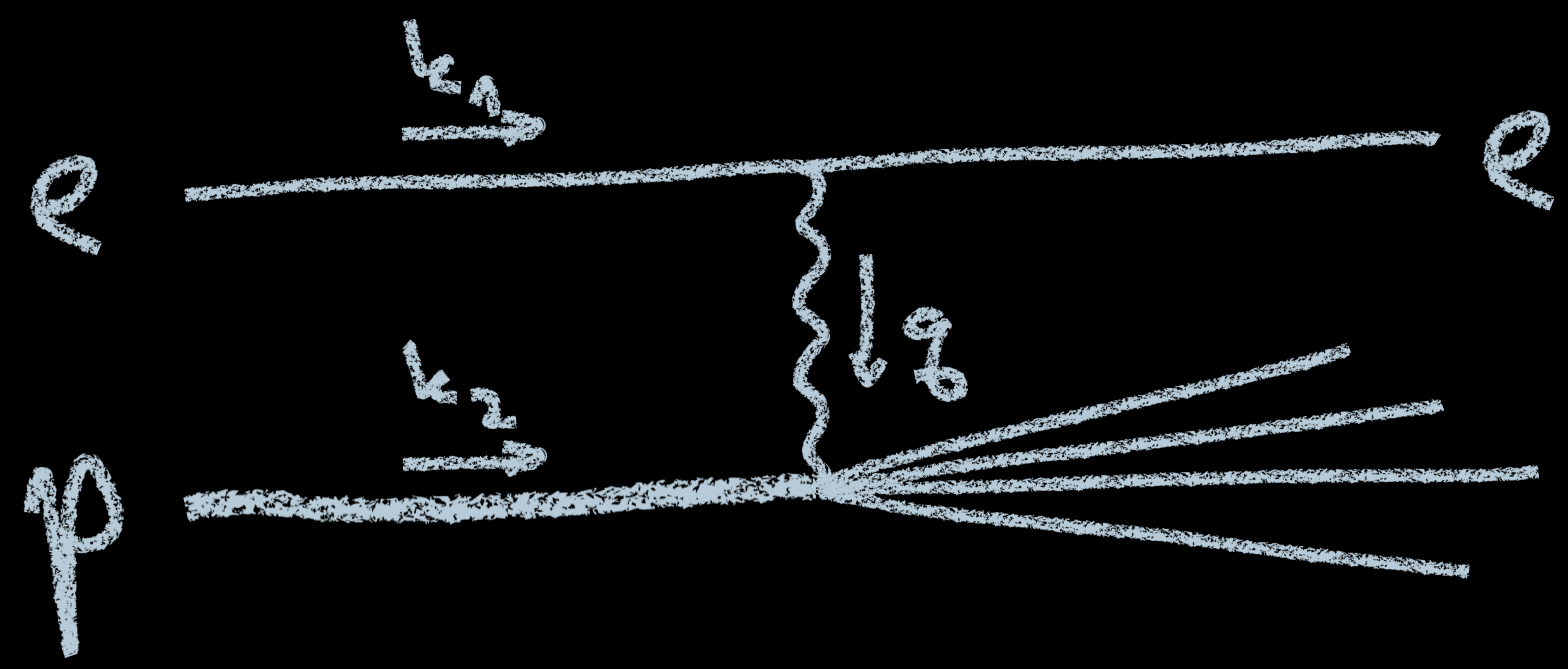
$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}Q^2\mathrm{d}x} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{m_p^2 y^2}{Q^2}\right)\frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2)\right]$$



Electron-proton scattering

deep inelastic scattering

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}Q^2\mathrm{d}x} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1-y\right)\frac{F_2(x,Q^2)}{x} + y^2F_1(x,Q^2)\right]$$

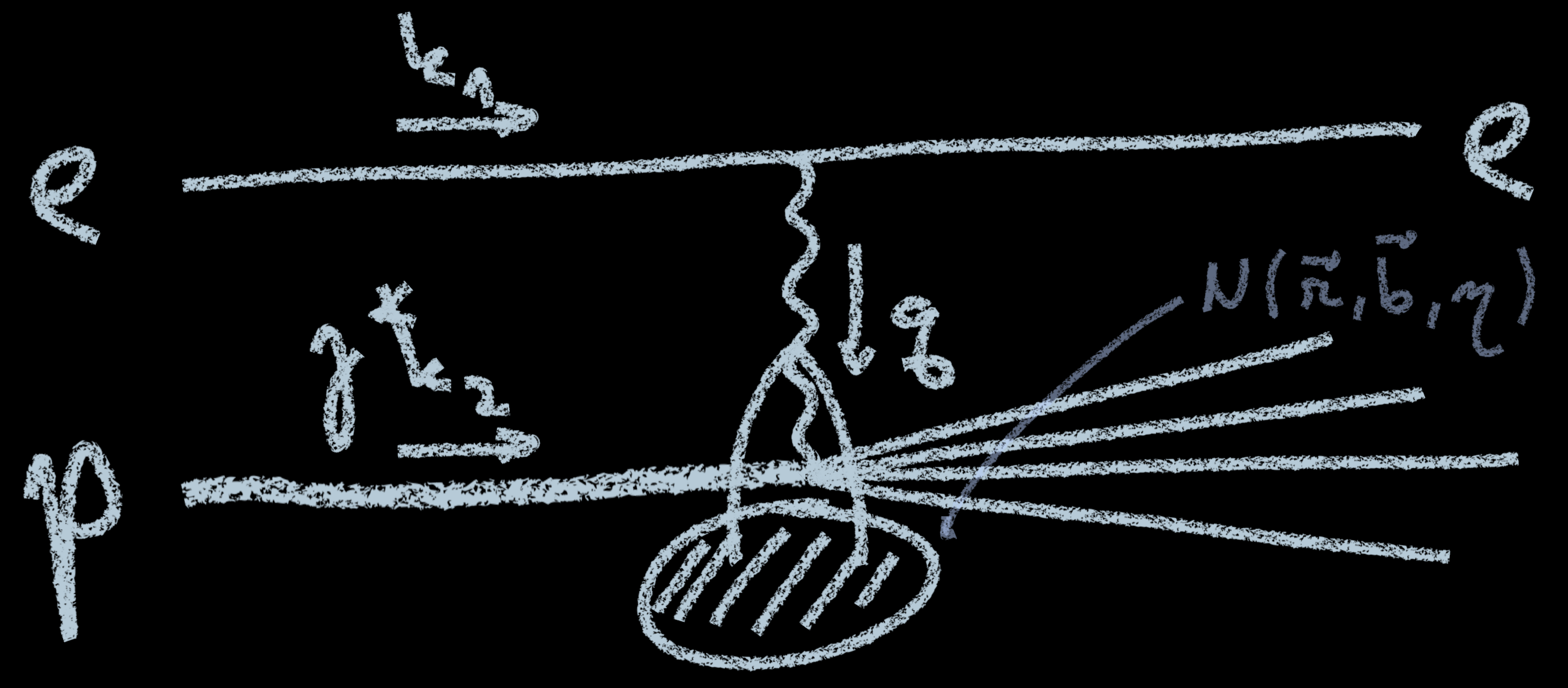


Study of non-linear evolution of the hadron structure within quantum chromodynamics

Electron-proton scattering

deep inelastic scattering

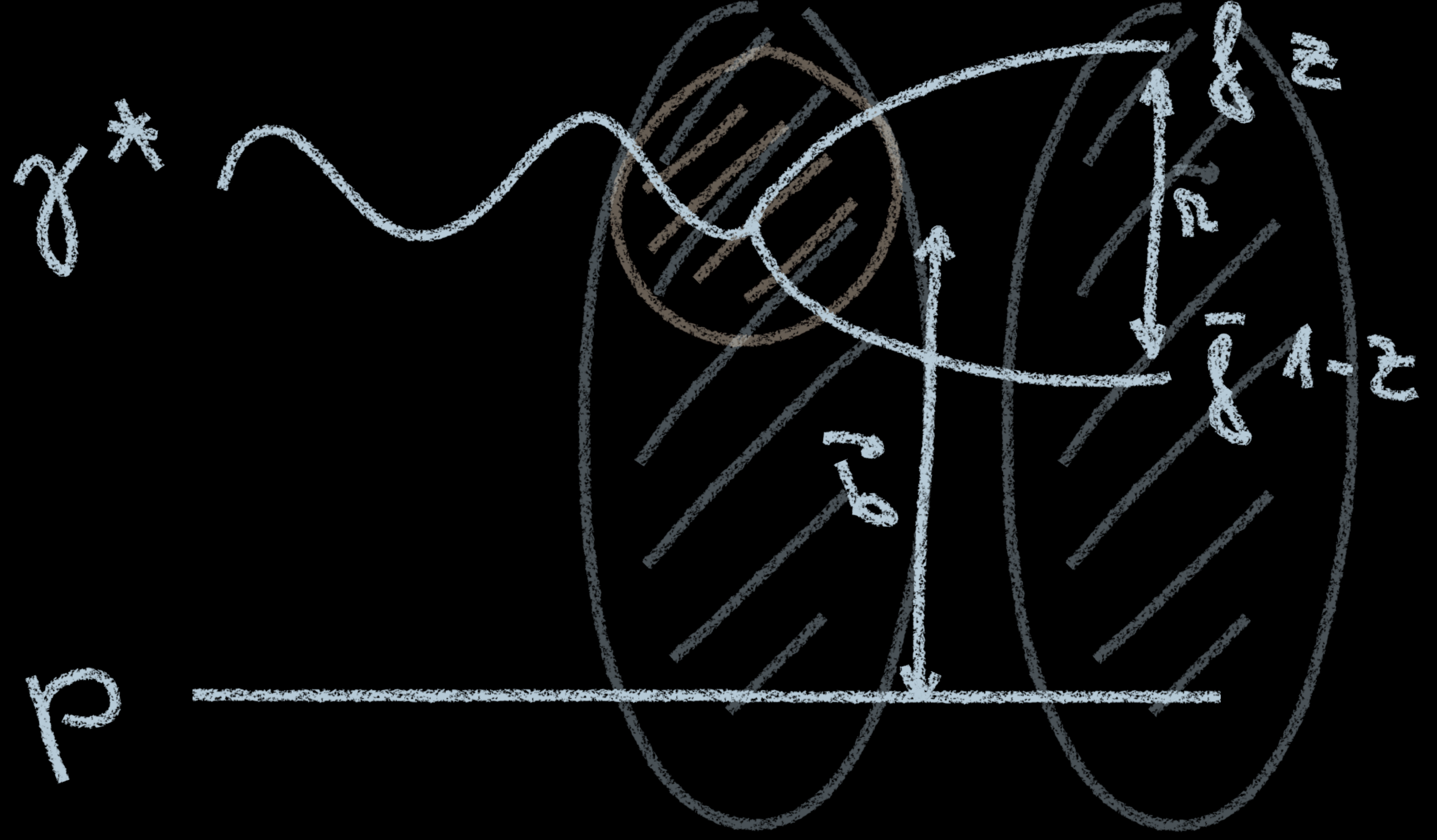
$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}Q^2\mathrm{d}x} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1-y\right)\frac{F_2(x,Q^2)}{x} + y^2F_1(x,Q^2)\right]$$



Dipole scattering amplitude

$$F_2(x,Q^2) = \frac{Q^2}{4\pi\alpha_{em}}\left(\sigma_L^{\gamma^*p}\left(x,Q^2\right) + \sigma_T^{\gamma^*p}\left(x,Q^2\right)\right)$$

$$F_L(x,Q^2) = \frac{Q^2}{4\pi\alpha_{em}}\sigma_L^{\gamma^*p}\left(x,Q^2\right)$$



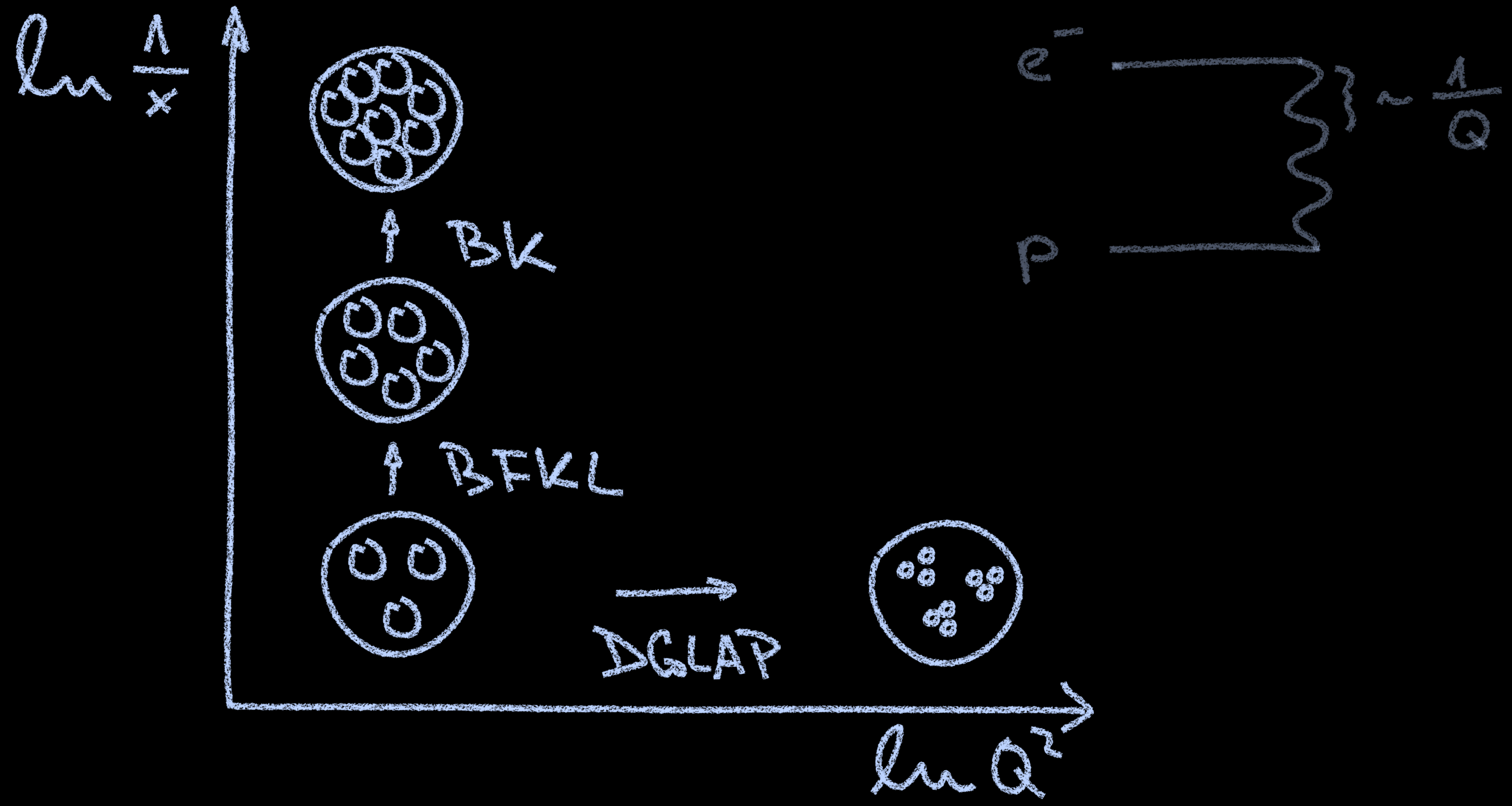
- the photon-proton cross-section

$$\sigma_{L,T}^{\gamma^*p}(x,Q^2) = \sum_f \int \mathrm{d}^2\vec{r}\int_0^1 \mathrm{d}z\,|\psi_{T,L}^{(f)}(\vec{r},Q^2,z)|^2\, 2\int \mathrm{d}^2\overrightarrow{b}\, N(\vec{r},\overrightarrow{b},\tilde{x}_f(x))$$

$\tilde{x}_f = x\left(1+\frac{4m_f^2}{Q^2}\right)$

Study of non-linear evolution of the hadron structure within quantum chromodynamics

Study of non-linear **evolution** of the hadron structure within quantum chromodynamics



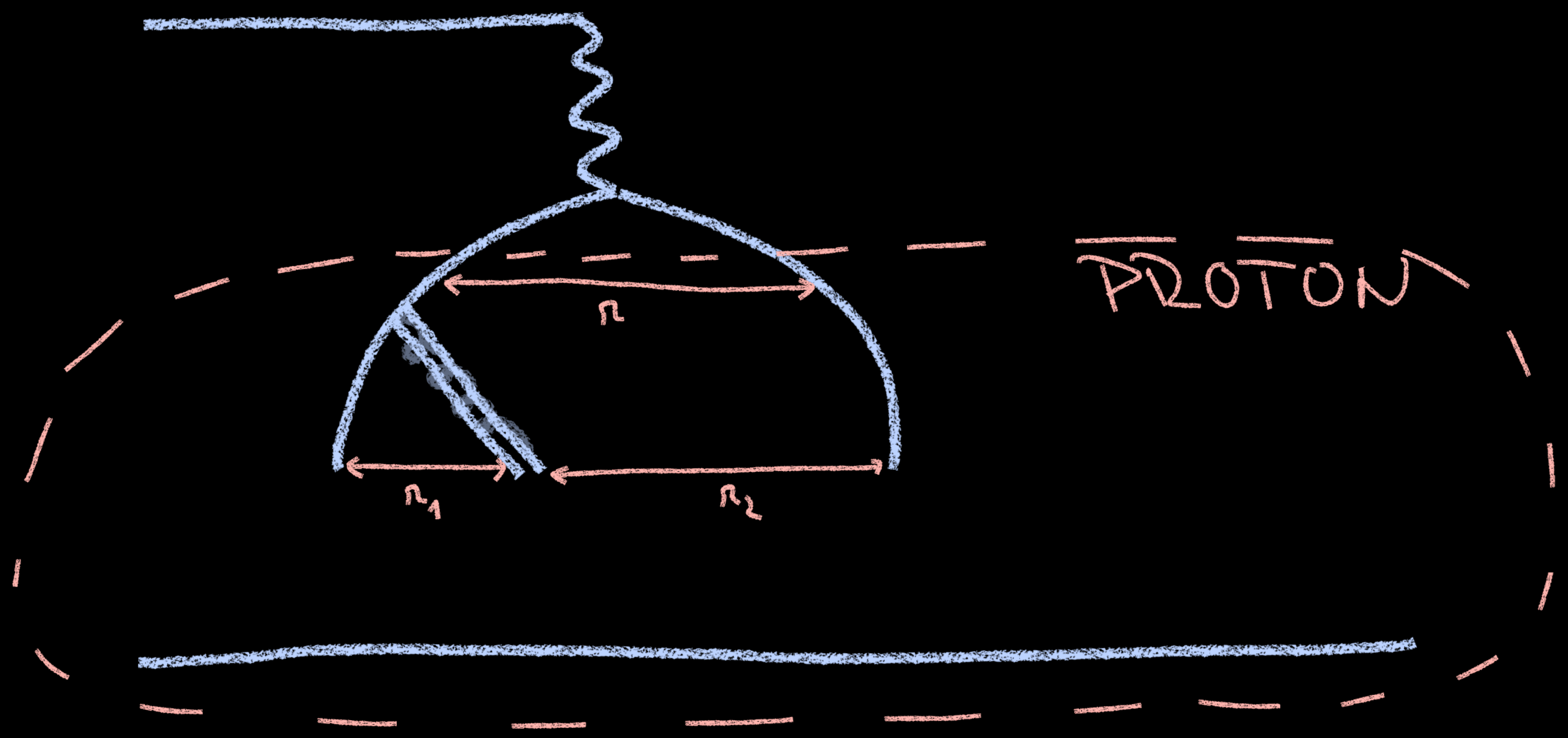
Study of non-linear **evolution** of the hadron structure within quantum chromodynamics

$$\frac{\partial N(\vec{r}, b, Y)}{\partial Y} = \int \mathrm{d}\vec{r}_1 K(r, r_1, r_2) \left[N(r_1, b_1, Y) + N(r_2, b_2, Y) - N(r, b, Y) - N(r_1, b_1, Y) N(r_2, b_2, Y) \right]$$

kernel

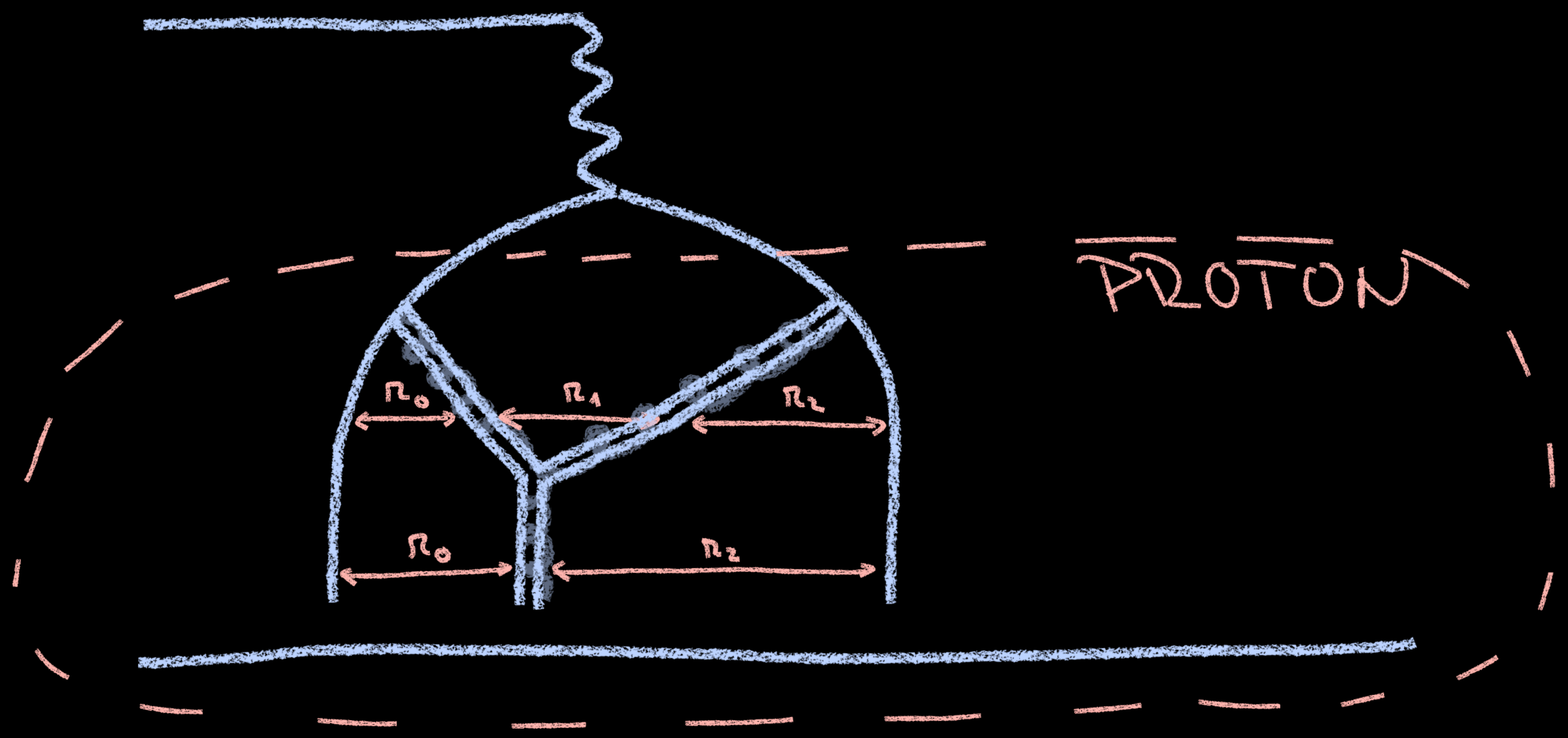
Y(x)

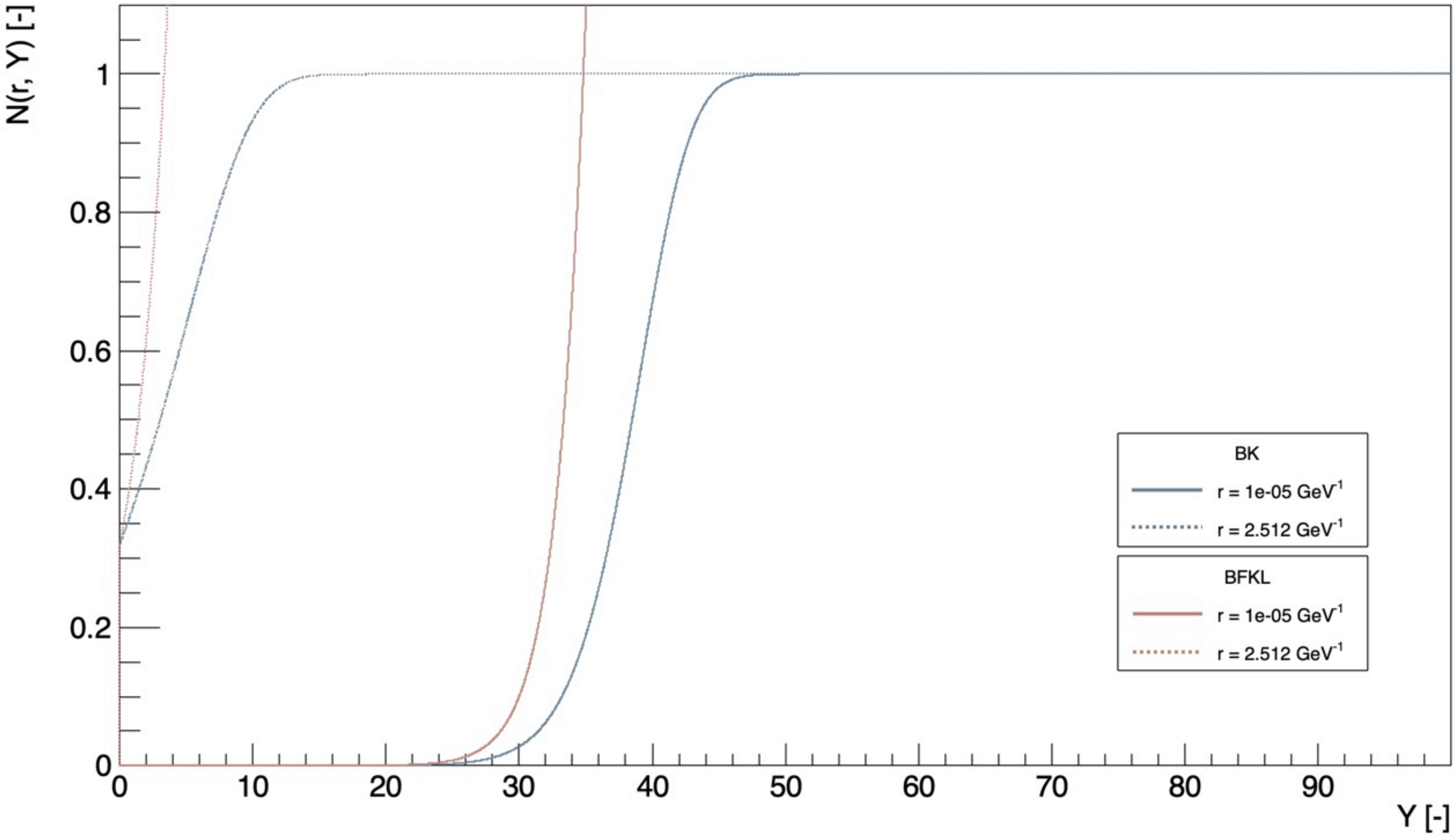
BFKL



Study of **non-linear evolution** of the hadron structure within quantum chromodynamics

$$\frac{\partial N(r,b,Y)}{\partial Y} = \int \mathrm{d}\vec{r}_1 K(r,r_1,r_2) \; \left[N(r_1,b_1,Y) + N(r_2,b_2,Y) - N(r,b,Y) - N(r_1,b_1,Y)N(r_2,b_2,Y)\right]$$



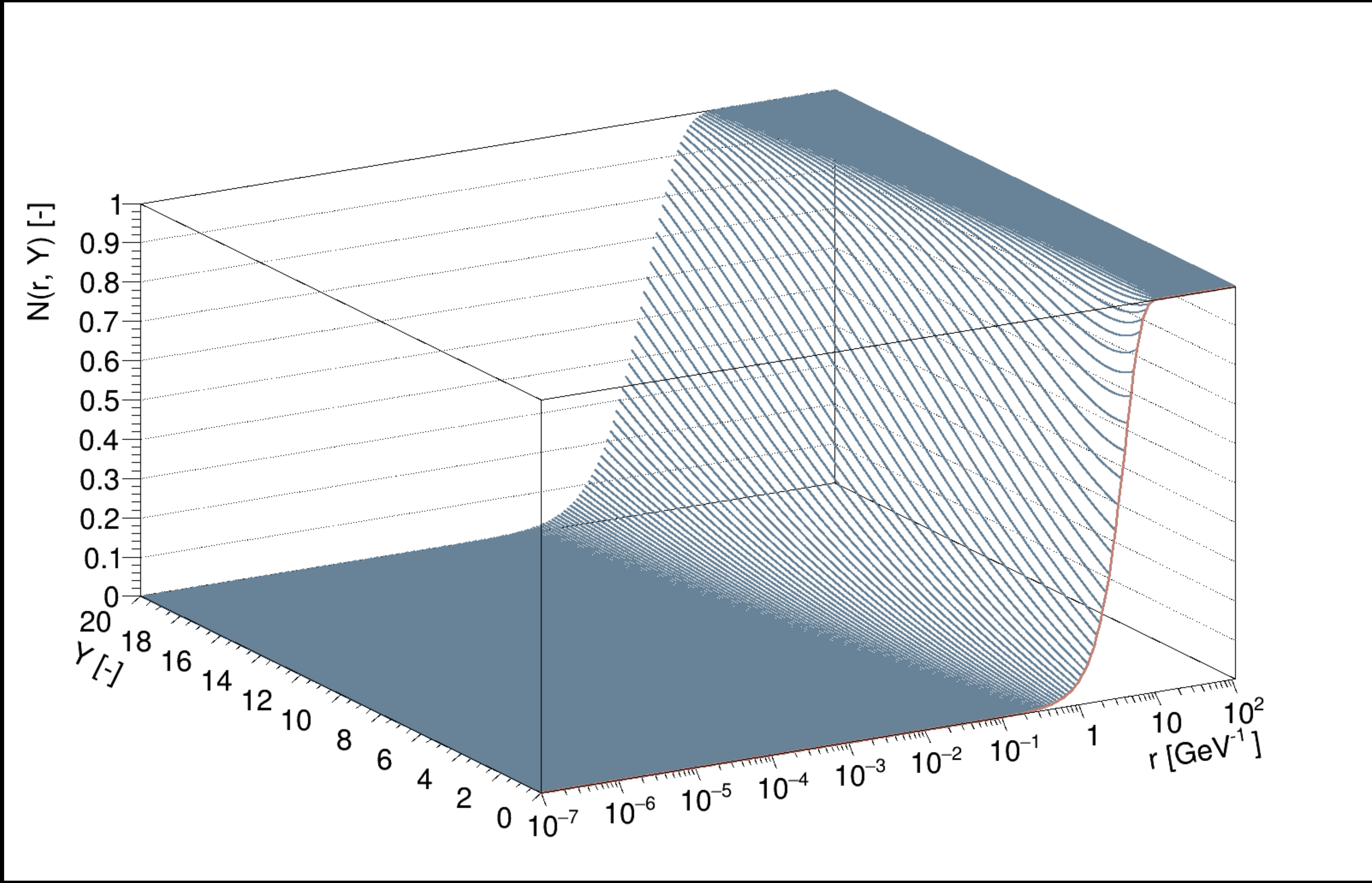

N(r, Y) [-]
1
0.8
0.6
0.4
0.2
0
0
10
20
30
40
50
60
70
80
90
Y [-]
BK
r = 1e-05 GeV-1
r = 2.512 GeV-1
BFKL
r = 1e-05 GeV-1
r = 2.512 GeV-1

1D solution

$$2\int \mathrm{d}\overrightarrow{b}\, N(\underbrace{\vec{r}, \overrightarrow{b}}_{\text{4 dim}}, Y) \approx \sigma_0 N(r, Y)$$

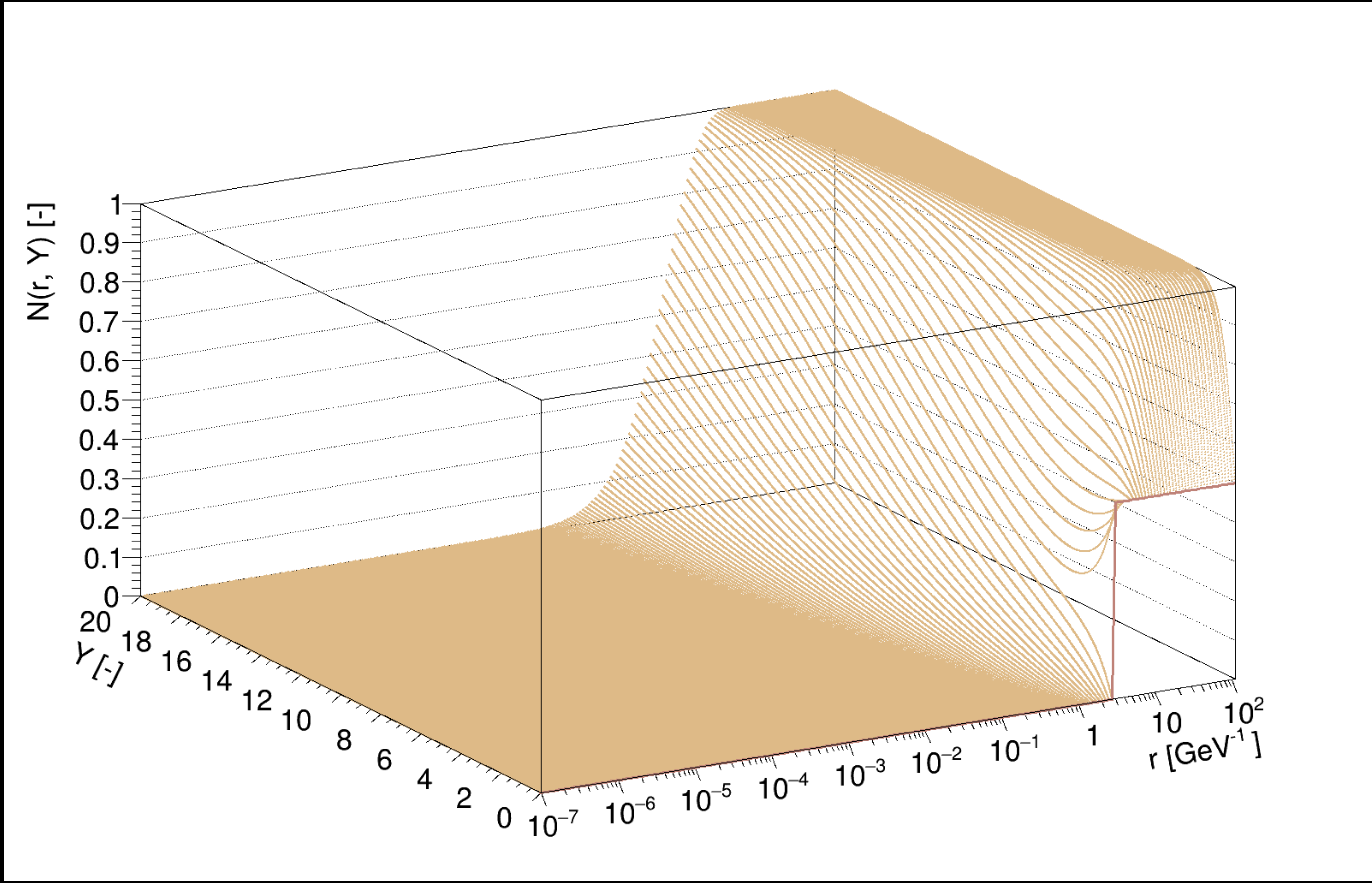
4 dim

1D solution



N(r, Y) [-]
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Y [-]
20
18
16
14
12
10
8
6
4
2
0
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
10
10^{2}
r [GeV^{-1}]

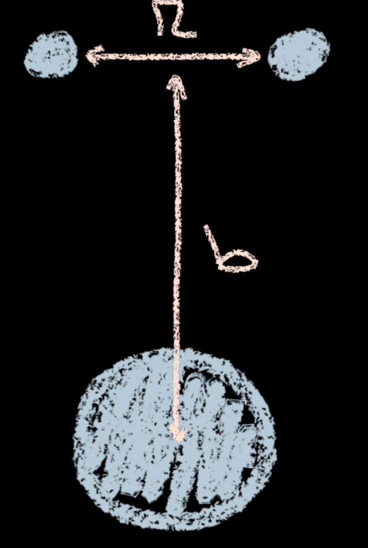
1D solution



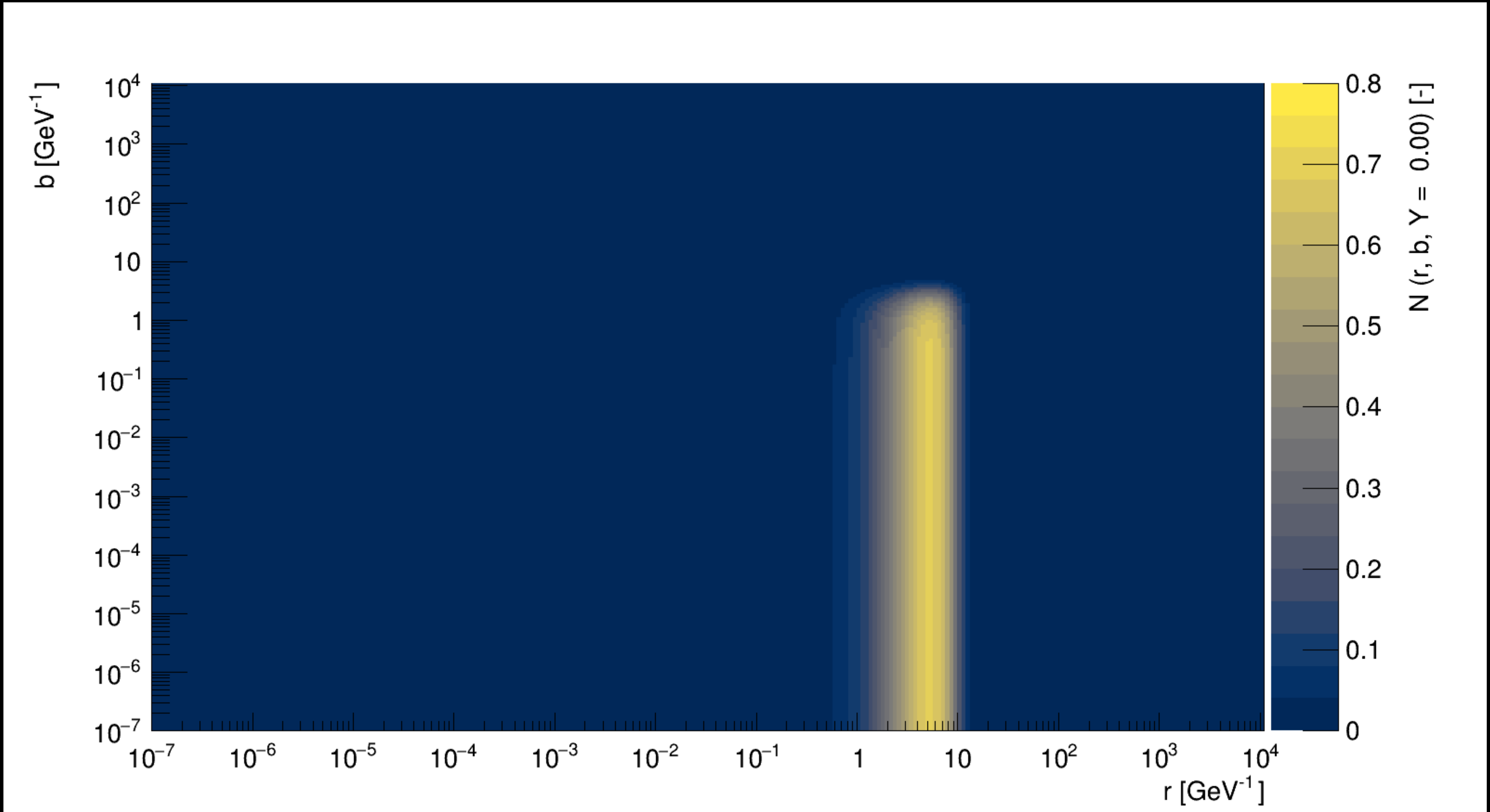
N(r, Y) [-]
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Y [-]
20
18
16
14
12
10
8
6
4
2
0
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
10
10^{2}
r [GeV^{-1}]

2D solution

$$2\int \mathrm{d}\overrightarrow{b}\, N(\vec{r}, \overrightarrow{b}, Y) \approx 4\pi \int \mathrm{d}b N(r, b, Y)$$

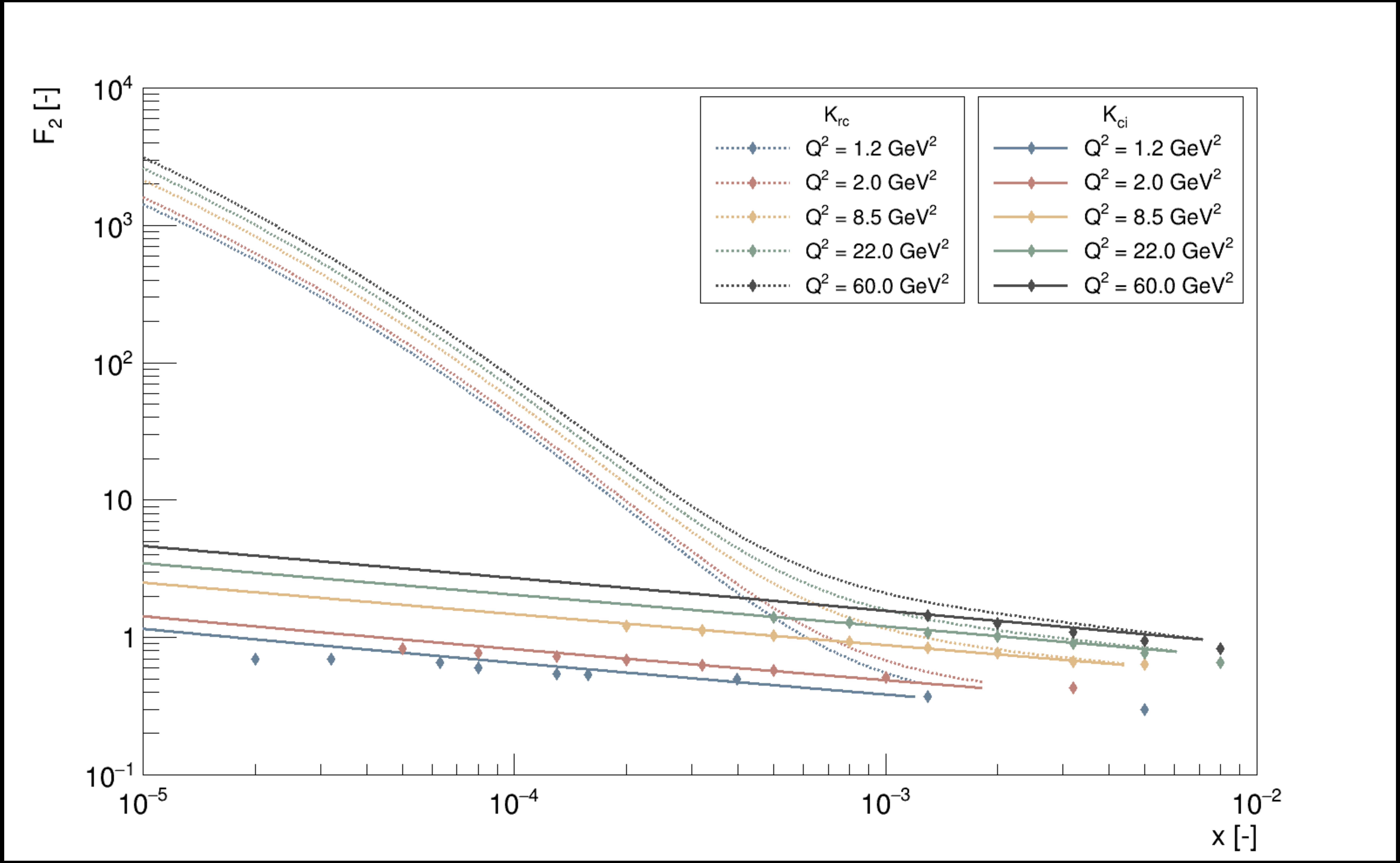


2D solution



b [GeV^{-1}]
10^4
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 10 10^2 10^3 10^4
r [GeV^{-1}]
N (r, b, Y = 0.00) [-]
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0

Structure functions

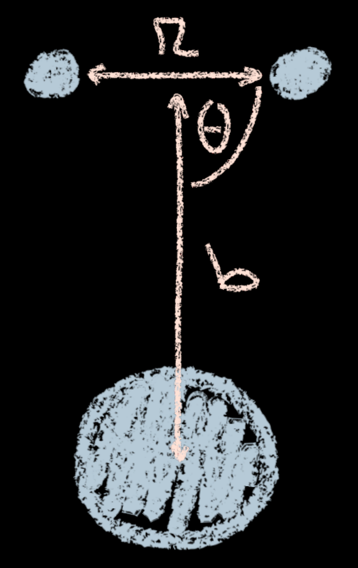


Future goals

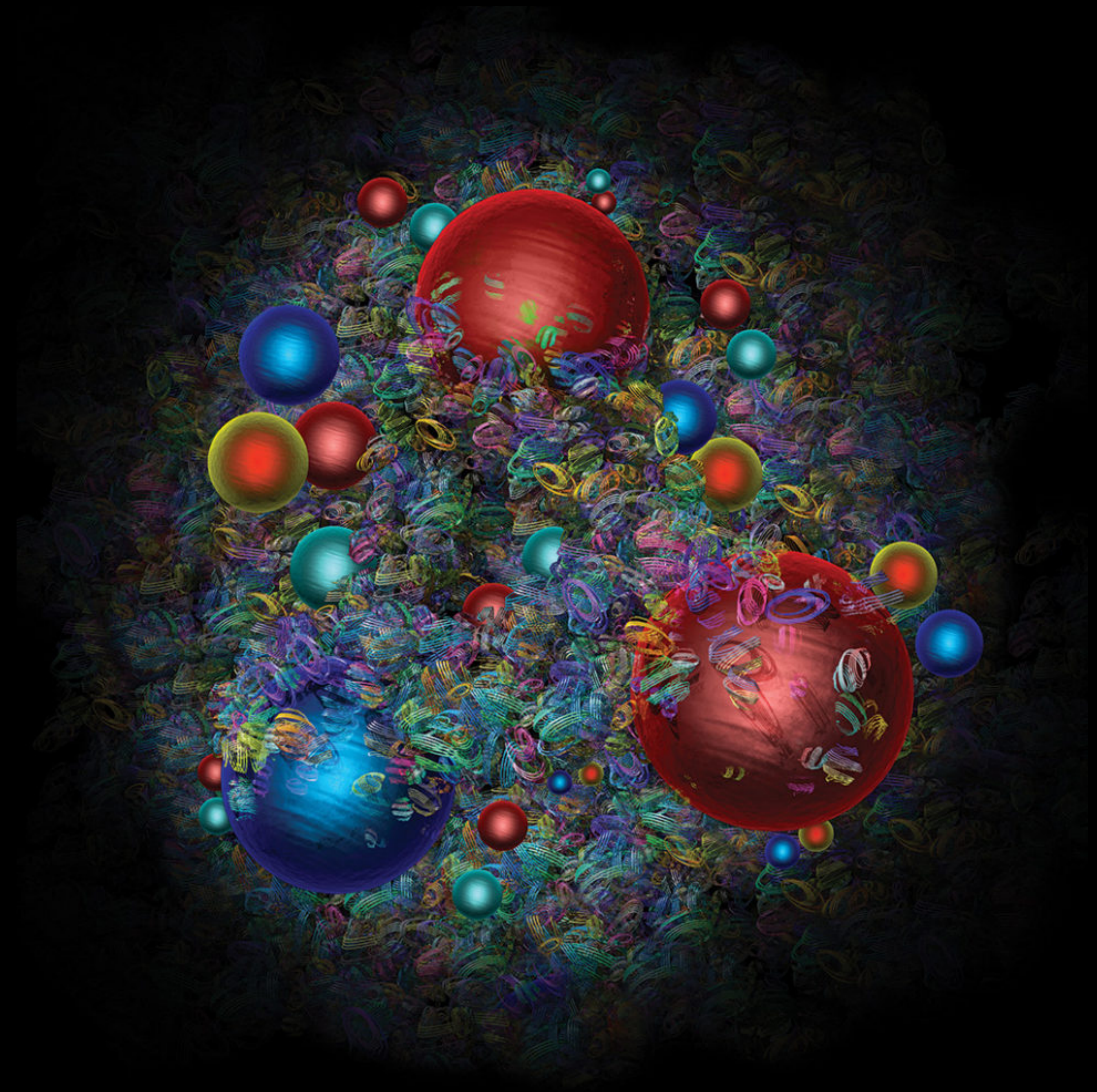
- the BK equation in full dimension

- numerical calculation optimization

- higher orders of the perturbation theory



Thank you for your attention



Daniel Dominguez. “An artist’s impression of the mayhem of quarks and gluons inside the proton.” In: (June 2019). General Photo. url: https: //cds.cern.ch/record/2678286

Kernels

- BFKL kernel

$$K_{BFKL} = \frac{\alpha_s N_C}{2\pi} \frac{r^2}{r_1^2 r_2^2}$$

- Running coupling kernel

$$K_{rc} = \frac{\alpha_s(r) N_C}{2\pi} \left[\frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_1^2} \left(\frac{\alpha_s(r_1)}{\alpha_s(r_2)} - 1 \right) + \frac{1}{r_2^2} \left(\frac{\alpha_s(r_2)}{\alpha_s(r_1)} - 1 \right) \right]$$

- Collinearly improved kernel

$$K_{ci} = \frac{\bar{\alpha}_s}{2\pi} \frac{r^2}{r_1^2 r_2^2} \left[\frac{r^2}{\min\{r_1^2, r_2^2\}} \right]^{\pm \bar{\alpha}_s A_1} \frac{J_1(2\rho\sqrt{\bar{\alpha}_s})}{\rho\sqrt{\bar{\alpha}_s}}$$

$$\rho = \sqrt{\left| \ln\left(\frac{r_1^2}{r^2}\right) \ln\left(\frac{r_2^2}{r^2}\right) \right|}$$

$$\bar{\alpha}_s = \frac{N_c}{\pi} \alpha_s(\min\{r^2, r_1^2, r_2^2\})$$

Initial conditions

- GBW initial condition

$$N_{GBW}(r, Y=0) = 1 - \exp\left[-\frac{(r^2Q_{s0}^2)^\gamma}{4}\right]$$

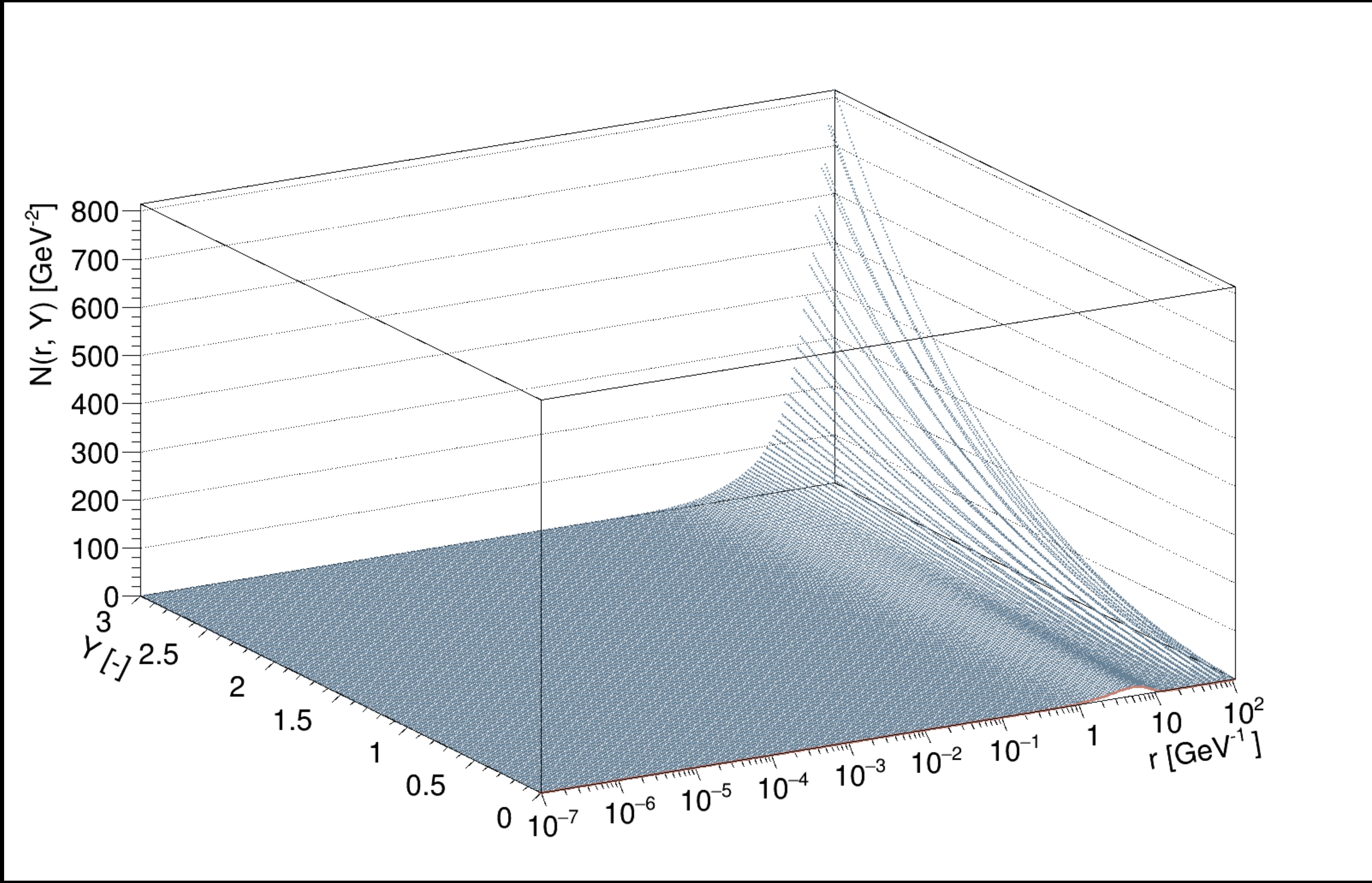
- MV initial condition

$$N_{MV}(r, Y=0) = 1 - \exp\left[-\frac{(r^2Q_{s0}^2)^\gamma}{4}\ln\left(\frac{1}{r\Lambda_{QCD}} + e\right)\right]$$

- b-dependent initial condition

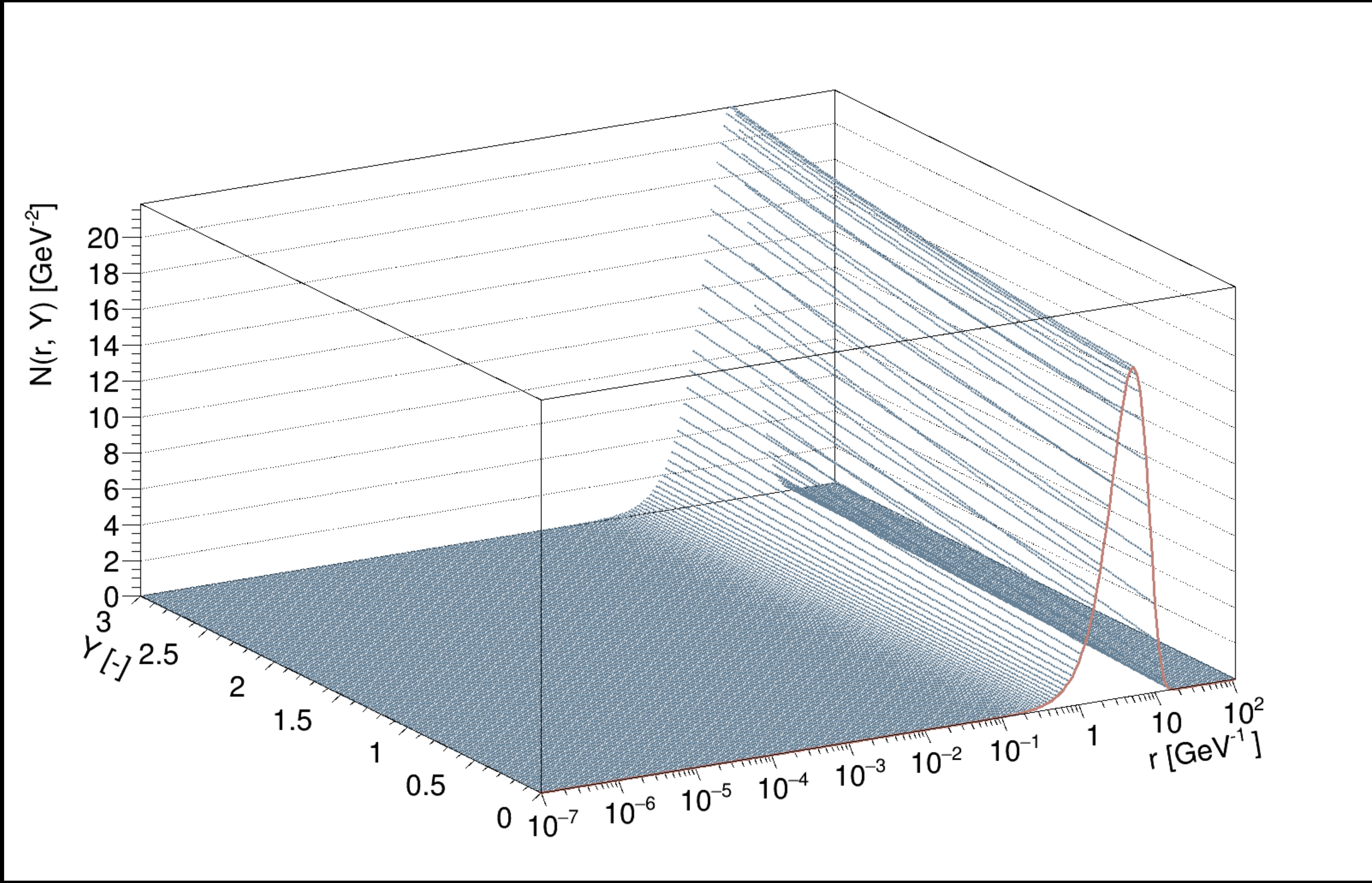
$$N_b(r, b, Y=0) = 1 - \exp\left[-\frac{1}{2}\frac{r^2Q_{s0}^2}{4}\left(e^{-\frac{d_1(\vec{r},\vec{b})}{2B}} + e^{-\frac{d_2(\vec{r},\vec{b})}{2B}}\right)\right]$$

2D mixed



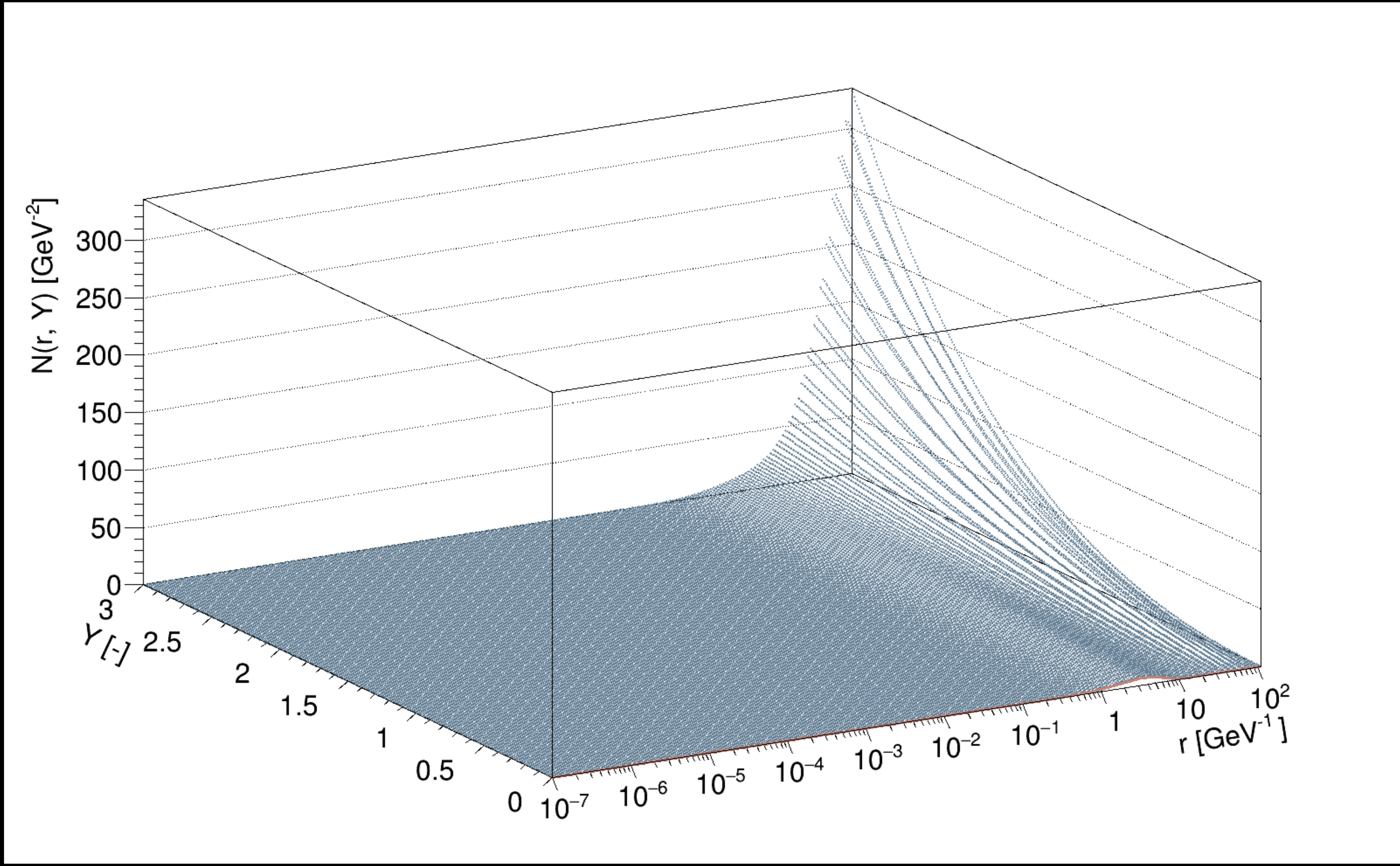
N(r, Y) [GeV^-2]
800
700
600
500
400
300
200
100
0
Y [-]
3
2.5
2
1.5
1
0.5
0
10^-7
10^-6
10^-5
10^-4
10^-3
10^-2
10^-1
1
10
10^2
r [GeV^-1]

2D $\theta = \pi/2$



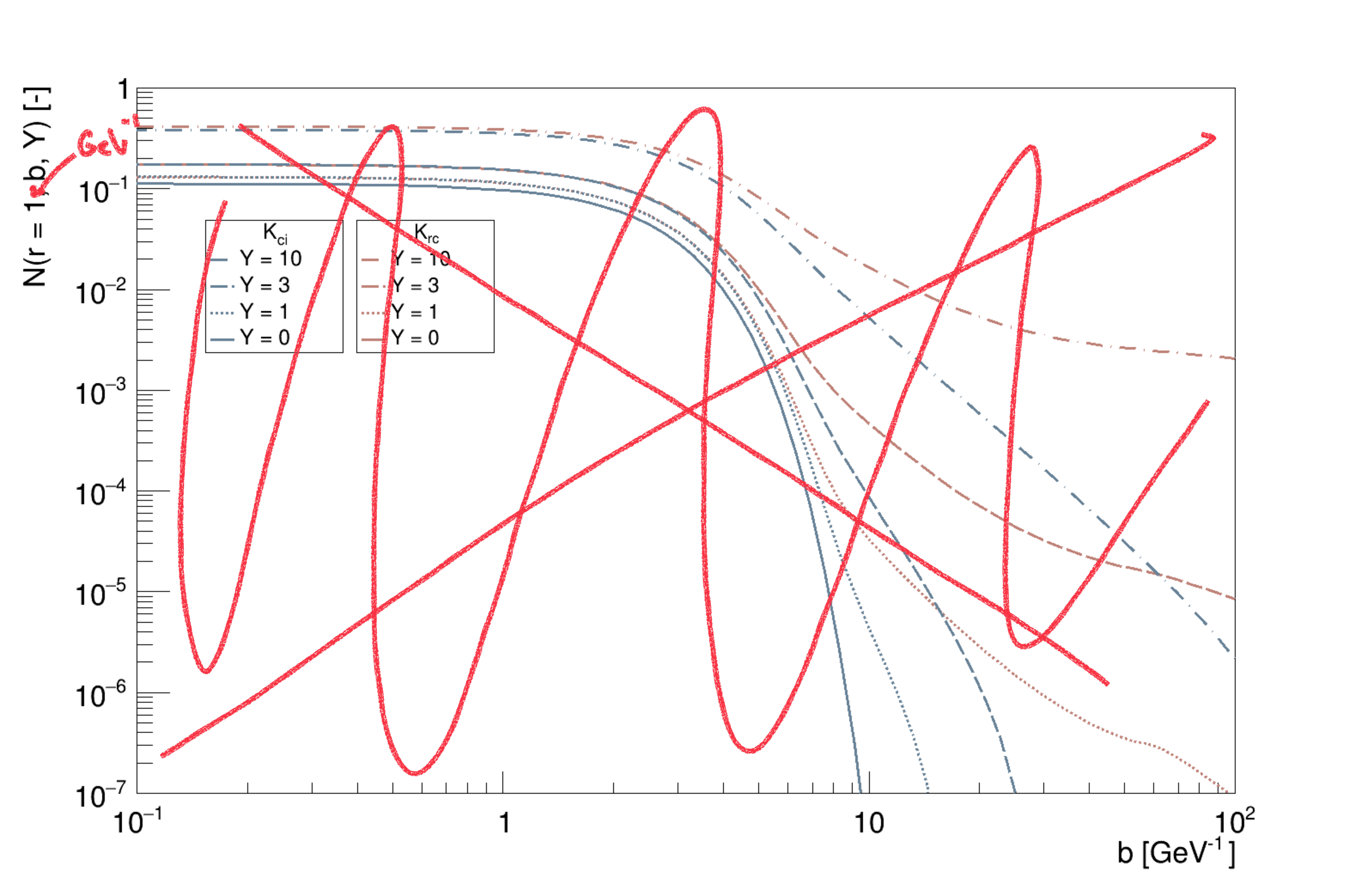
N(r, Y) [GeV^-2]
20
18
16
14
12
10
8
6
4
2
0
Y [-]
3
2.5
2
1.5
1
0.5
0
10^-7
10^-6
10^-5
10^-4
10^-3
10^-2
10^-1
1
10
10^2
r [GeV^-1]

2D $\theta = \pi$

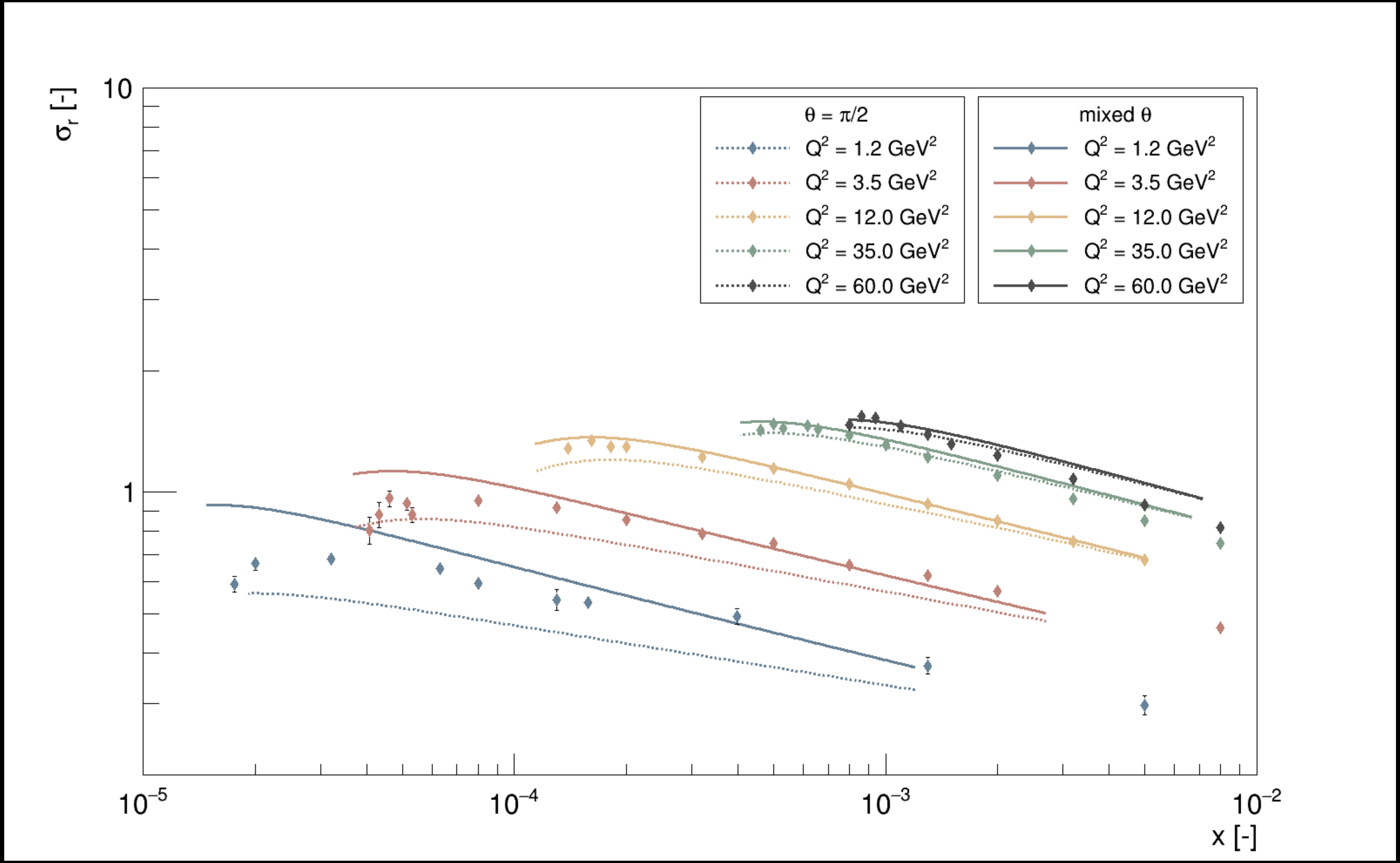


N(r, Y) [GeV^-2]
300
250
200
150
100
50
0
Y [-]
3
2.5
2
1.5
1
0.5
0
10^-7
10^-6
10^-5
10^-4
10^-3
10^-2
10^-1
1
10
10^2
r [GeV^-1]

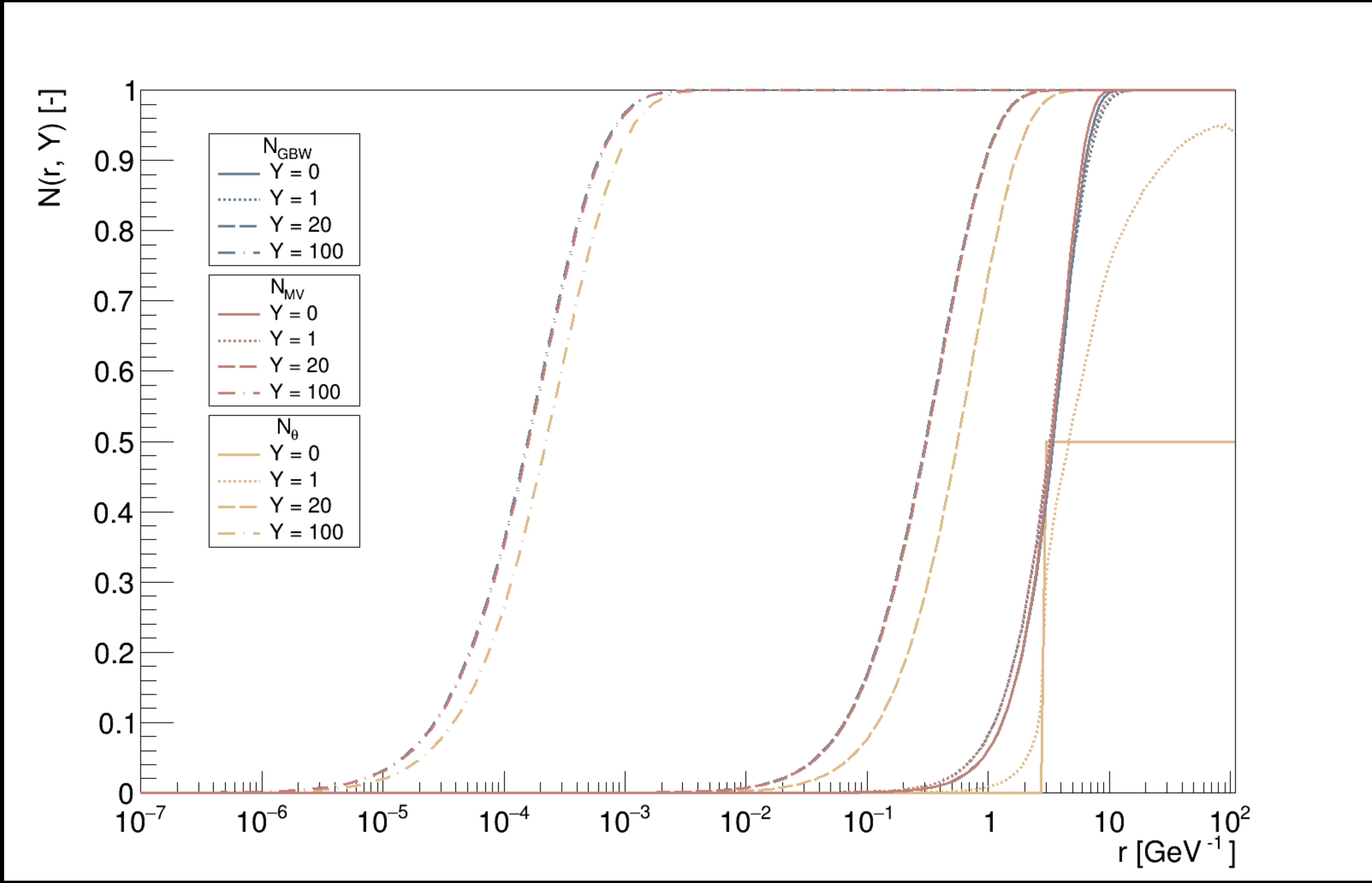
2D Coulomb tails



2D $\sigma_r(x, Q^2)$

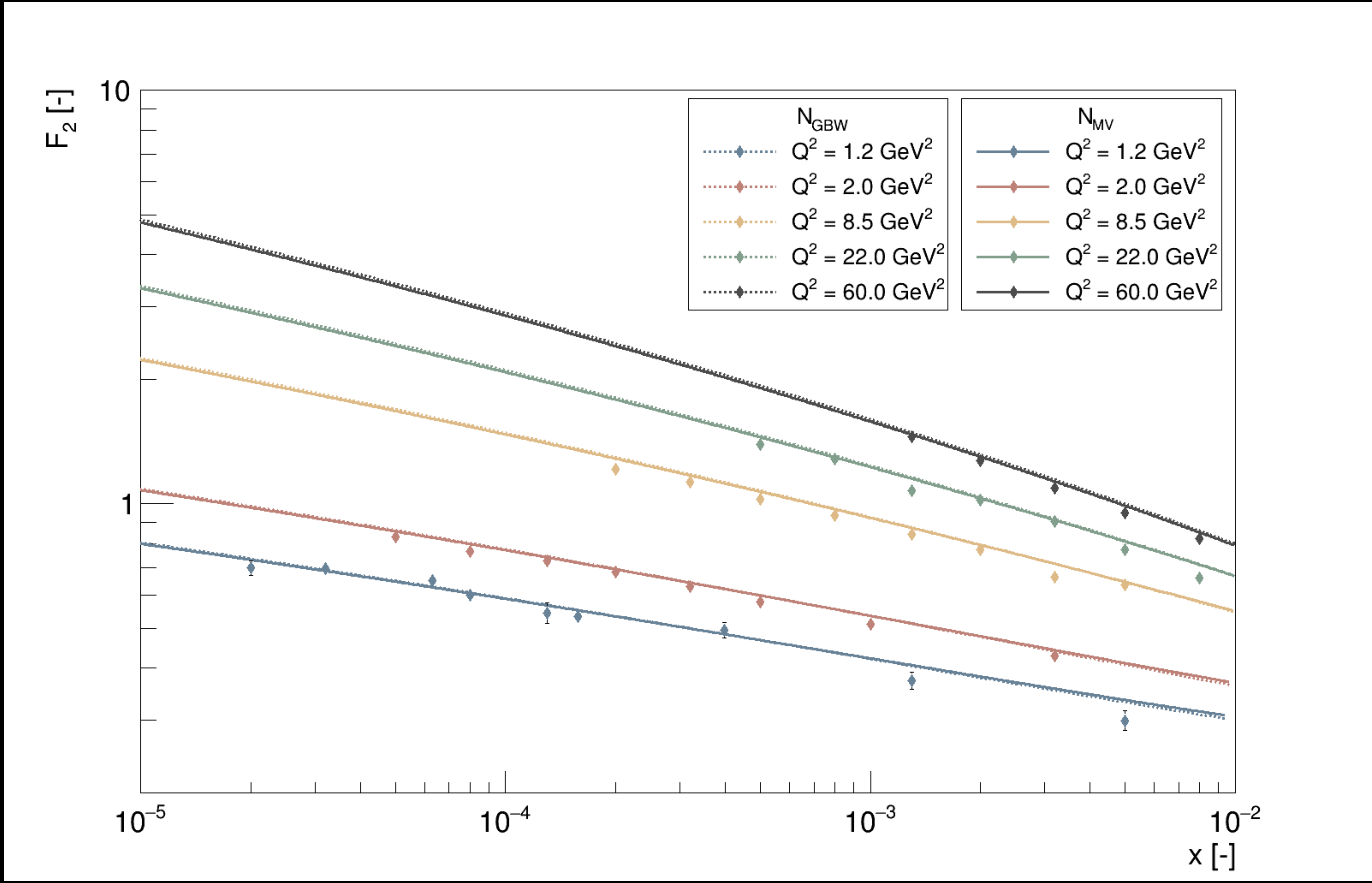


1D



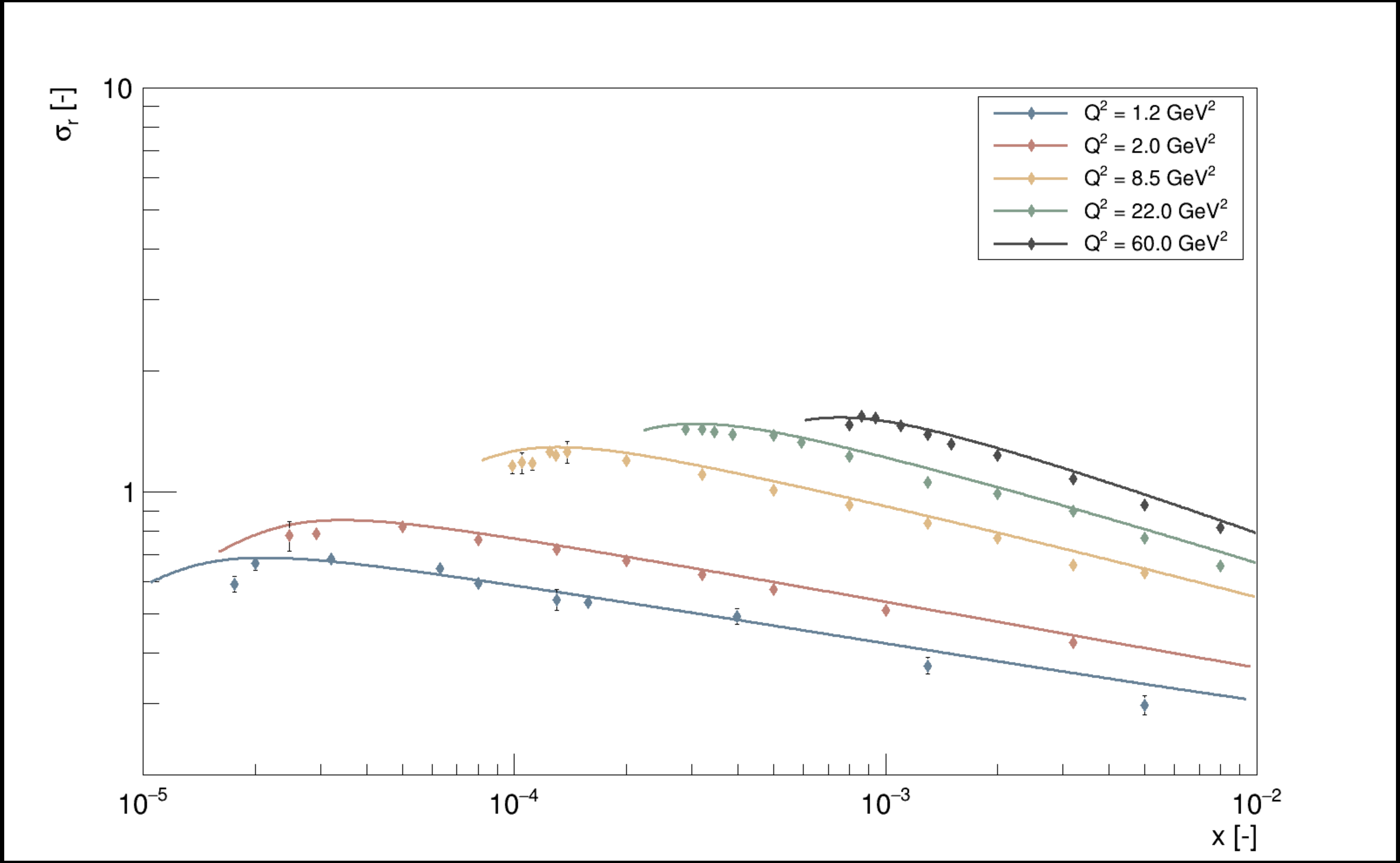
N(r, Y) [-]
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
N_{GBW}
Y = 0
Y = 1
Y = 20
Y = 100
N_{MV}
Y = 0
Y = 1
Y = 20
Y = 100
N_{θ}
Y = 0
Y = 1
Y = 20
Y = 100
10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 10 10^{2}
r [GeV^{-1}]

1D $F_2(x, Q^2)$



10
1
F_2 [-]
10^{-5}
10^{-4}
10^{-3}
10^{-2}
x [-]
N_{GBW}
Q^2 = 1.2 GeV^2
Q^2 = 2.0 GeV^2
Q^2 = 8.5 GeV^2
Q^2 = 22.0 GeV^2
Q^2 = 60.0 GeV^2
N_{MV}
Q^2 = 1.2 GeV^2
Q^2 = 2.0 GeV^2
Q^2 = 8.5 GeV^2
Q^2 = 22.0 GeV^2
Q^2 = 60.0 GeV^2

1D $\sigma_r(x, Q^2)$



Electron-proton deep inelastic scattering

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}Q^2\mathrm{d}x} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y\right)\frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2)\right]$$

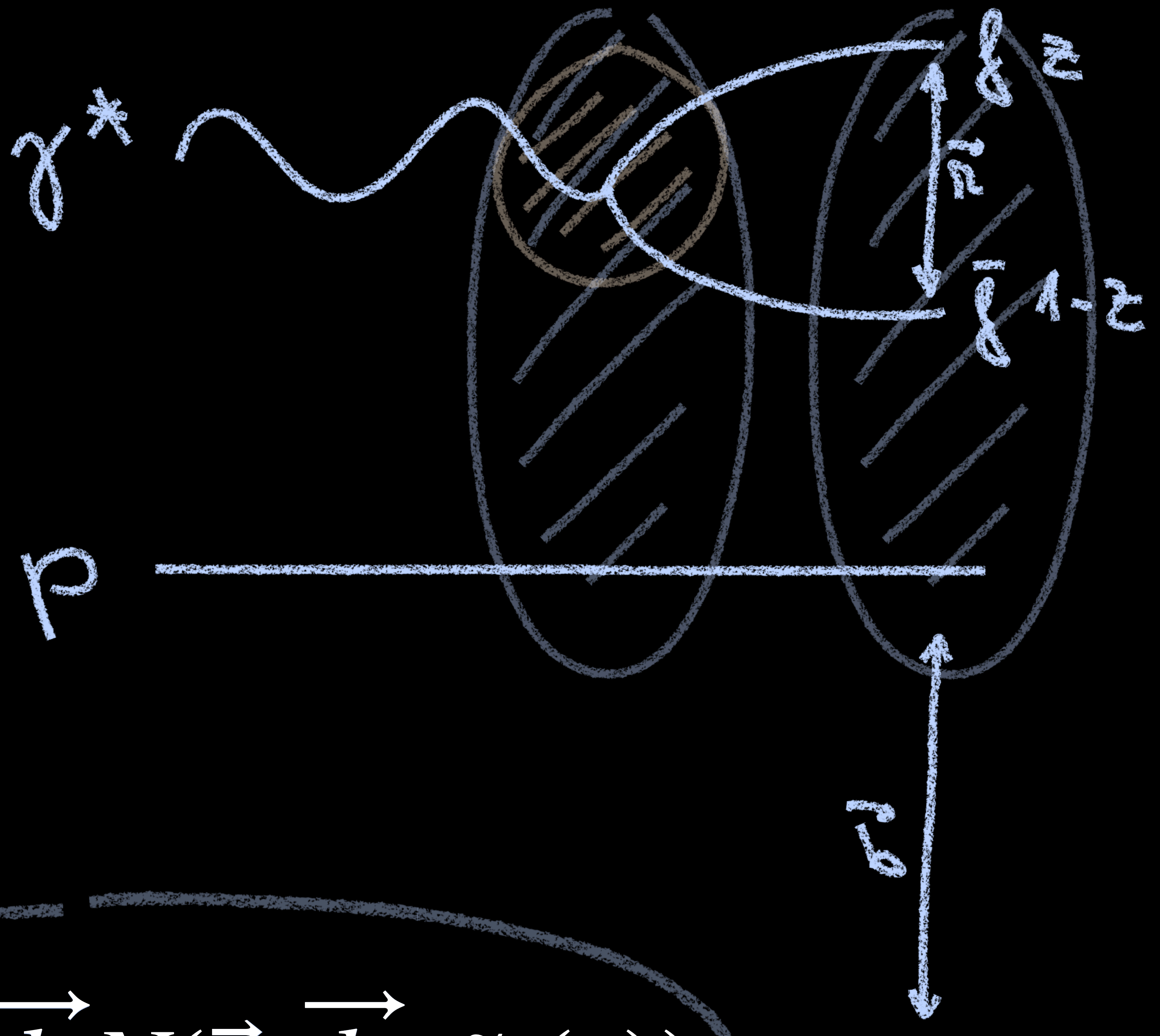
Balitsky-Kovchegov equation

1) the color dipole model
2) the Color Glass Condensate

- a way to predict the structure functions

$$F_2(x,Q^2) = \frac{Q^2}{4\pi\alpha_{em}}\left(\sigma_L^{\gamma^* p}\left(x,Q^2\right) + \sigma_T^{\gamma^* p}\left(x,Q^2\right)\right)$$

$$F_L(x,Q^2) = \frac{Q^2}{4\pi\alpha_{em}}\sigma_L^{\gamma^* p}\left(x,Q^2\right)$$



- the photon-proton cross-section

$$\sigma_{L,T}^{\gamma^* p}(x,Q^2) = \sum_f \int \mathrm{d}^2\vec{r}\int_0^1 \mathrm{d}z\,|\psi_{T,L}^{(f)}(\vec{r},Q^2,z)|^2\, 2\int \mathrm{d}^2\overrightarrow{b}\, N(\vec{r},\overrightarrow{b},\tilde{x}_f(x))$$

Balitsky-Kovchegov equation
2) the Color Glass Condensate

- effective high energy QCD

- JIMWLK equations $\xrightarrow[\text{large } N_C]{}$ BK equation