

# Strangeness production vs. charged particle multiplicity with PYTHIA8 & Pion interferometry with CorAL

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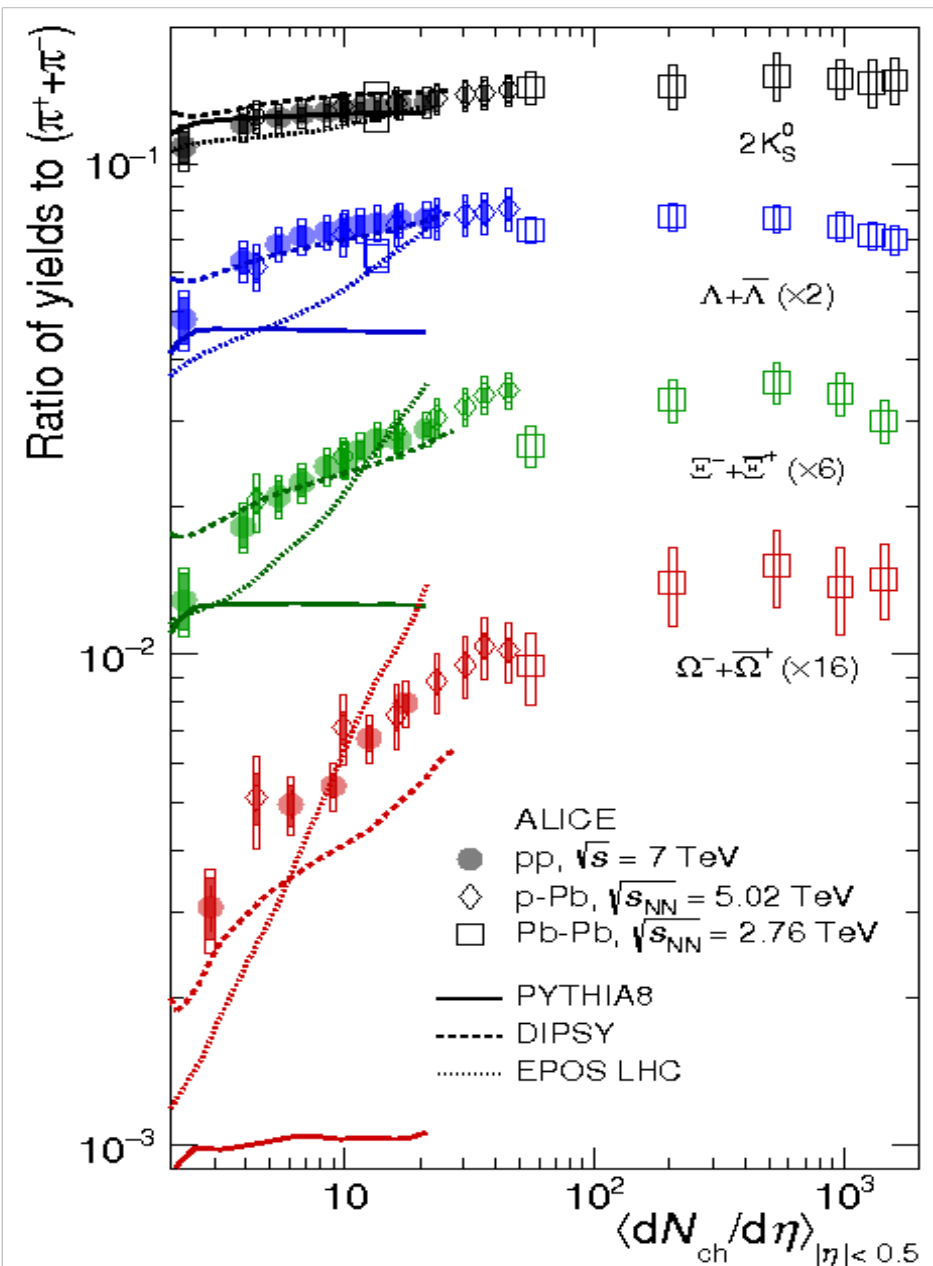
# Overview

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- **Investigation of Strangeness Enhancement at ALICE experiment using PYTHIA event generator**
- **Analysis of transverse momentum ( $p_T$ ) distribution of strange hadrons using Tsallis-Weibull formalism**
- **Spherocity dependent study of Rope Hadronization for p-p collisions**
- **Analysis of the trend of enhanced production of Strange baryons in p-p collisions at  $\sqrt{S} = 7$  TeV**
- **Pion interferometry with Correlation Algorithm Library (CorAL)**

Investigation of Strangeness  
Enhancement at  
ALICE experiment using PYTHIA

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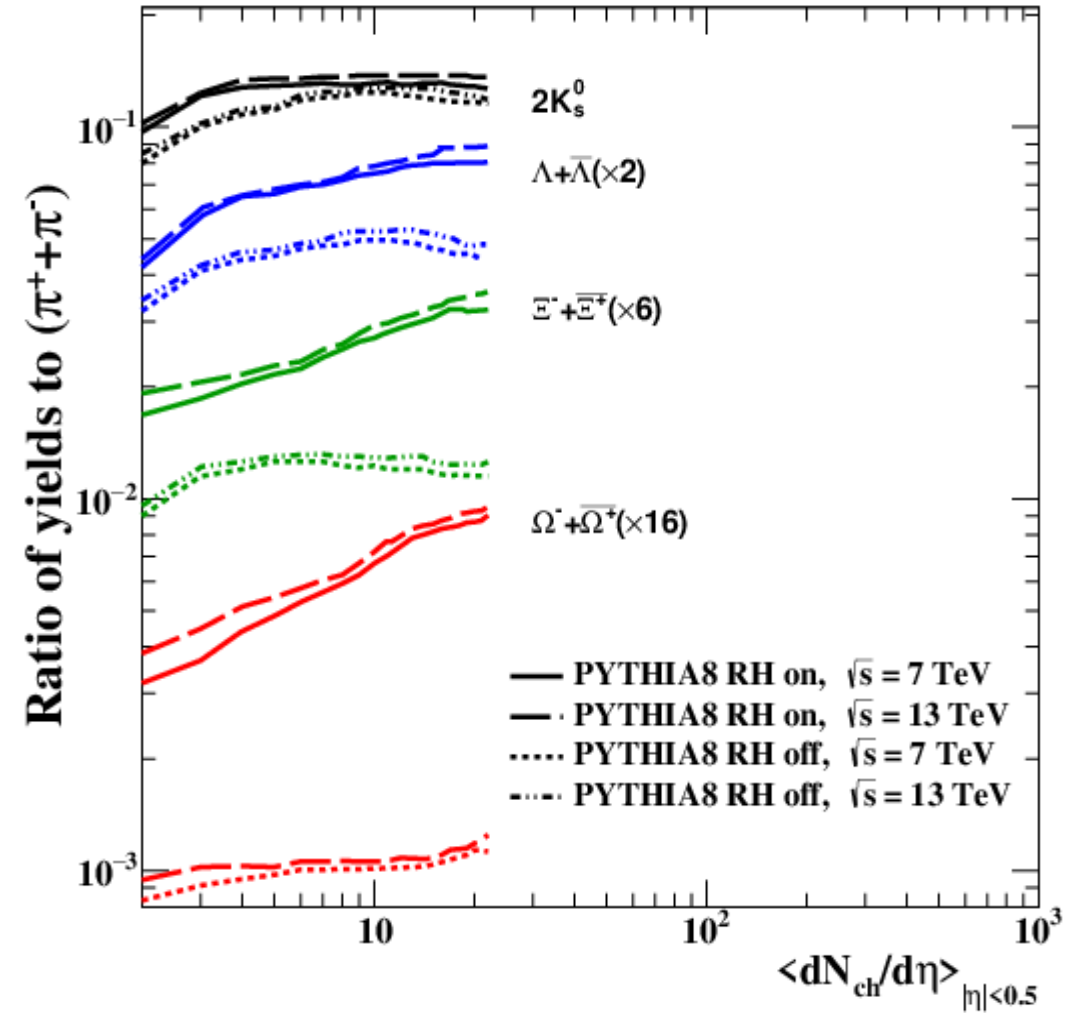
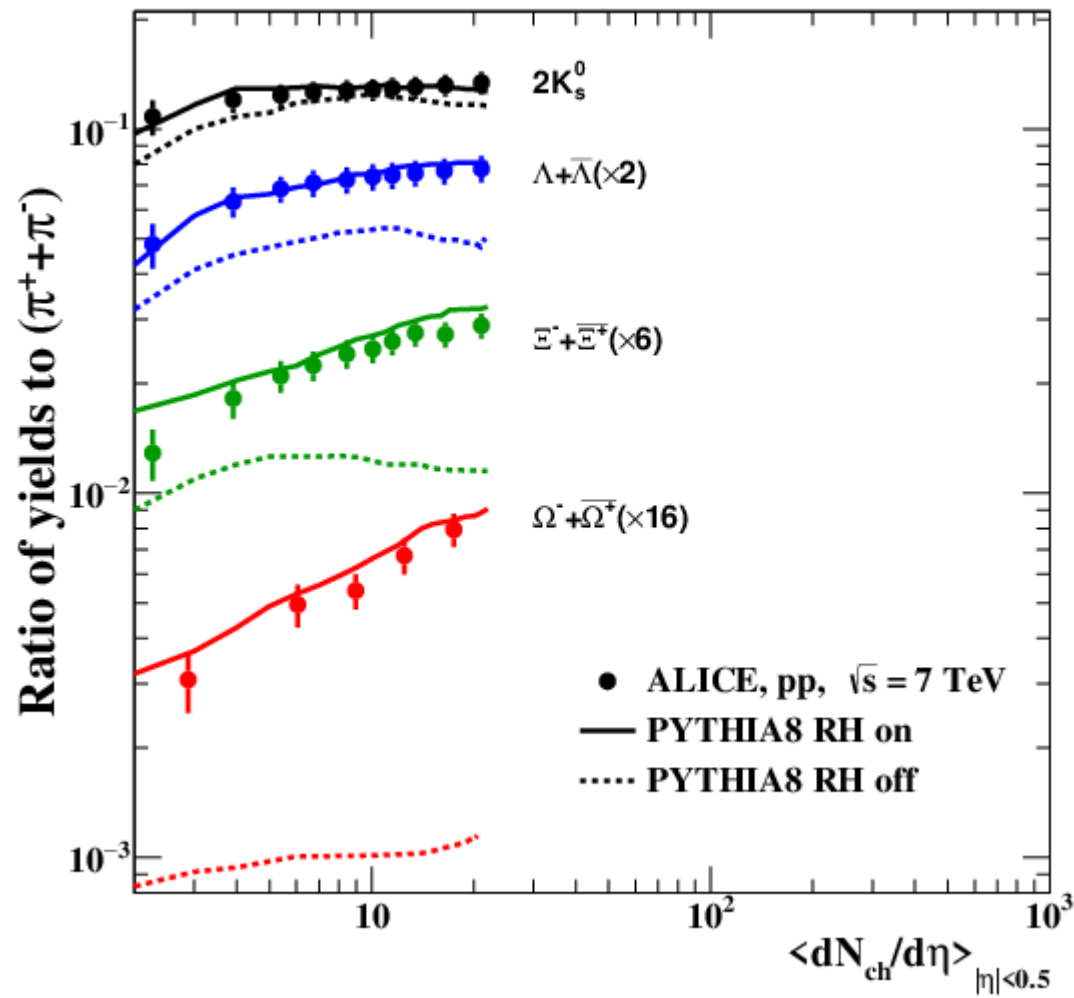


In heavy ion collisions, the enhanced production of strange particles in central and midcentral collisions have been attributed to the abundance of strange and anti-strange quarks in the deconfined QGP medium.

Rate of enhancement increases with the strangeness content of the Baryons

# Color Reconnection and Rope Hadronization

- CR address the question: between which partons do strings form?
- Rope Hadronization is a model extending the Lund string hadronization model to describe environments such as high multiplicity pp collisions or AA collisions.
- The key point of the Rope Hadronization model, is the increase of local string tension.



Analysis of transverse momentum  
distribution of strange hadrons  
using Tsallis-Weibull formalism

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Bulk properties of the system created in relativistic heavy-ion collisions can be studied via the transverse momentum ( $p_T$ ) distribution through statistical approach.

### Dynamical non-equilibrium effects :

- Hard QCD scattering processes in the initial stage of hadronisation

$$\frac{1}{2\pi p_T} \frac{d^2N}{dp_T dy} = \frac{gV m_T}{(2\pi)^3} \exp\left(-\frac{m_T}{T}\right)$$

## ***Boltzmann-Gibbs Blast-Wave Model***

### Assumptions :

- Local thermal equilibrium → Boltzmann distribution
- Longitudinal and transverse expansions (1+2)
- Temperature and  $\langle\beta_T\rangle$  are global quantities

### Limitations :

- Strong assumption on local thermal equilibrium
- Limited to low  $p_T$  regime



# Tsallis-Weibull Distribution

## Weibull Distribution

Processes governed by sequential branching and fragmentation is described by Weibull Distribution

$$P(x; \lambda, k) = \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k}$$

## Tsallis q Statistics

Generalization of Gibbs-Boltzmann statistics

$$P_q(x; q, T) = \frac{1}{T} e_q^{-\left(\frac{x}{T}\right)},$$

where

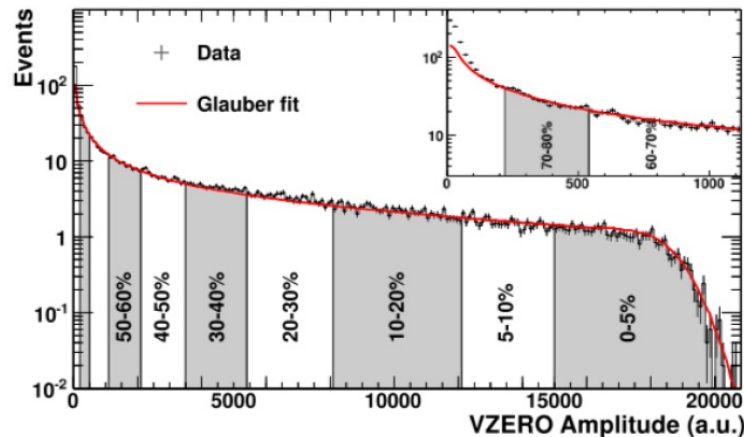
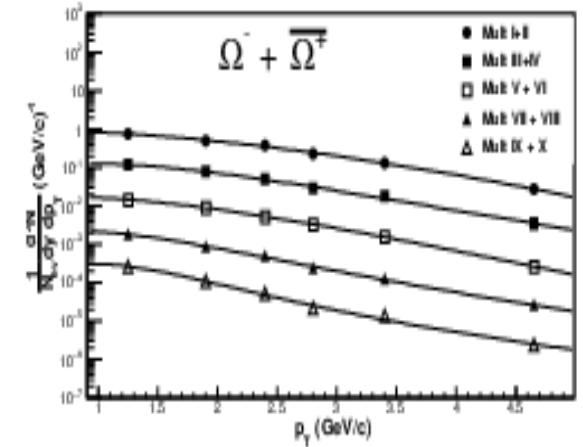
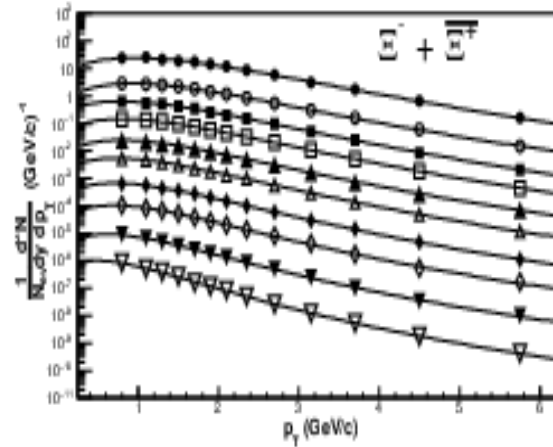
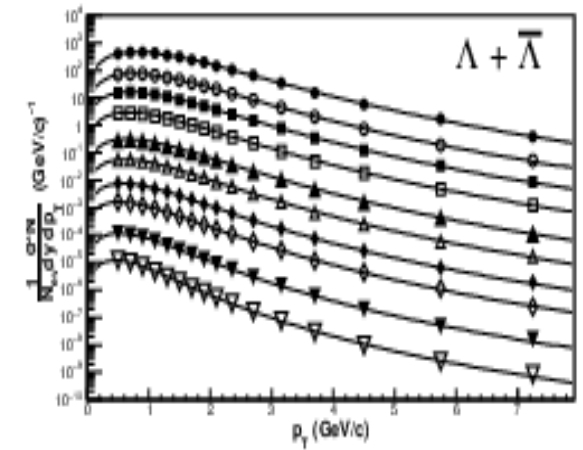
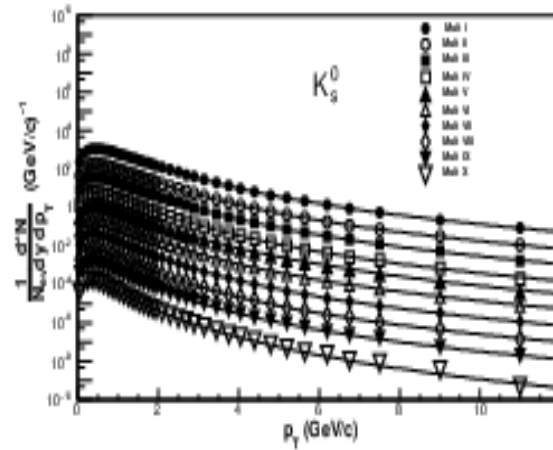
$$e_q^{-\left(\frac{x}{T}\right)} = \left(1 - (1 - q) \left(\frac{x}{T}\right)\right)^{\left(\frac{1}{1-q}\right)}$$

# q-Weibull distribution

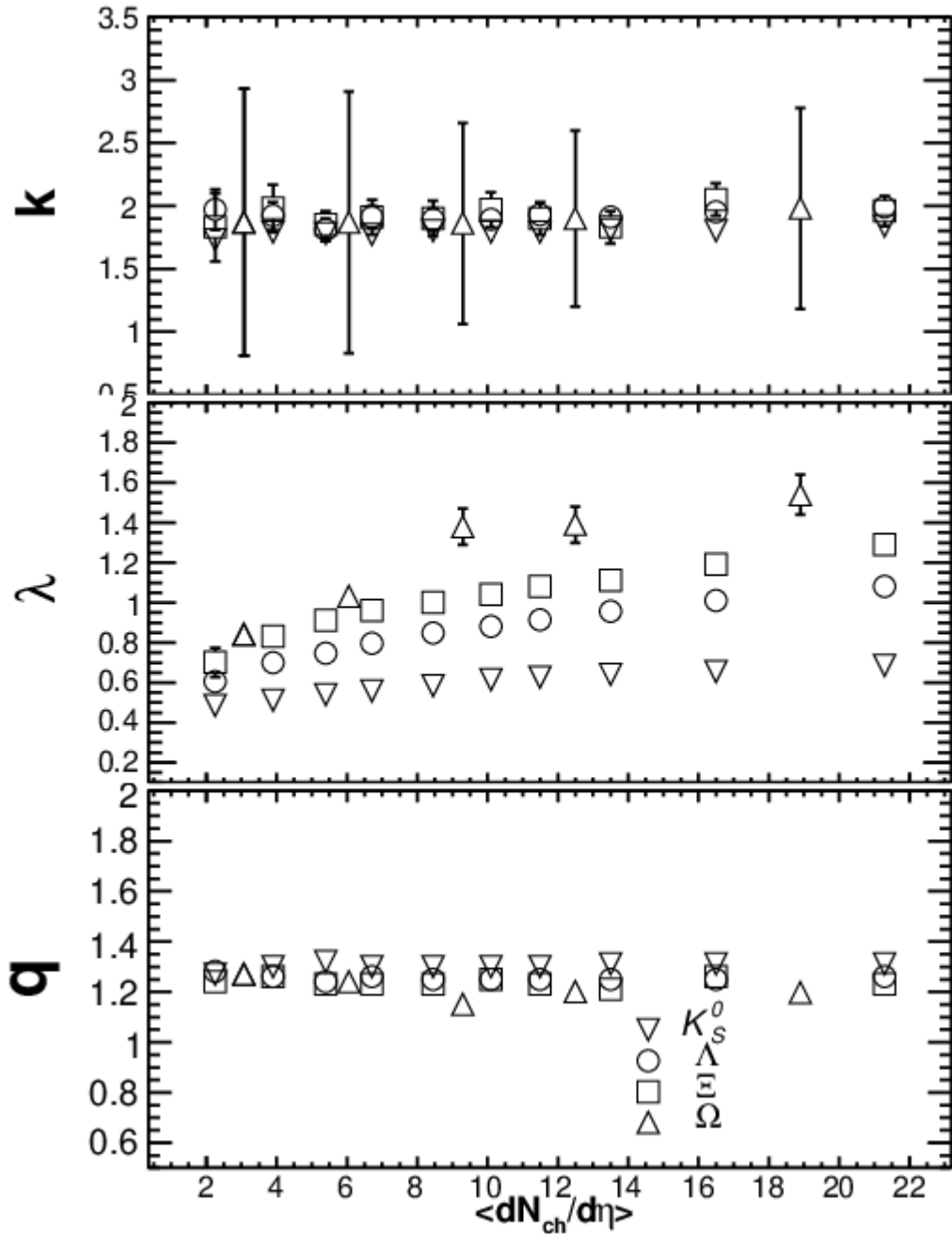
$$P_q(x; q, \lambda, k) = \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} e_q^{-\left(\frac{x}{\lambda}\right)^k}$$

Where,  $e_q^{-\left(\frac{x}{\lambda}\right)^k} = \left(1 - (1 - q) \left(\frac{x}{\lambda}\right)^k\right)^{\frac{1}{1-q}}$

The transverse momentum ( $p_T$ ) distribution of strange hadrons measured in p-p collisions at LHC energies has been studied for different **multiplicity classes**



Multiplicity class	$\langle dN_{ch}/d\eta \rangle$	Multiplicity class	$\langle dN_{ch}/d\eta \rangle$
I	$21.5 \pm 0.6$	VI	$8.45 \pm 0.25$
II	$16.5 \pm 0.5$	VII	$6.72 \pm 0.21$
III	$13.5 \pm 0.4$	VIII	$5.40 \pm 0.17$
IV	$11.5 \pm 0.3$	IX	$3.90 \pm 0.14$
V	$10.1 \pm 0.3$	X	$2.26 \pm 0.12$



For  $q \rightarrow 1$  and  $k = 1$ , the  $q$ -exponential


$$e_q^{-\left(\frac{x}{\lambda}\right)^k} = \left(1 - (1 - q) \left(\frac{x}{\lambda}\right)^k\right)^{\frac{1}{1-q}}$$

becomes  $e^{-\left(\frac{x}{\lambda}\right)}$  i.e. Gibbs-Boltzmann distribution.  $\lambda$  can be associated with the temperature of the system.  $q$  represents the deviation from thermal equilibrium.

$\lambda$  shows an increment from peripheral to central collisions and mass hierarchy .

$q > 1$  ;  collective expansion

 non-equilibrium

$k$  almost constant ;  Initial state effects

# Sphericity dependent study of Rope Hadronization for p-p collisions

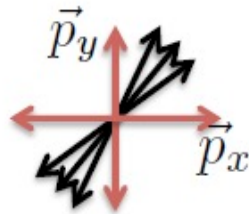
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# Transverse Spherocity

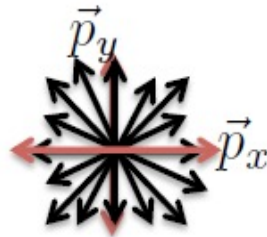
Spherocity is defined for a unit vector  $\hat{n} = (n_x, n_y, 0)$

such that 
$$S_o = \frac{\pi^2}{4} \min_{\hat{n}=(n_x, n_y, 0)} \left( \frac{\sum_i |\vec{p}_{Ti} \times \hat{n}|}{\sum_i p_{Ti}} \right)^2$$

Jetty (pencil like):  $S_o \rightarrow 0$

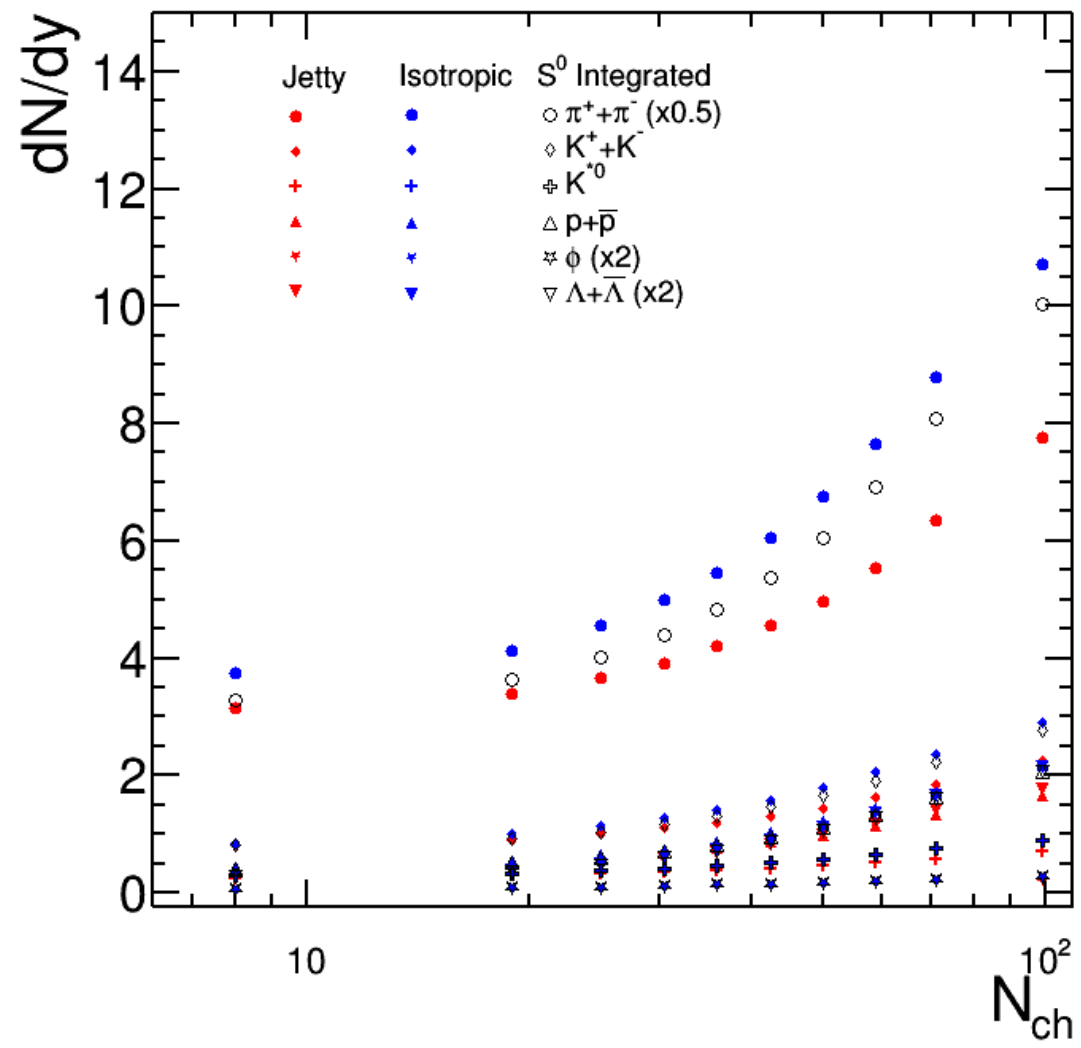
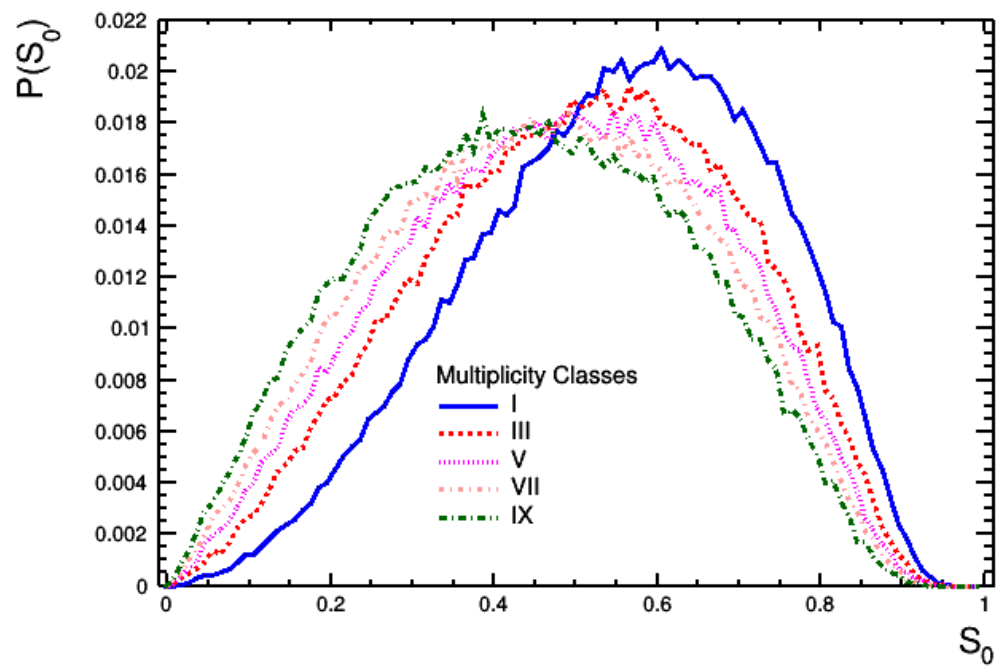


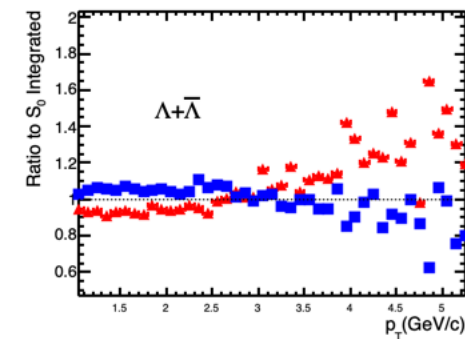
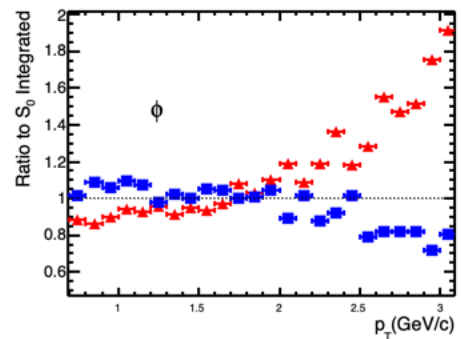
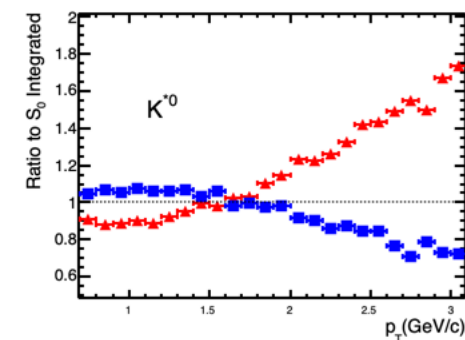
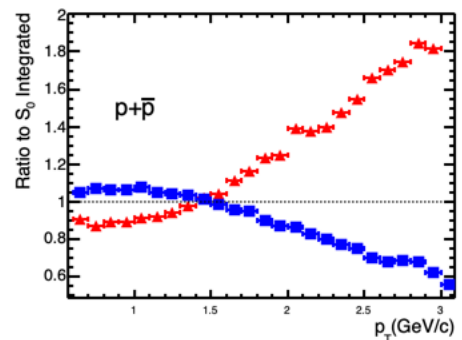
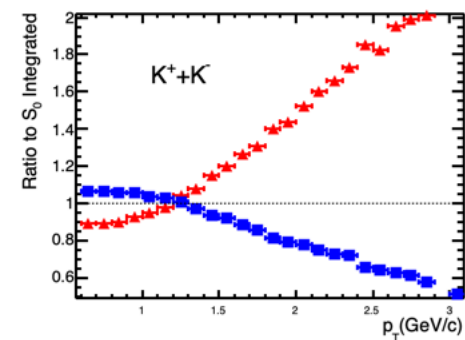
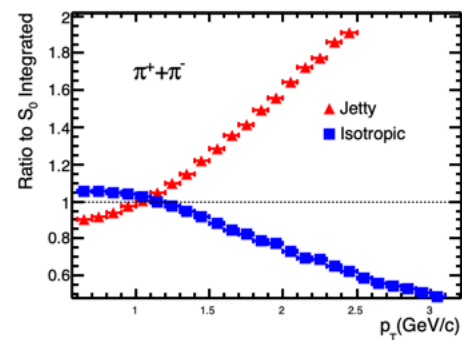
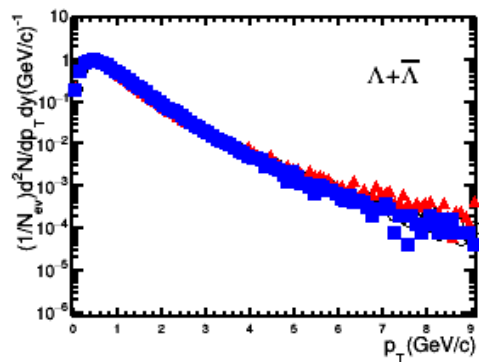
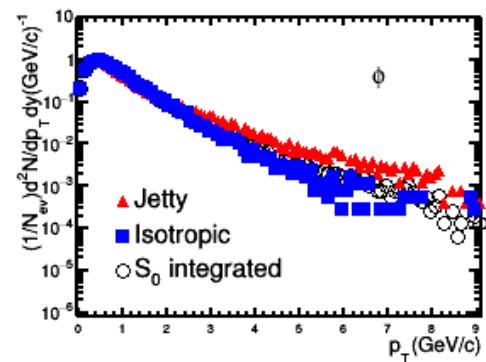
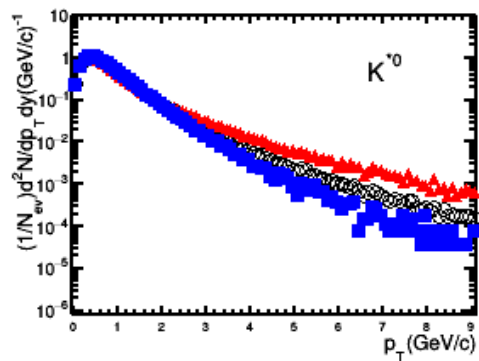
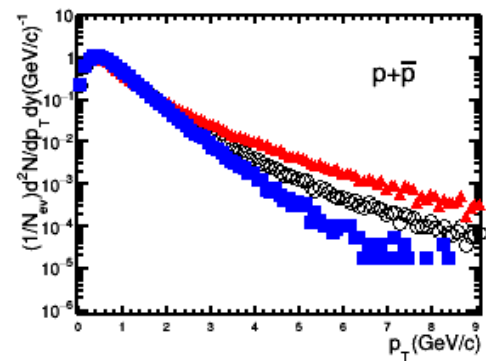
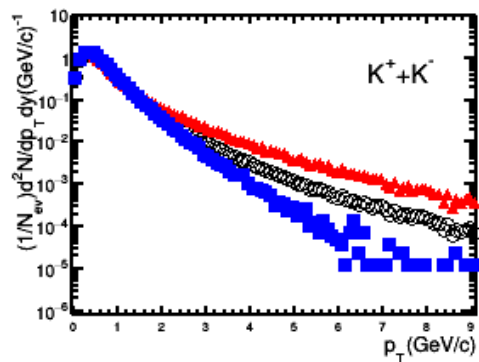
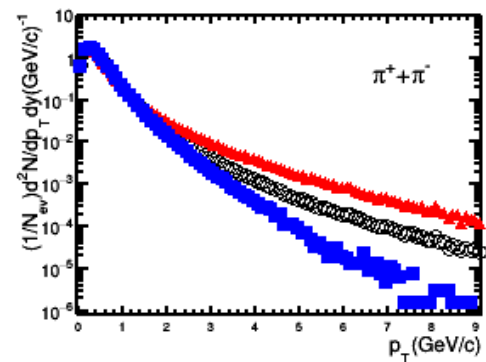
Isotropic (spherically symmetric):  $S_o \rightarrow 1$



**Spherocity can help to discriminate hard and soft processes.**

- Jetty events :  
hard QCD  
low multiplicity events
- Isotropic events :  
soft QCD  
high multiplicity events



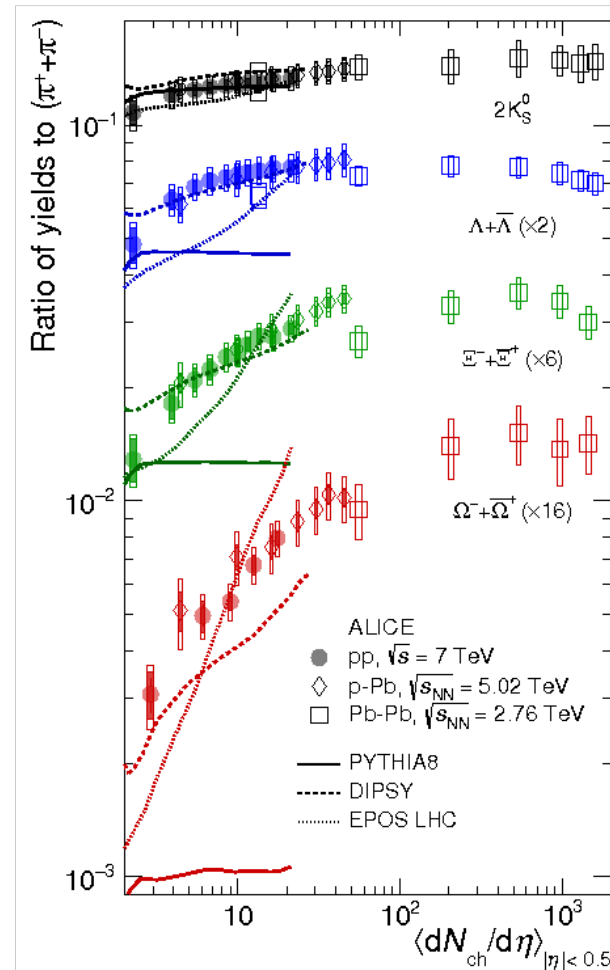


Analysis of the trend of enhanced  
production of Strange baryons in  
p-p collisions at  $\sqrt{S} = 7 \text{ TeV}$

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Rate of enhancement increases with the strangeness content of the Baryons



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# Yield Ratio Function

$$r = 0.37 e^{-2.64s} N_{ch}^{0.16s - 0.001}$$

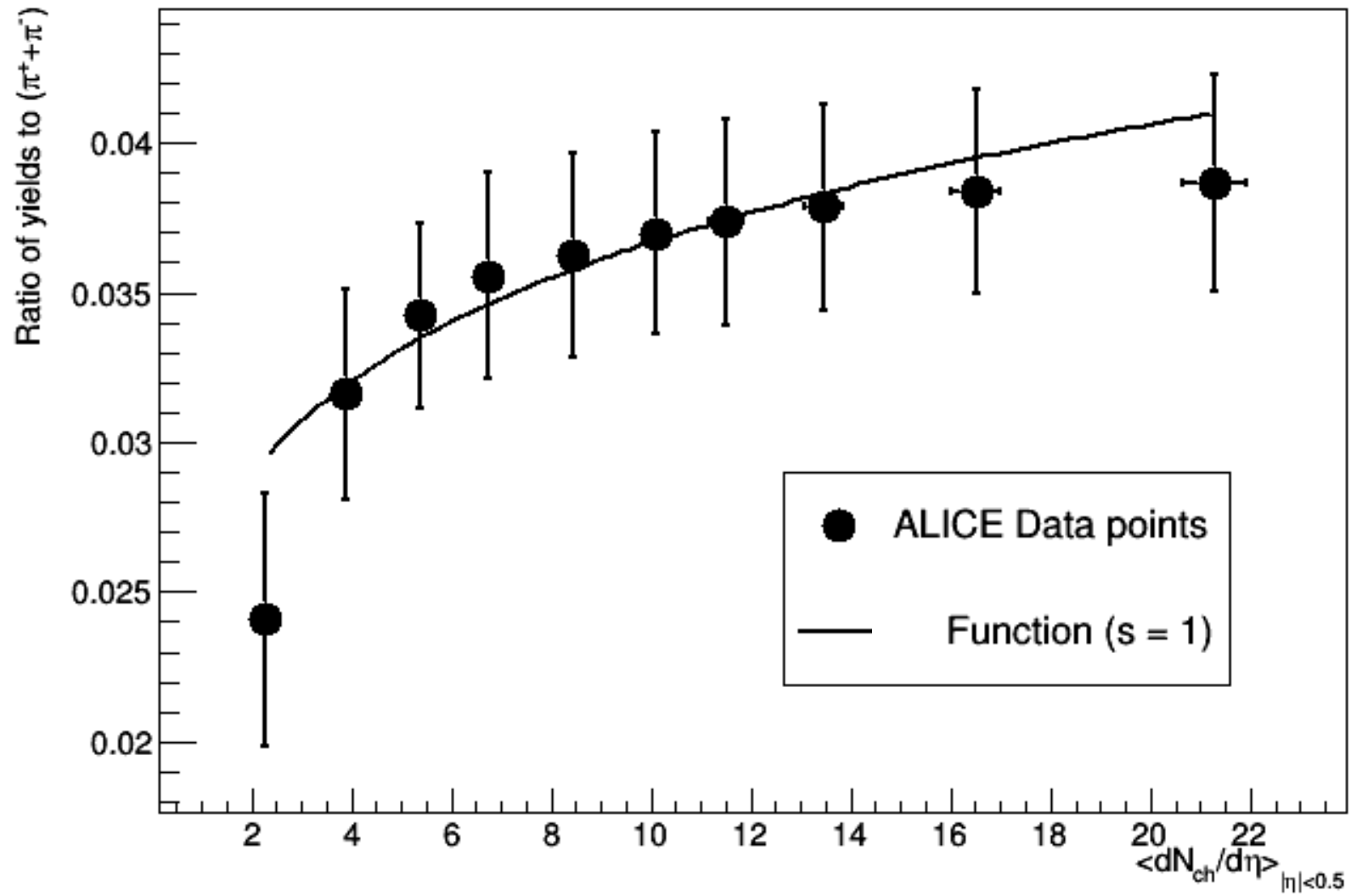
Where ,

r is the ratio of yield of the Baryon to  $(\pi^+ + \pi^-)$  measured at midrapidity

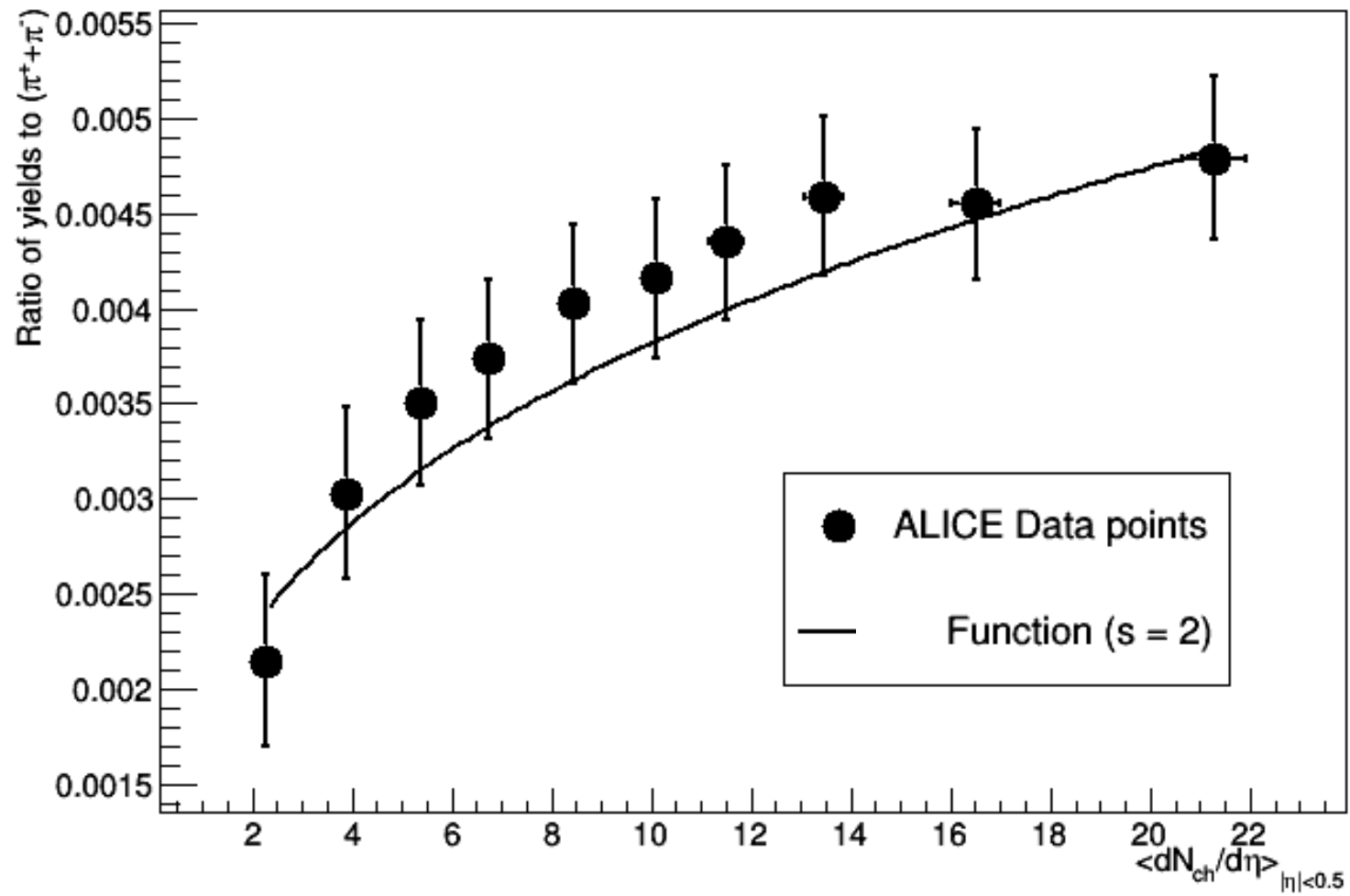
$N_{ch}$  is the charged particle multiplicity of the event detected at midrapidity ( $|\eta| < 0.5$ )

s is the strangeness content of the Baryon

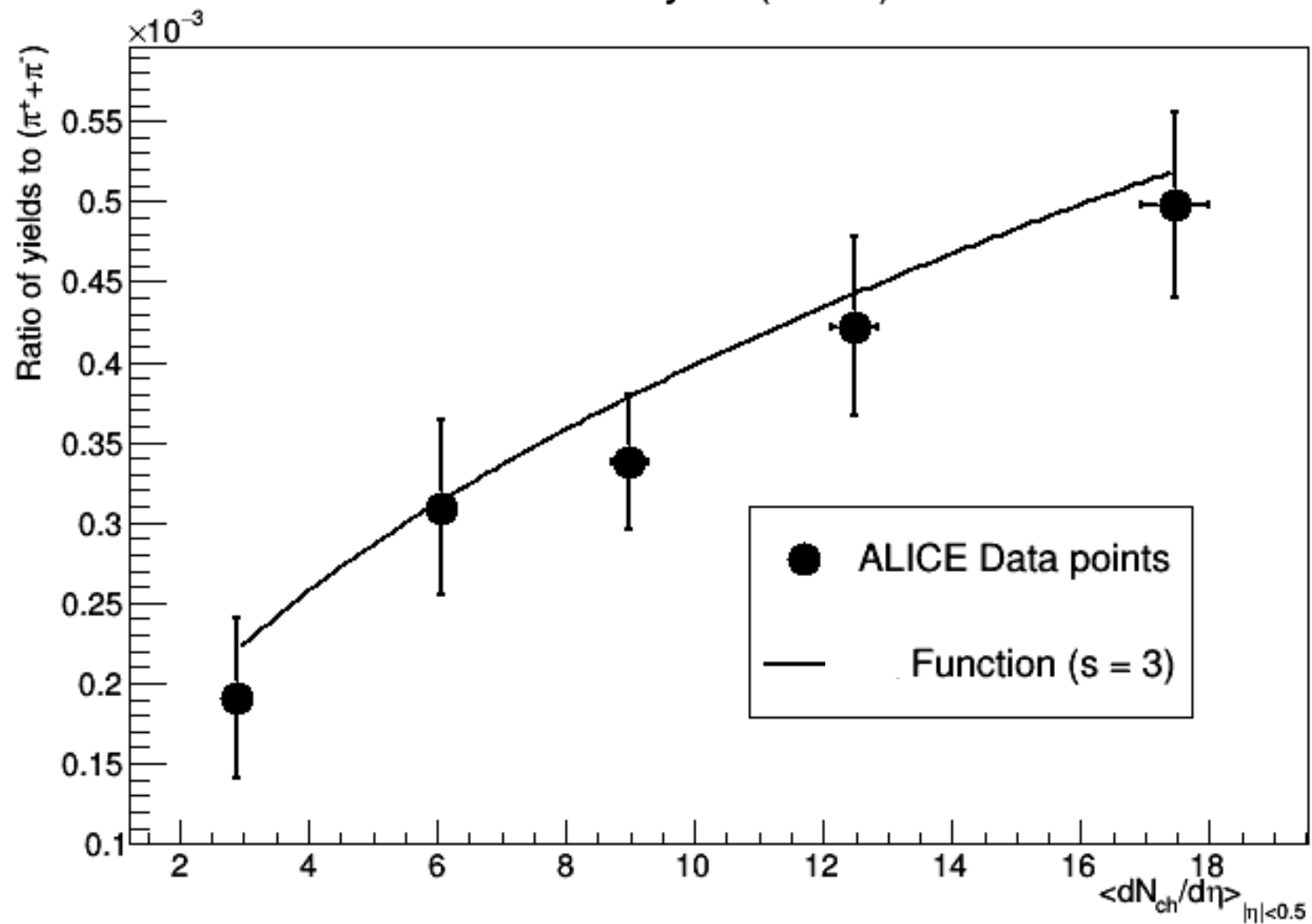
# $\Lambda$ baryon ( $s = 1$ )



### $\Xi$ baryon ( $s = 2$ )



### $\Omega$ baryon ( $s = 3$ )



HBT Inteferometry in Heavy-Ion collisions:  
Explore space-time evolution of system

Measuring the size of subatomic and  
nuclear collisions

# What is Intensity Interferometry ?

## Young's double slit Experiment

Interference of the amplitudes of two light waves from two slit openings which travel different paths to arrive at the same detection point

## Intensity Interferometry

Interference of intensities when identical particles are detected at different points measured in coincidence.

When a particle has been detected in one detector, the probability for the detection of a second particle in coincidence is found to exhibit a correlation.

i.e. these two events may not be independent!!!



# Correlation Function :

$$C_2(k_1, k_2) = \frac{P(k_1, k_2)}{P(k_1) P(k_2)}$$

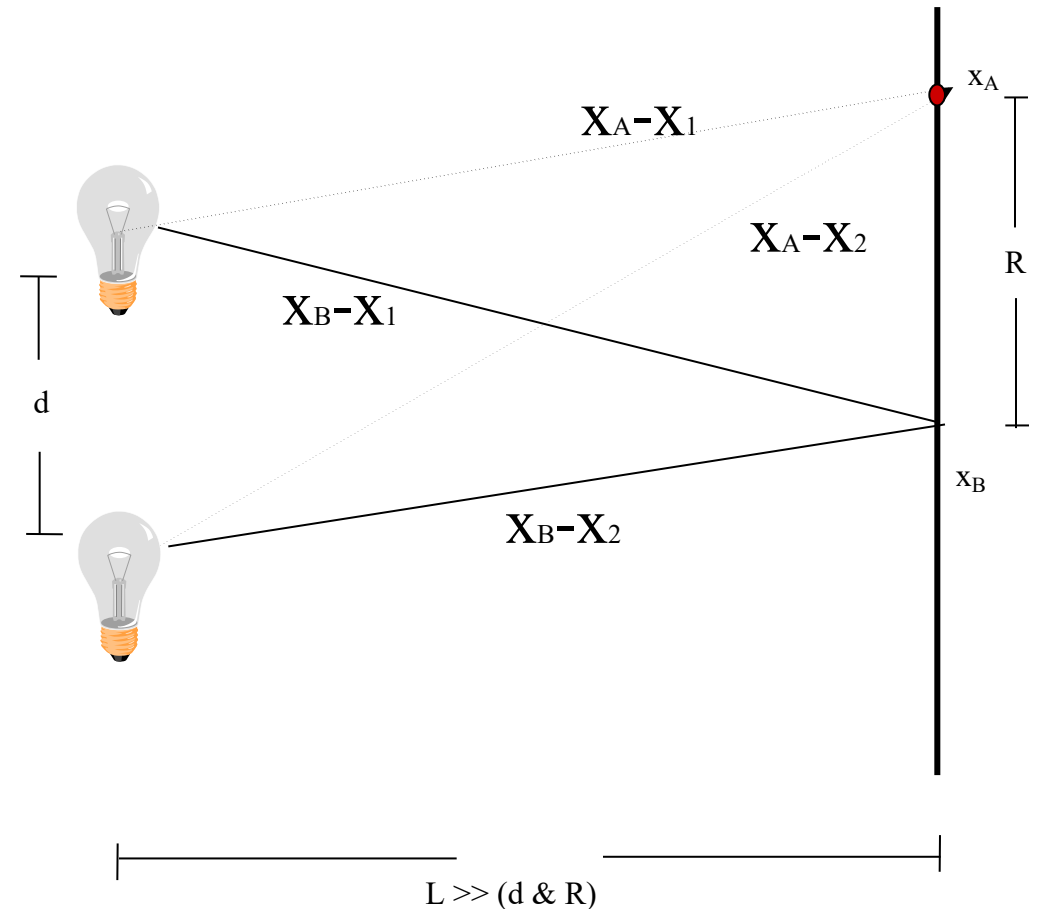
$P(k_1, k_2)$  = probability of the coincidence of a particle pair with momentum  $k_1$  and  $k_2$ .

$P(k_i)$  = probability of observing a particle with momentum  $k_i$ .

Probability amplitude for 2 particles with momenta  $k_1$  and  $k_2$ , emitted from points  $x_1$  and  $x_2$  and detected at  $x_A$  and  $x_B$  is

$$A(k_1, k_2) = \frac{1}{\sqrt{2}} \left[ e^{-ik_1 \cdot (x_A - x_1)} e^{i\phi_1} e^{-ik_2 \cdot (x_B - x_2)} e^{i\phi_2} \pm e^{-ik_1 \cdot (x_A - x_2)} e^{i\phi_2'} e^{-ik_2 \cdot (x_B - x_1)} e^{i\phi_1'} \right]$$

$$\begin{aligned} P_2(k_1, k_2) &= \langle |A(k_1, k_2)|^2 \rangle = \\ &= \frac{1}{2} \left[ 2 \pm (e^{i(k_1 - k_2) \cdot (x_1 - x_2)} \langle e^{\pm i(\phi_1 + \phi_2 - \phi_1' - \phi_2')} \rangle + c.c.) \right] \\ &= 1 \pm \cos[(k_1 - k_2) \cdot (x_1 - x_2)] \end{aligned}$$



Now, for the single particle momentum distribution, we have

$$A(k_i) = \frac{1}{\sqrt{2}} [e^{-ik_1 \cdot (x_A - x_1)} e^{i\phi_1} \pm e^{-ik_1 \cdot (x_A - x_2)} e^{i\phi_2}] \quad \langle e^{\pm i(\phi_1 - \phi_2)} \rangle = 0$$

$$P_1(k_i) = \langle |A(k_i)|^2 \rangle = \frac{1}{2} [2 \pm e^{ik_i \cdot (x_1 - x_2)} \langle e^{\pm i(\phi_1 - \phi_2)} \rangle + c.c.]$$

$$C(k_1, k_2) = \frac{P_2(k_1, k_2)}{P_1(k_1)P_1(k_2)} = 1 \pm \cos[(k_1 - k_2) \cdot (x_1 - x_2)]$$

For an extended source of emission,  
we have:

$$\begin{aligned}
 P(k_1, k_2) &= \int dx_1 dx_2 \rho(x_1) \rho(x_2) |\Phi(k_1 k_2 : x_1 x_2 \rightarrow x'_1 x'_2)|^2 \\
 &= P(k_1) P(k_2) + \left| \int dx e^{i(k_1 - k_2) \cdot x} \rho(x) A(k_1, x) A(k_2, x) \right|^2
 \end{aligned}$$

$$\rho_{\text{eff}}(x; k_1, k_2) = \frac{\rho(x) A(k_1, x) A(k_2, x)}{\sqrt{P(k_1) P(k_2)}} \quad C_2(k_1, k_2) = 1 + |\tilde{\rho}_{\text{eff}}(q; k_1 k_2)|^2$$

$\rho(x)$  is the normalised space-time  
distribution .

# Gaussian profile :

$$\rho_{eff}(x) = e^{x^\mu x_\mu / (2R)^2}$$

$$C(k_1, k_2) = 1 \pm \lambda e^{-q^2 R^2}$$

$$\tilde{\rho}_{eff}(q) = e^{-q^2 R^2 / 2}$$

$$C(p_1, p_2) = (1 \pm \lambda |\tilde{\rho}(q)|^2)$$

$\lambda$  is called the chaoticity parameter.

# Correlation Analysis Library (CorAL)

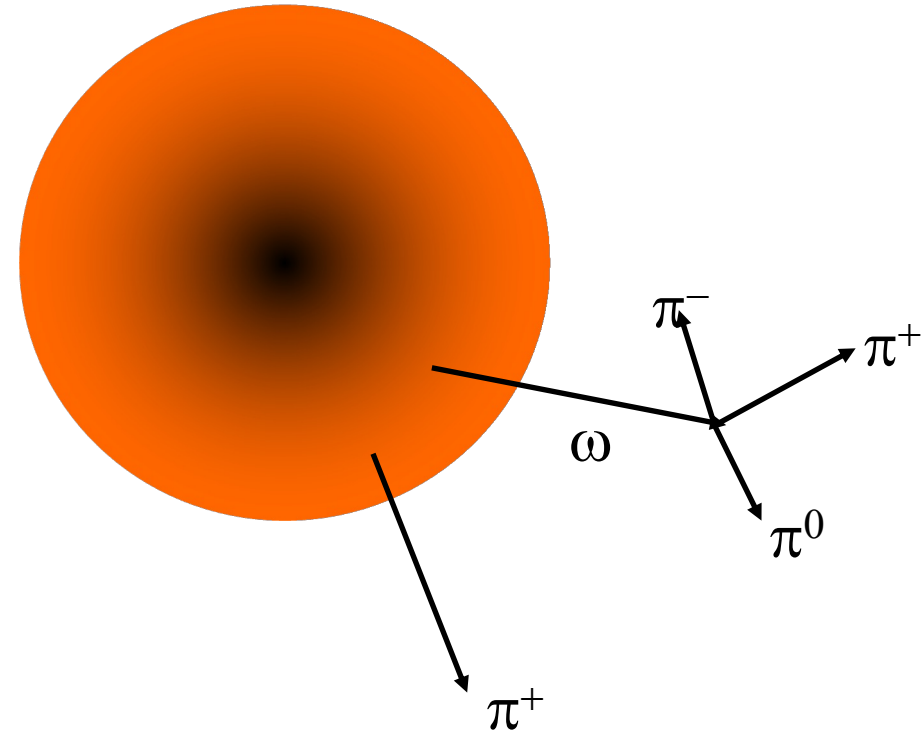
- Code base for analysis of 2-particle correlations at small relative momentum.
  
- Generate pions based on the Core-Halo picture of Bose-Einstein correlations.

# Core Halo Model

*Boson source is a superposition of central core and the surrounding extended halo.*

$$D(r, t, p) = fD_{core}(r, t, p) + (1-f)D_{halo}(r, t, p)$$

↙  
Emission Function



$$S_{\vec{P}}(\vec{r}') = \int dr'_0 \int d^4 R D(R + r/2, \vec{P}/2) D(R - r/2, \vec{P}/2)$$



The *source function*, it gives the probability of producing a pair of total momentum  $\vec{P}$  and a distance  $r$  apart.

$$C_P(\vec{q}) = \int d^3 r |\Phi_{\vec{q}}(\vec{r})|^2 S_P(\vec{r})$$



The pair's final state wave-function, we can compute this if we know the interaction. This is called the kernel function  $K(q,r)$ .

Kernel is affected by the interactions.

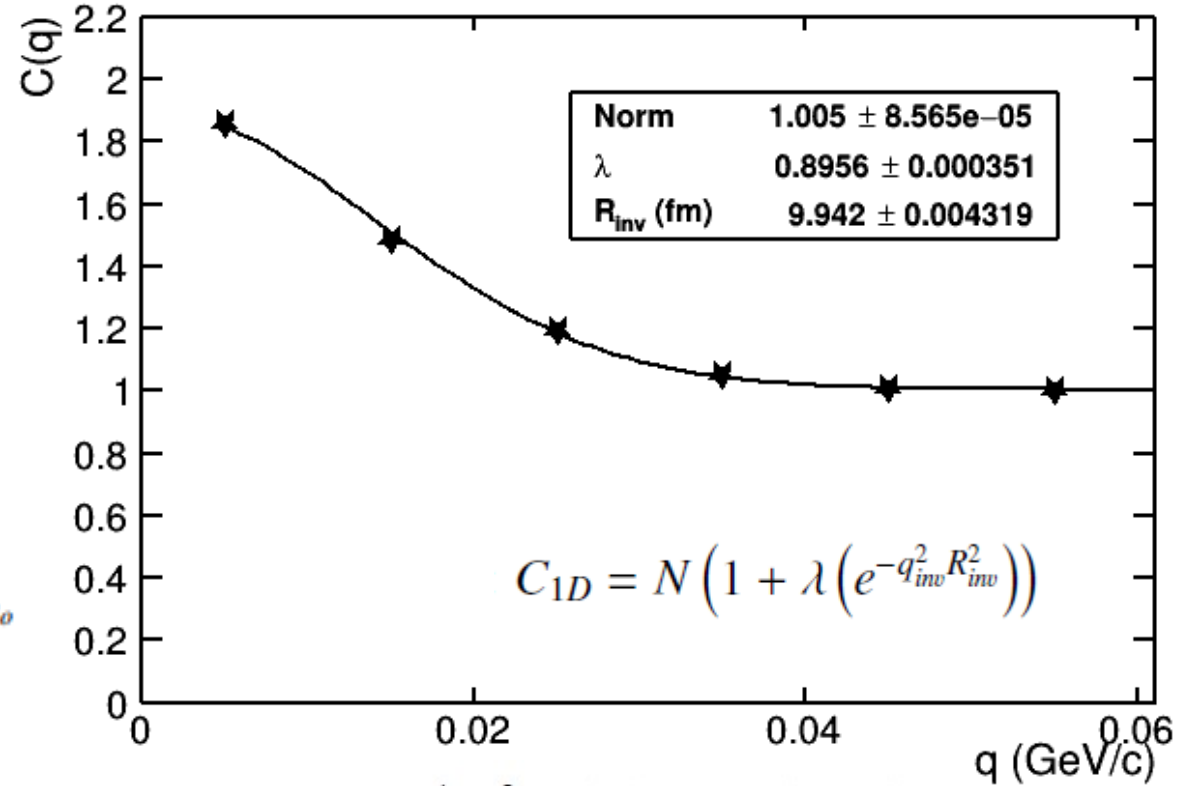


# 1-D Parameterization of the source

- Rx 4.00 fm
- Ry 3.00 fm
- Rz 6.00 fm
- Freeze-out time 13.00 fm/c
- f 0.8
- Temperature 100.0 MeV
- Flow-profile radial

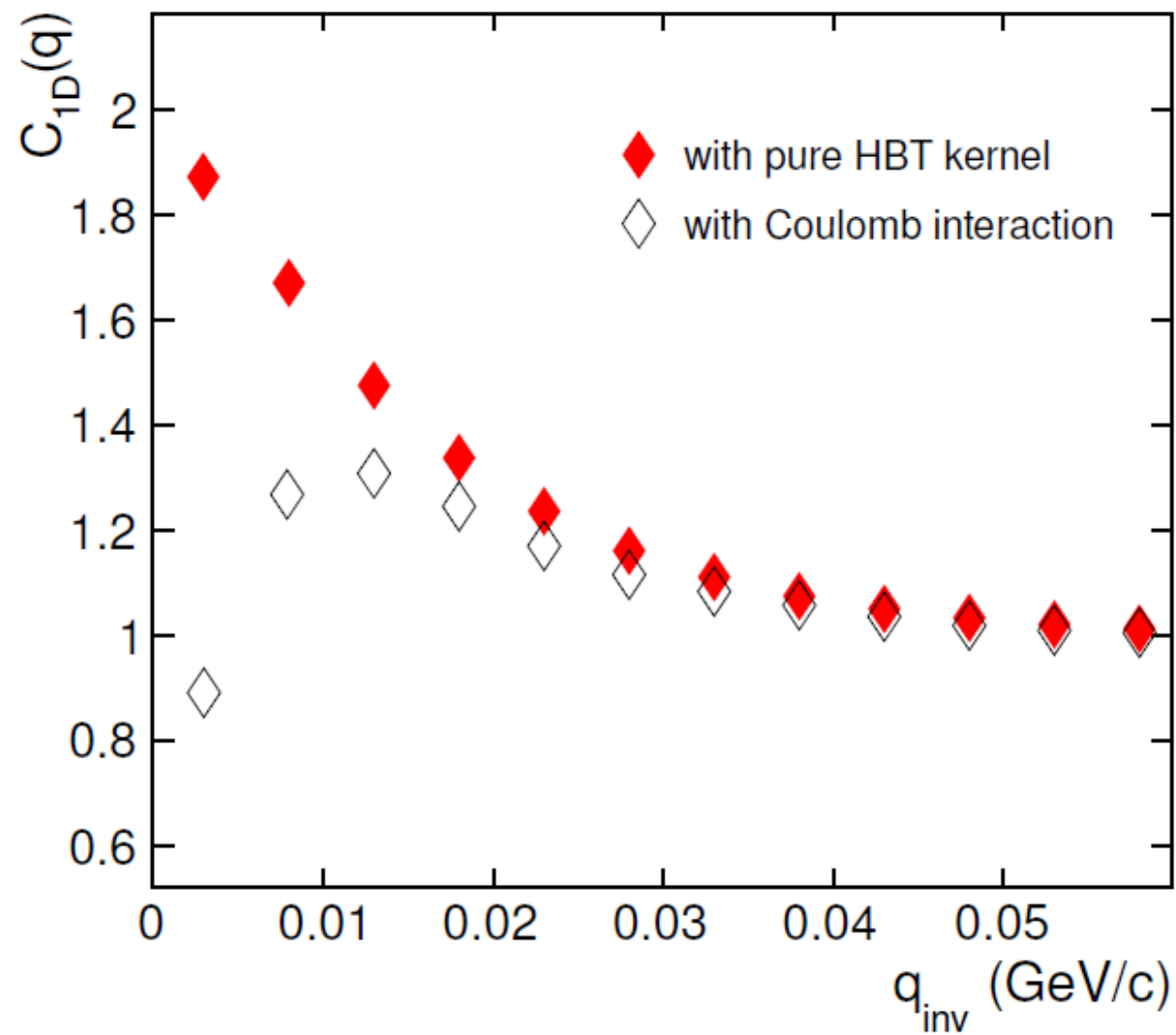
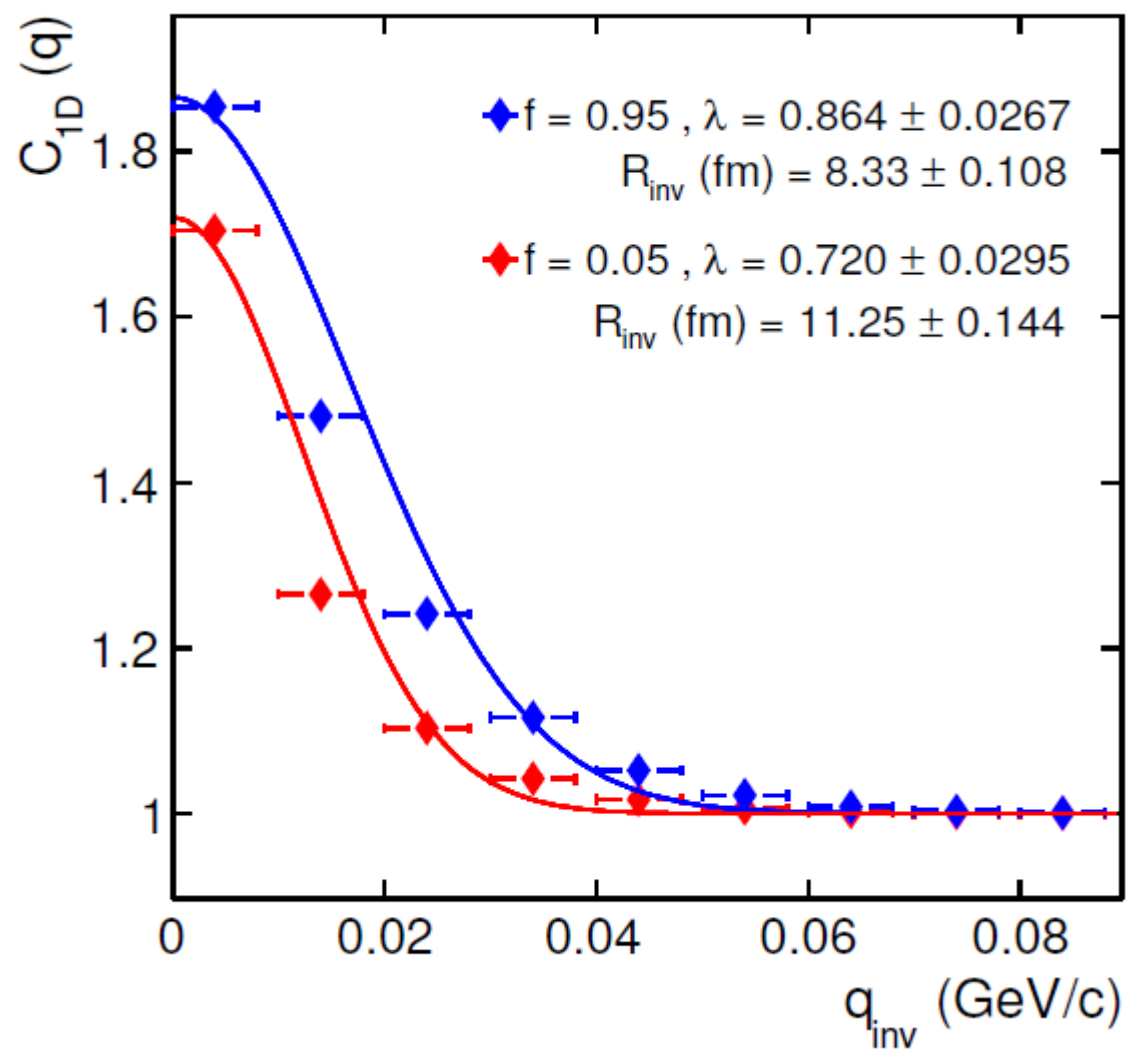
$$D_{\text{core}}(\mathbf{r}, t, \mathbf{p}) \propto e^{-p_\mu u^\mu / T - r_x^2 / 2R_x^2 - r_y^2 / 2R_y^2 - r_z^2 / 2R_z^2 - t / \tau_{fo}}$$

$$D_{\text{halo}}(\mathbf{r}, t, \mathbf{p}) \propto \int d\Delta t d^3 p_\omega P(\mathbf{p}_\omega, \mathbf{p}) e^{-\Delta t / \tau_\omega} \\ \times D_{\text{core}}\left(\mathbf{r} - \frac{\mathbf{p}_\omega}{E_\omega} \Delta t, t - \Delta t, \mathbf{p}_\omega\right)$$



$$C_{1D}(q) = \frac{1}{4\pi} \int_{-1}^1 \int_0^{2\pi} C(\vec{q}) d\phi_{\hat{q}} d(\cos \theta_{\hat{q}})$$

$$S_{1D}(r) = \frac{1}{4\pi} \int_{-1}^1 \int_0^{2\pi} S(\vec{r}) d\phi_{\hat{r}} d(\cos \theta_{\hat{r}})$$



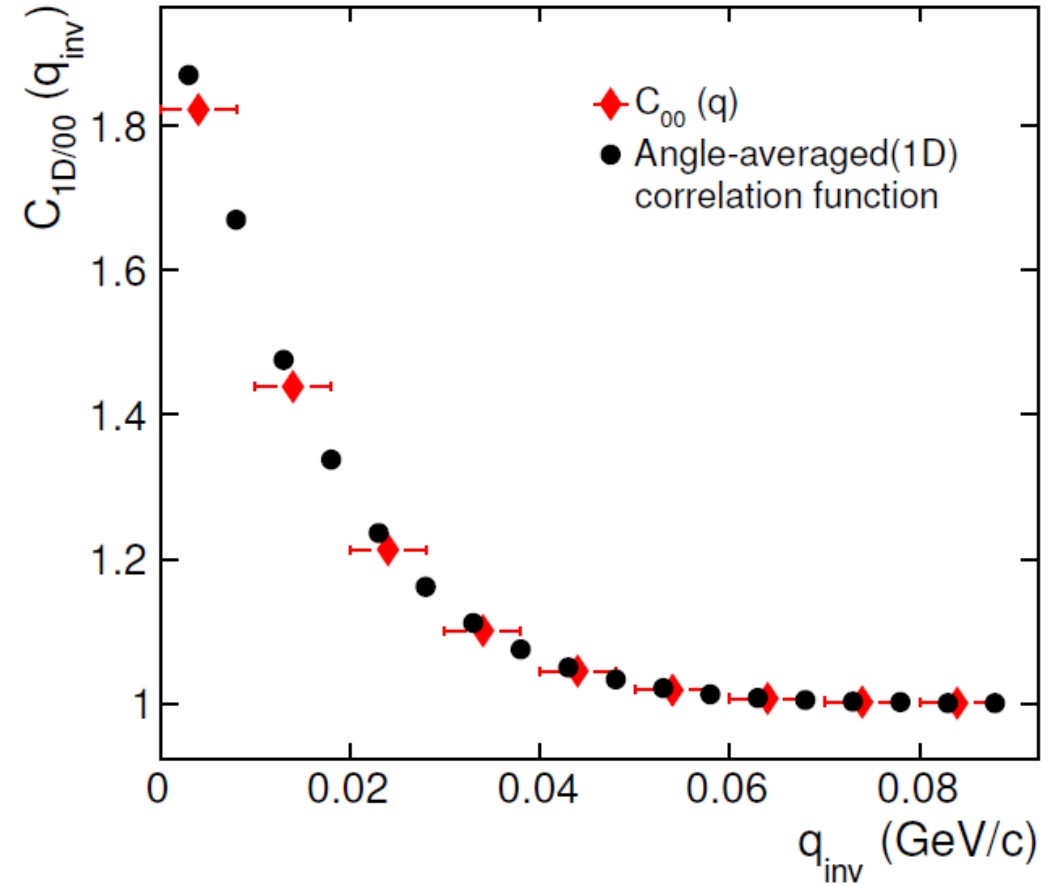
$$C(\vec{q}) = \sqrt{4\pi} \sum_{l_1, m_1} C_{l_1, m_1}(q) Y_{l_1, m_1}(\Omega_{\hat{q}})$$

$$S(\vec{r}) = \sqrt{4\pi} \sum_{l_2, m_2} S_{l_2, m_2}(r) Y_{l_2, m_2}(\Omega_{\hat{r}})$$

$$\sqrt{4\pi} C_{lm}(q) = \int_{-1}^1 \int_0^{2\pi} Y_{l,m}^*(\Omega_{\hat{q}}) C(\vec{q}) d\phi_{\hat{q}} d(\cos \theta_{\hat{q}})$$

$$C_{00}(q) = \frac{1}{\sqrt{4\pi}} \int_{-1}^1 \int_0^{2\pi} Y_{0,0}^*(\Omega_{\hat{q}}) C(\vec{q}) d\phi_{\hat{q}} d(\cos \theta_{\hat{q}})$$

$$= \frac{1}{4\pi} \int_{-1}^1 \int_0^{2\pi} C(\vec{q}) d\phi_{\hat{q}} d(\cos \theta_{\hat{q}})$$

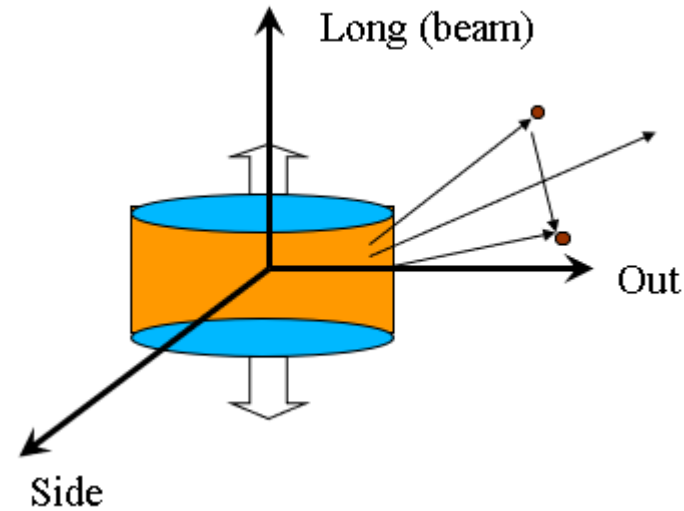


# 3D Parameterization

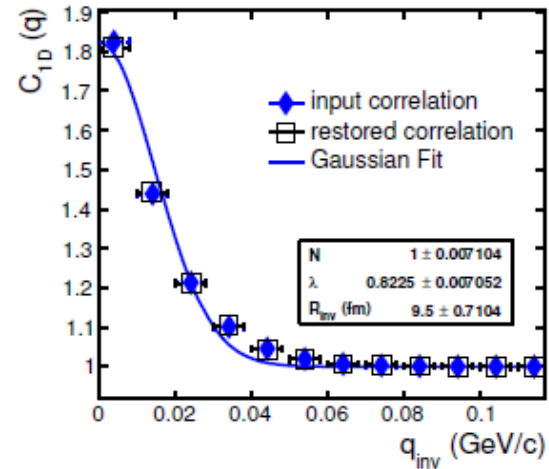
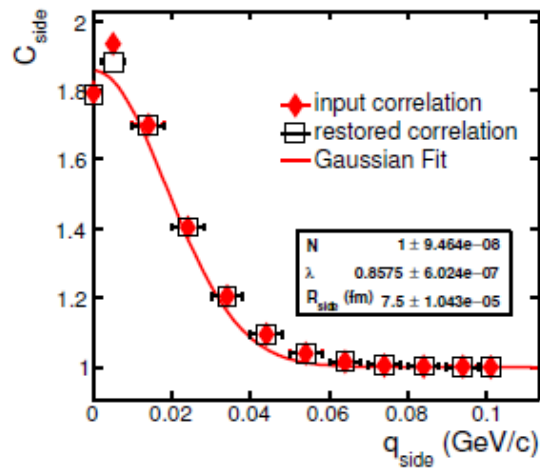
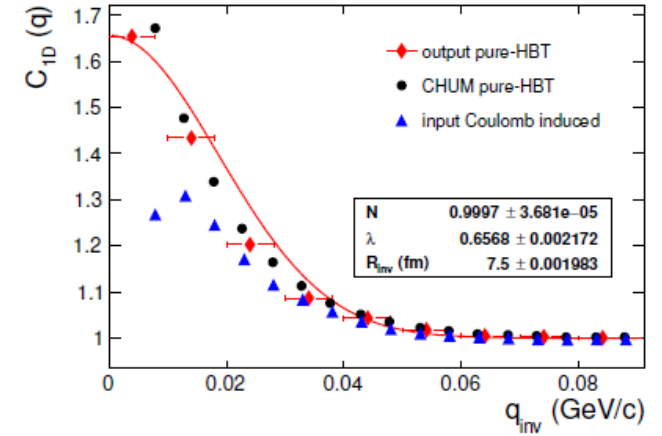
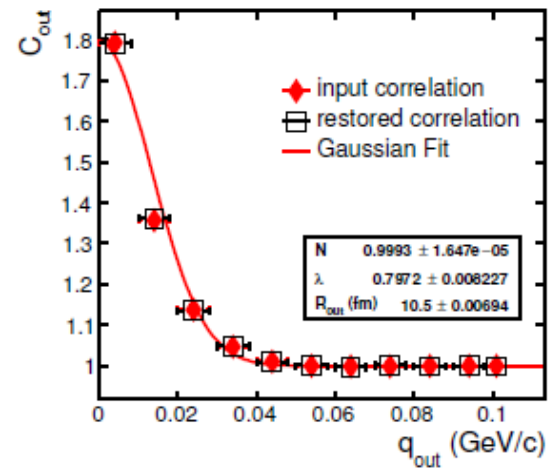
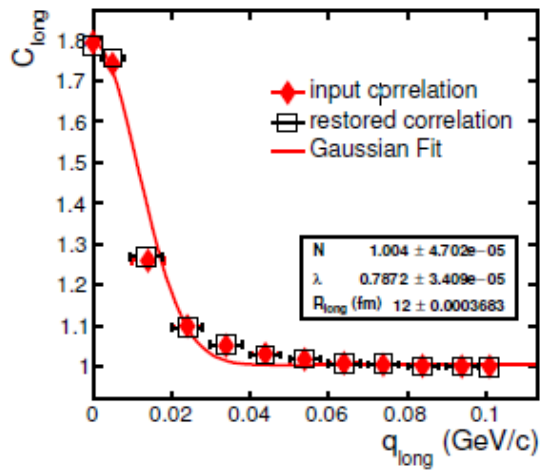
$R_{\text{long}}$  – along beam direction

$R_{\text{out}}$  – along “line of sight”

$R_{\text{side}}$  –  $\perp$  “line of sight”



$$C(\vec{q}, \vec{K}) = 1 + \lambda(\vec{K}) \exp(-R_{\text{out}}^2(\vec{K})q_{\text{out}}^2 - R_{\text{side}}^2(\vec{K})q_{\text{side}}^2 - R_{\text{long}}^2(\vec{K})q_{\text{long}}^2)$$



$$C_i = N \left( 1 + \lambda \left( e^{-q_i^2 R_i^2} \right) \right)$$

# Summary

- The combined effect of color reconnection and the Rope Hadronization was able to describe the enhanced production of strange hadrons in proton-proton collisions at ALICE.
- The q-Weibull function successfully describes the  $p_T$  distribution of the strange particles. And the strange particles were emitted from a source which is not fully equilibrated.
- In low-multiplicity classes, the effect of sphericity is minimal, whereas towards higher-multiplicity classes, it starts playing a role in making a separation of jetty to isotropic events for all identified particles.
- At small relative momentum, long-range Coulomb repulsion effects caused a suppression of the correlation function for  $(\pi^+ \pi^+)$  pair.

THANK YOU

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