Contribution ID: **4**

Type: **Talk**

Application of quantum and microwave graphs in investigations of spectral invariants and non-Weyl systems

Friday, 27 May 2022 11:00 (1 hour)

We introduce a new spectral invariant: the generalized Euler characteristic $\mathcal{E}$ [1]. The commonly used Euler characteristic i.e., the difference between the number of vertices $|V|$ and edges $|E|$ is the most important topological characteristic of a graph. However, to describe spectral properties of differential equations with mixed Dirichlet and Neumann vertex conditions it is necessary to introduce a new spectral invariant, the generalized Euler characteristic is $\mathcal{E} = |V| - |V_D| - |V|$, with $|V_D|$ denoting the number of Dirichlet vertices. We demonstrate theoretically and experimentally that the generalized Euler characteristic $\mathcal{E}$ of quantum and microwave graphs [2] can be determined from small sets of the lowest eigen-frequencies. We discuss a relationship between the generalized Euler characteristic of the original graph $\mathcal{E}_o$ which was split at vertices into two disconnected subgraphs $i = 1, 2$ and their generalized Euler characteristics $\mathcal{E}_i$.

Another important characteristic of a quantum graph is the average density of resonances, $\rho = (L/\pi)$, where L denotes the length of the graph. This is a robust measure. It does not depend on the number of vertices in a graph and holds also for most of the boundary conditions at the vertices. Graphs obeying this characteristic are called Weyl graphs. Using microwave graphs (networks) that simulate quantum graphs we show that there exist graphs, called non-Weyl graphs, that do not adhere to this characteristic [3]. For standard coupling conditions we demonstrate that the transition from a Weyl graph to a non-Weyl graph occurs if we introduce a balanced vertex. A vertex is called balanced if the numbers of infinite leads and internal edges meeting at a vertex are the same.

Acknowledgements: This work was supported by the National Science Centre, Poland, Grant No. UMO-2018/30/Q/ST2/00324.

References:

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Session Classification: Talk