

FACULTY OF NUCLEAR SCIENCES AND PHYSICAL ENGINEERING CTU IN PRAGUE

# Small angle asymptotics for Robin Laplacians on infinite circular cones 

Wednesday, 25 May 2022 16:10 (20 minutes)

For $\varepsilon>0$ and $n \in \mathbb{N}$ consider the infinite cone $\Omega_{\varepsilon}:=\left\{\left(x_{1}, x^{\prime}\right) \in(0, \infty) \times \mathbb{R}^{n}:\left|x^{\prime}\right|<\varepsilon x_{1}\right\}$ and the operator $Q_{\varepsilon}^{\alpha}$ acting as the Laplacian $u \mapsto-\Delta u$ on $\Omega_{\varepsilon}$ with the Robin boundary condition $\partial_{\nu} u=\alpha u$ at $\partial \Omega_{\varepsilon}$, where $\partial_{\nu}$ is the outward normal derivative and $\alpha>0$. It is known from numerous earlier works that the essential spectrum of $Q_{\varepsilon}^{\alpha}$ is $\left[-\alpha^{2},+\infty\right)$ and that the discrete spectrum is finite for $n=1$ and infinite for $n \geq 2$, but the behavior of individual eigenvalues with respect to the geometric parameter $\varepsilon$ was only addressed for $n=1$ so far. In the present work we consider arbitrary $n \geq 2$ and look at the spectral asymptotics as $\varepsilon$ becomes small, i.e. as the cone becomes "sharp" and collapses to its central axis. Our main result is as follows: if $n \geq 2, \alpha>0$ and $j \in \mathbb{N}$ are fixed, then the $j$ th eigenvalue $E_{j}\left(Q_{\varepsilon}^{\alpha}\right)$ of $Q_{\varepsilon}^{\alpha}$ behaves as $E_{j}\left(Q_{\varepsilon}^{\alpha}\right)=-\frac{n^{2} \alpha^{2}}{(2 j+n-2)^{2} \varepsilon^{2}}+O\left(\frac{1}{\varepsilon}\right)$ as $\varepsilon \rightarrow 0^{+}$.

Primary authors: Prof. PANKRASHKIN, Konstantin (Carl von Ossietzky Universität); VOGEL, Marco (Carl von Ossietzky Universität)

Session Classification: Poster

