Workshop on Modern Trends in Quantum Theory



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## Small angle asymptotics for Robin Laplacians on infinite circular cones

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For  $\varepsilon > 0$  and  $n \in \mathbb{N}$  consider the infinite cone  $\Omega_{\varepsilon} := \{(x_1, x') \in (0, \infty) \times \mathbb{R}^n : |x'| < \varepsilon x_1\}$  and the operator  $Q_{\varepsilon}^{\alpha}$  acting as the Laplacian  $u \mapsto -\Delta u$  on  $\Omega_{\varepsilon}$  with the Robin boundary condition  $\partial_{\nu} u = \alpha u$  at  $\partial \Omega_{\varepsilon}$ , where  $\partial_{\nu}$  is the outward normal derivative and  $\alpha > 0$ . It is known from numerous earlier works that the essential spectrum of  $Q_{\varepsilon}^{\alpha}$  is  $[-\alpha^2, +\infty)$  and that the discrete spectrum is finite for n = 1 and infinite for  $n \ge 2$ , but the behavior of individual eigenvalues with respect to the geometric parameter  $\varepsilon$  was only addressed for n = 1 so far. In the present work we consider arbitrary  $n \ge 2$  and look at the spectral asymptotics as  $\varepsilon$  becomes small, i.e. as the cone becomes "sharp" and collapses to its central axis. Our main result is as follows: if  $n \ge 2$ ,  $\alpha > 0$  and  $j \in \mathbb{N}$  are fixed, then the *j*th eigenvalue  $E_j(Q_{\varepsilon}^{\alpha})$  of  $Q_{\varepsilon}^{\alpha}$  behaves as  $E_j(Q_{\varepsilon}^{\alpha}) = -\frac{n^2 \alpha^2}{(2j+n-2)^2 \varepsilon^2} + O\left(\frac{1}{\varepsilon}\right)$  as  $\varepsilon \to 0^+$ .

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