The properties of scaled GIG distribution

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GENERALIZED INVERSE GAUSSIAN DISTRIBUTION

Formula:

$$f_{GIG}(x) = Ax^{\alpha} e^{-\frac{\beta}{x}} e^{-\lambda x}$$

$A > 0,$
$\alpha \in \mathbb{R},$
$\beta > 0,$
$\lambda > 0$

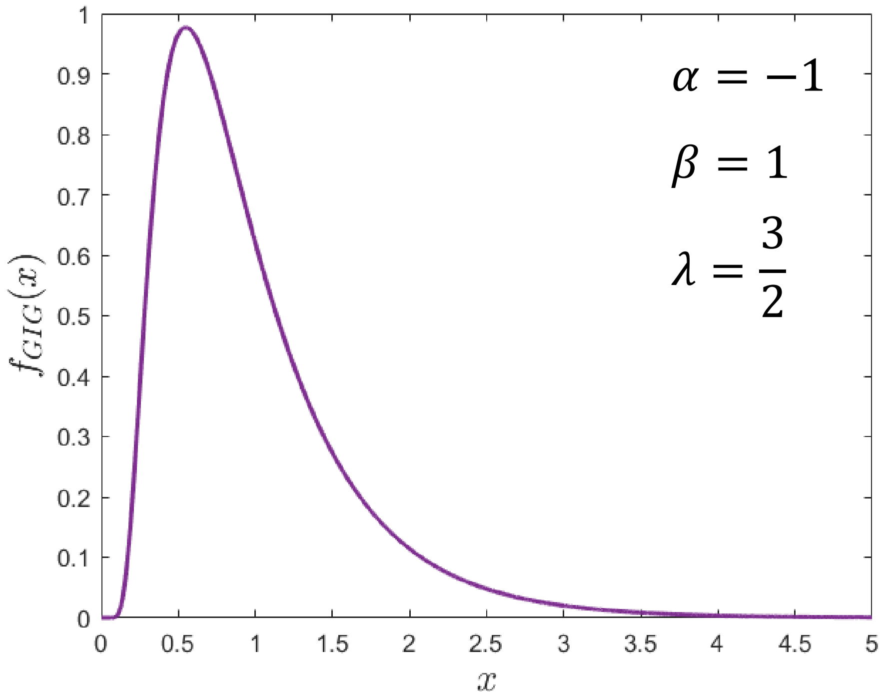


Fig 1: The shape of GIG distribution.

BASIC PROPERTIES

- GIG is balanced density:

$$f_{GIG}(x) \in \mathcal{B} \iff A > 0, \alpha \in \mathbb{R}, \beta > 0, \lambda > 0$$

- Normalization constant:

$$A = \frac{\left(\sqrt{\frac{\lambda}{\beta}}\right)^{\alpha+1}}{2\, K_{\alpha+1}(2\sqrt{\beta\lambda})}$$

($K_a(x)$ reffers to Macdonalds function of the a-kind)

- Scaling problem:

(viz [1])

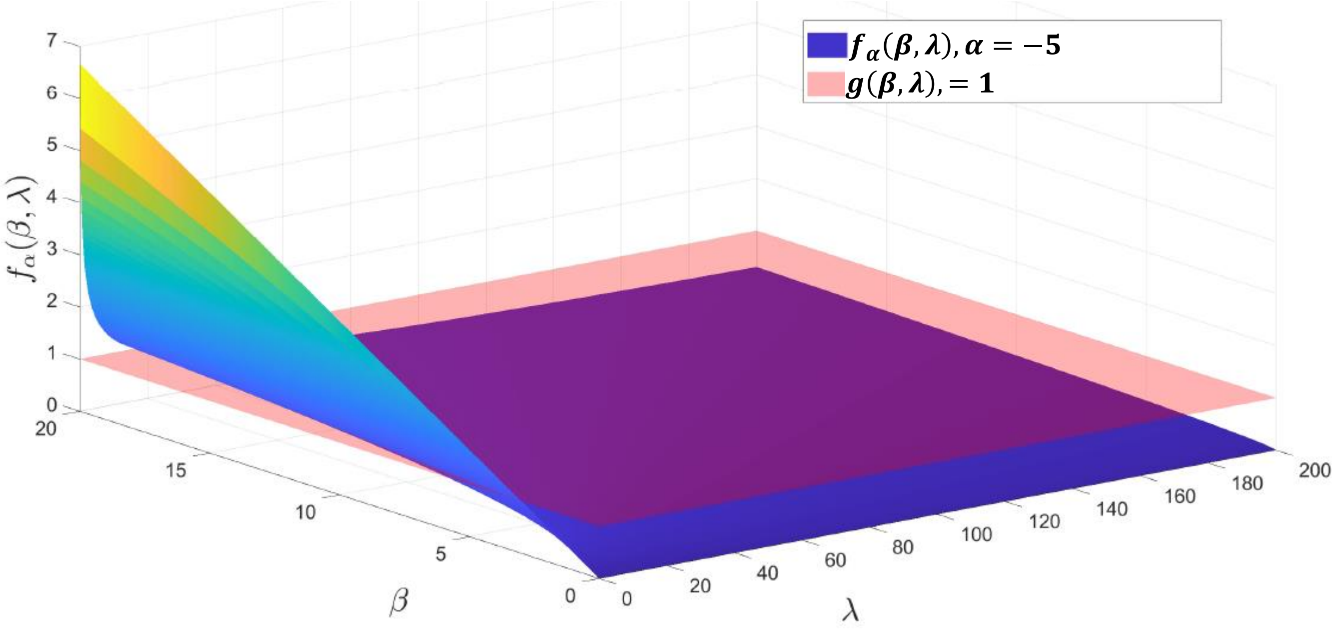
  - scaling condition: $\alpha + \beta + 2 > 0$
  - asymptotic approximation of scaling function: $\lambda(\beta) \approx \alpha + \beta + \frac{3}{2}$

MORE ABOUT SCALING PROBLEM

- Scaling equation:

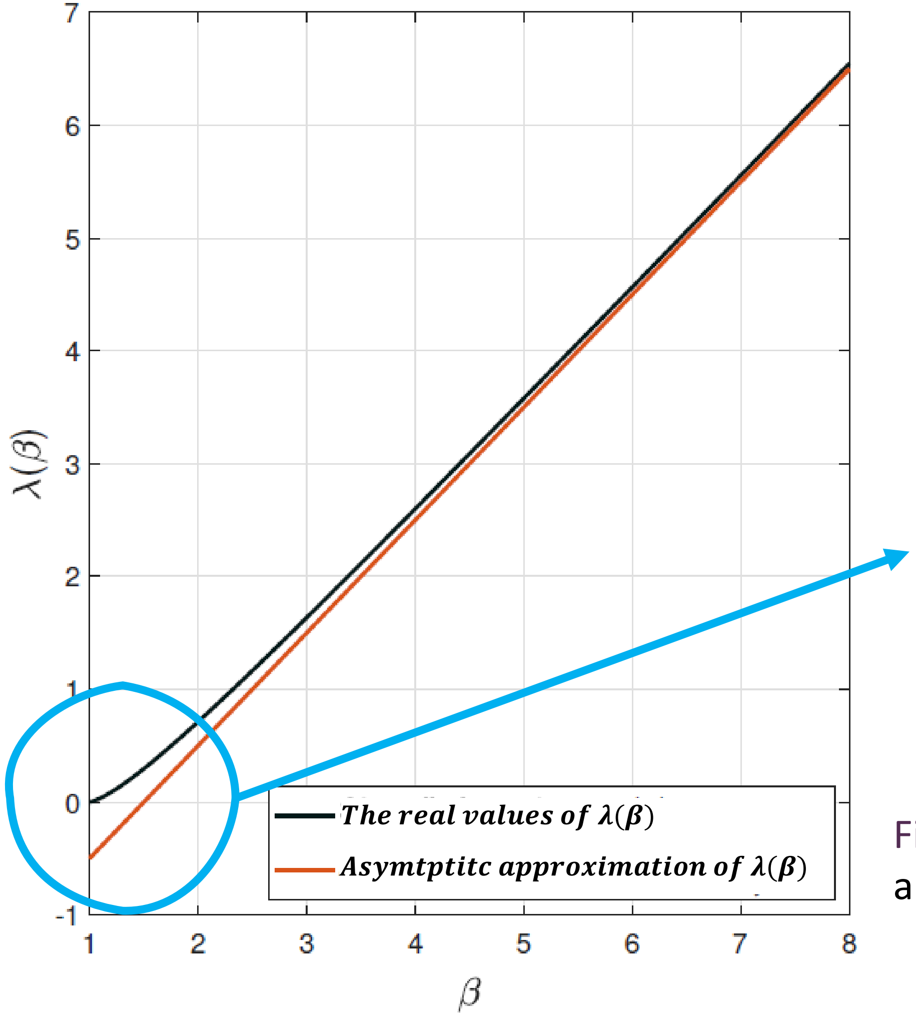
$$1 = \frac{\left(\sqrt{\frac{\lambda}{\beta}}\right)^{\alpha+1}}{2\ K_{\alpha+1}(2\sqrt{\beta\lambda})} \int_0^\infty x^{\alpha+1} e^{-\frac{\beta}{x}} e^{-\lambda x}\ dx = \underbrace{\sqrt{\frac{\beta}{\lambda}} \frac{K_{\alpha+2}(2\sqrt{\beta\lambda})}{K_{\alpha+1}(2\sqrt{\beta\lambda})}}_{f_\alpha(\beta,\lambda)}$$

Fig 2: Example of the need for scaling condition.



$$\boldsymbol{\lambda(\beta) =?}$$

ASYMPTOTIC APPROXIMATION OF $\lambda(\beta)$



We found asymptotic approximation of the scaling function.

$$\boldsymbol{\lambda(\beta) \approx \alpha + \beta + \frac{3}{2}}$$

However, we aimed for better approximation.

Fig 3: The difference between the real values of scaling function and its asymptotic approximation for $\alpha = -3$.

IMPORTANT DISCOVERIES

- $\lambda(\beta)$ is **increasing** function

- lower and upper **bound**:

$$\alpha + \beta - \lambda(\beta) + 1 < 0$$
$$\alpha + \beta - \lambda(\beta) + 2 > 0$$

- **limits:**

$$\alpha > -1: \quad \lim_{\beta \to 0_+} \lambda(\beta) = \alpha + 1$$
$$\alpha < -2: \quad \lim_{\beta \to -\alpha-2} \lambda(\beta) = 0$$

- upgraded **approximation**:

$$\lambda(\beta) \approx \begin{cases} \alpha + \beta + \frac{3}{2} - \frac{1}{2} e^{-\sqrt{\frac{4\beta}{6+\alpha}}}, \alpha > -1 \\ \alpha + \beta + \frac{3}{2} + \frac{1}{2} e^{-\sqrt{\frac{4(\alpha+\beta+2)}{6-\alpha}}}, \alpha \leq -1 \end{cases}$$

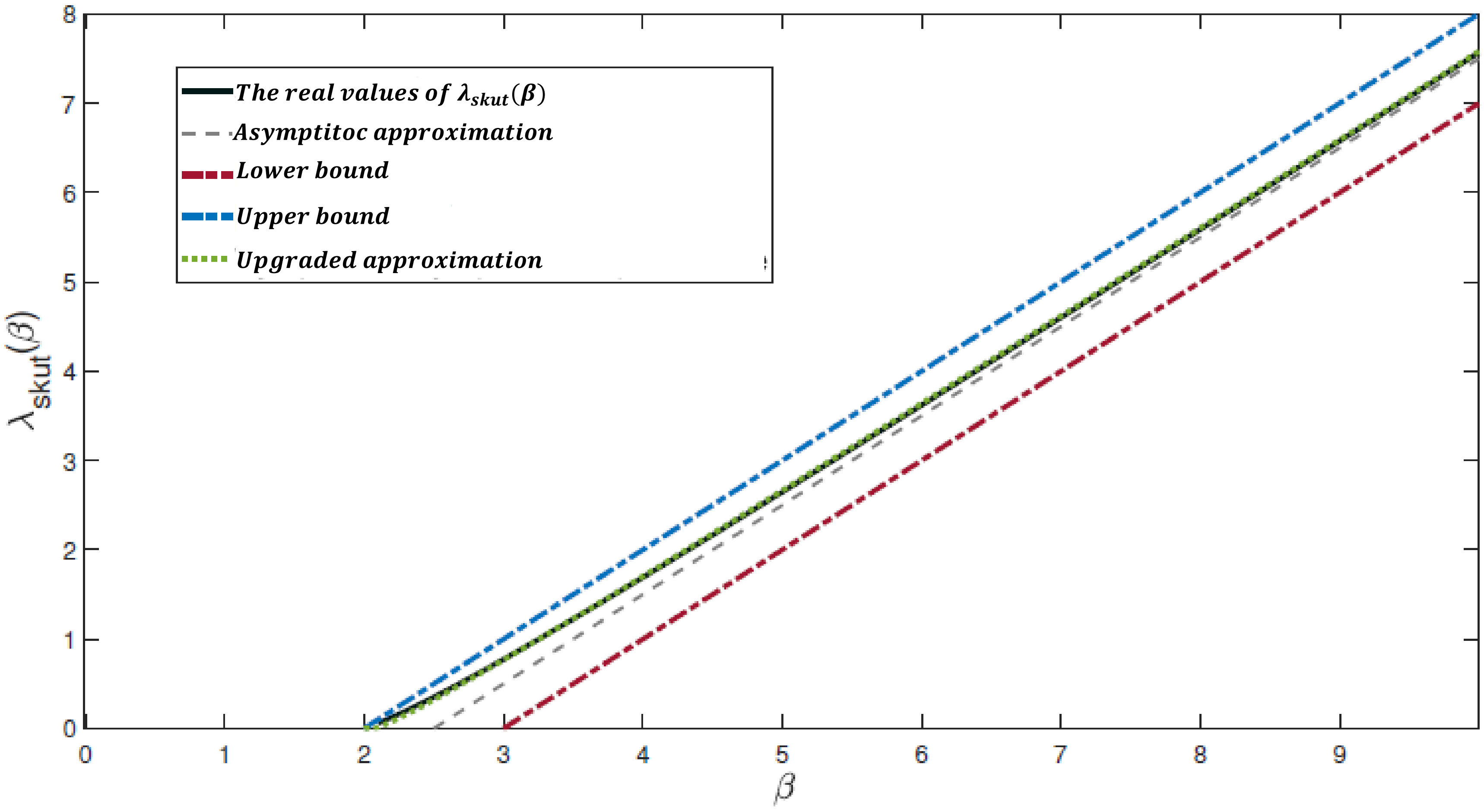
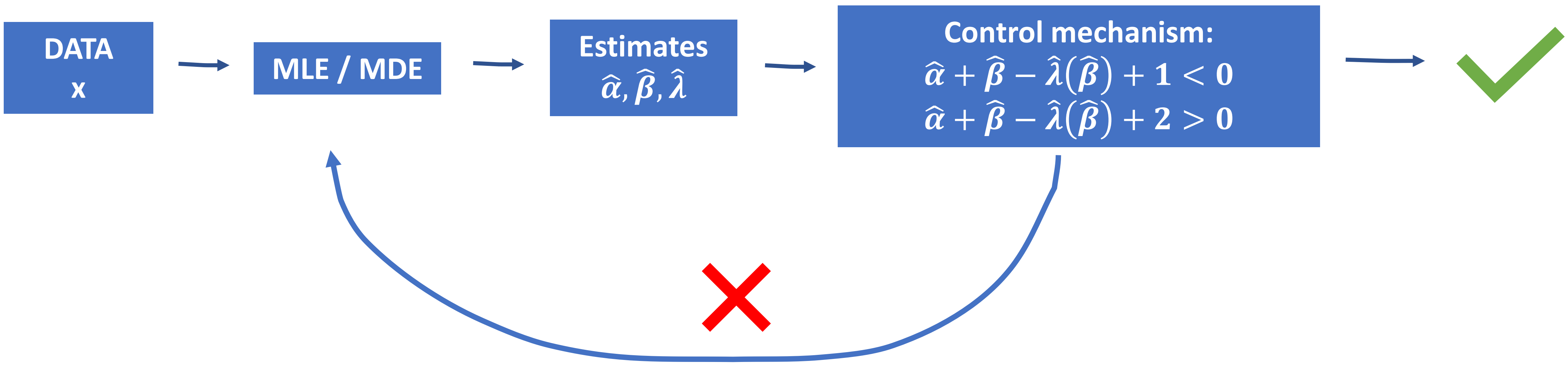


Fig 4: Example of found properties of the scaling function $\lambda(\beta)$, where $\alpha = -4$.

APPLICATION

Lower and upper bound can be used as **control mechanism** for the basic estimation methods (maximum likelihood est., minimum distance est.).



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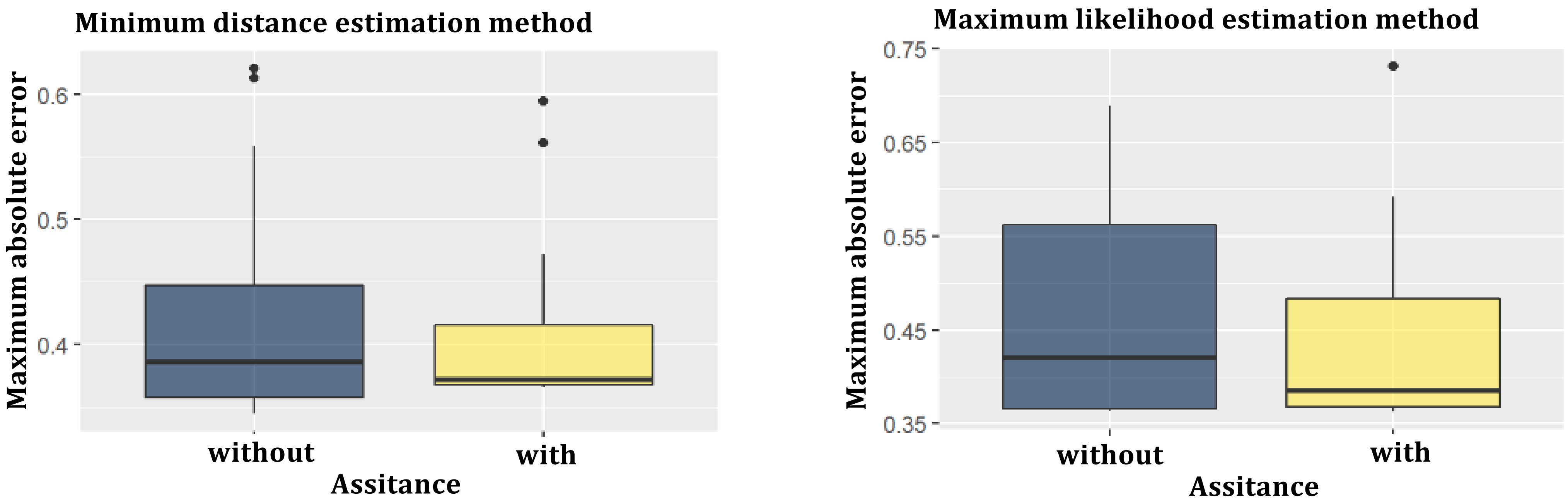

Fig 5: Example of the improving attempts for minimum distance est. and maximum likelihood est.

THANK YOU FOR YOUR ATTENTION!