Density Estimates in Cellular Automata Models of Pedestrian Dynamics

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SMPS

Density expectations

Standard definition

- $\rho = \frac{N}{A}$
- global version A covers whole area
- local version: A is fixed in space, i.e. detector approach
- individual version: A is defined as pedestrian surroundings

Density expectations

Standard definition

- $\rho = \dfrac{N}{A}$
- global version *A* covers whole area
- local version: *A* is fixed in space, i.e. detector approach
- individual version: *A* is defined as pedestrian surroundings
- works when particle size is neglectable comparing to size of area

$\rightarrow$ fulfilled when number of pedestrians is large

$\rightarrow$ generalization and/or redefinition

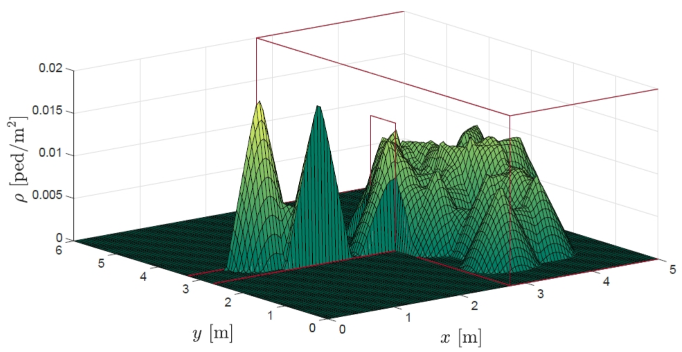
- close to fundamental definition (when comparable) ++
- mathematical requirements - roughness, featureless

$$\langle \dot{C} \rangle := \frac{1}{\#T} \sum_{i=2}^{\#T} \frac{|C(t_i) - C(t_{i-1})|}{t_i - t_{i-1}} \qquad \max_{t \in \mathbb{R}^+} C(t)$$

Density definition

Pedestrian distribution

$$\rho_B = \frac{N}{|B|} = \frac{\int_B p(\vec{x})\,\mathrm{d}\vec{x}}{|B|} = \frac{\int_B \sum_{\alpha=1}^{N} p_\alpha(\vec{x})\,\mathrm{d}\vec{x}}{|B|} = \sum_{\alpha=1}^{N} \frac{\int_B p_\alpha(\vec{x})\,\mathrm{d}\vec{x}}{|B|}$$



Density definition

Kernels

- **Point approximation**

$$p_\alpha(\boldsymbol{x}) = \delta_{\boldsymbol{x},\boldsymbol{x}_\alpha}$$

- **Stepwise function** .. cylindrical, Voronoi, Manhattan, ...

$$p_\alpha(\boldsymbol{x}, R) = 1_{A_\alpha(R)}(\boldsymbol{x})\,\frac{1}{|A_\alpha(R)|}$$

- **Conic kernel**

$$p_\alpha(\boldsymbol{x}, R) = \frac{3}{\pi R^3} 1_{A_\alpha(R)}(\boldsymbol{x})\,\left(R - \|\boldsymbol{x} - \boldsymbol{x}_\alpha\|\right)$$

- **Gaussian kernel** in a symmetric version, i.e. with diagonal covariance matrix

$$p_\alpha(\boldsymbol{x}, R) = \frac{1}{2\pi R^2}\,\exp\left\{-\frac{\|\boldsymbol{x} - \boldsymbol{x}_\alpha\|^2}{2\,R^2}\right\}$$

Density definition

Kernels 2

- **Borsalino kernel** from [Krbálek, Krbálková]

$$p(\boldsymbol{x}, R) = \frac{1}{Z}\frac{1}{R}\, 1_{A_\alpha(R)}(\boldsymbol{x}) \exp\left\{ \frac{R^2}{\|\boldsymbol{x} - \boldsymbol{x}_\alpha\|^2 - R^2} \right\}$$

Density definition

Kernels 2

- **Borsalino kernel** from [Krbálek, Krbálková]

$$p(\boldsymbol{x}, R) = \frac{1}{Z}\,\frac{1}{R}\,1_{A_\alpha(R)}(\boldsymbol{x})\,\exp\left\{\frac{R^2}{\|\boldsymbol{x}-\boldsymbol{x}_\alpha\|^2 - R^2}\right\}$$

Min dist

- scaled inverse value of appropriate distance

$$p(\vec{x}) = \frac{c_{md}}{\mathrm{dist}(\vec{x})}$$

- different ranges

$$\mathrm{dist}(\vec{x}) := D(\vec{x}) := \min_{\{\alpha\in\mathrm{N}:\measuredangle\left(\vec{x}_\alpha,\vec{s}_{\vec{x}}\right)\leq\frac{\pi}{3}\}} \|\vec{x}-\vec{x}_\alpha\|$$

- different distances

$$\mathrm{dist}(\vec{x}) := D(\vec{x})^2$$

Density definition

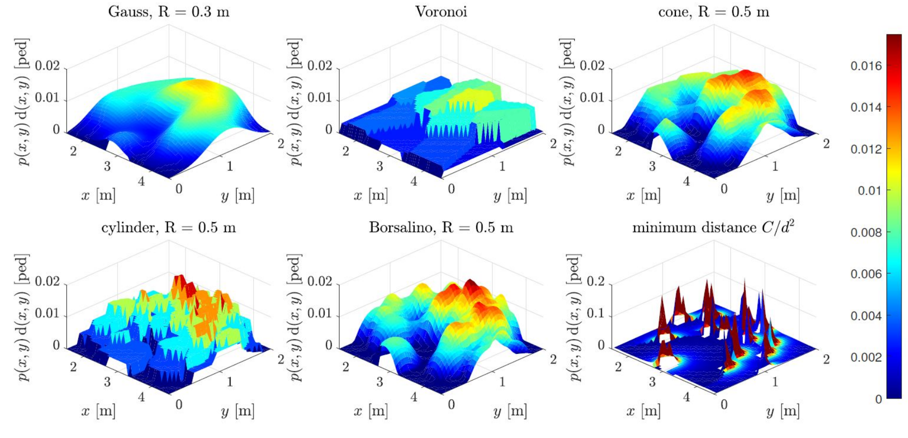
Existing methods

- Voronoi method
- weighted pedestrians based on distance to detector/pedestian
- weighted pedestrians based on time presence in detector
- minimum distance based estimates

Ref.	Year	Dim	Kernel	Details
Daamen, Hoogendoorn, and Bovy (2005); Daamen and Hoogendoorn (2007)	2005	2	-	density defined using travel times
Helbing et al. (2006)	2006	1	-	smooth particle count
Helbing, Johansson, and Al-Abideen (2007)	2007	2	exponential	weighted pedestrian contribution
Johansson et al. (2008); Johansson (2009)	2008	2	exponential	local density using weights
Krisp et al. (2009)	2009	2	Gauss	tool for visual choice of bandwidth
Steffen and Seyfried (2010)	2010	2	Voronoi	the concept used here
Schadschneider, Chowdhury, and Nishinari (2010); Schadschneider et al. (2018)	2010	2	exponential	concept of weighting pedestrian
Schadschneider and Seyfried (2011)	2011	2	-	no consensus for the estimates
Liddle et al. (2011)	2011	2	Voronoi	stabilization of the result
Tipakornkiat, Kim, and Limanond (2011)	2011	1	-	density at sidewalks (crossing lines)
Zhang et al. (2011)	2011	2	Voronoi	comparing of few methods
Plaue et al. (2012); Plaue, Bärwolff, and Schwandt (2014)	2012	2	Gauss	bandwidth depends on time
Fan (2013); Fan, Herty, and Seibold (2013)	2013	2	Gauss	density of vehicular streams
Tordeux et al. (2015)	2015	1	Voronoi, Gauss	comparing of few methods
Duives, Daamen, and Hoogendoorn (2015)	2015	2	Voronoi, exp.	comparing of few methods
Krbálek and Krbálkova (2018)	2018	1	Borsalino	smooth particle count
Mollier et al. (2019)	2019	2	Gauss	opt. bandwidth for vehicles Fan (2013)
Bukáček and Vacková (2019)	2019	2	Cone	individual density study
Hillebrand, Hoogeveen, and Geraerts (2020)	2020	2	Voronoi, Gauss	comparing of few methods
Vacková and Bukáček (2020)	2020	2	Cone	application of individual density

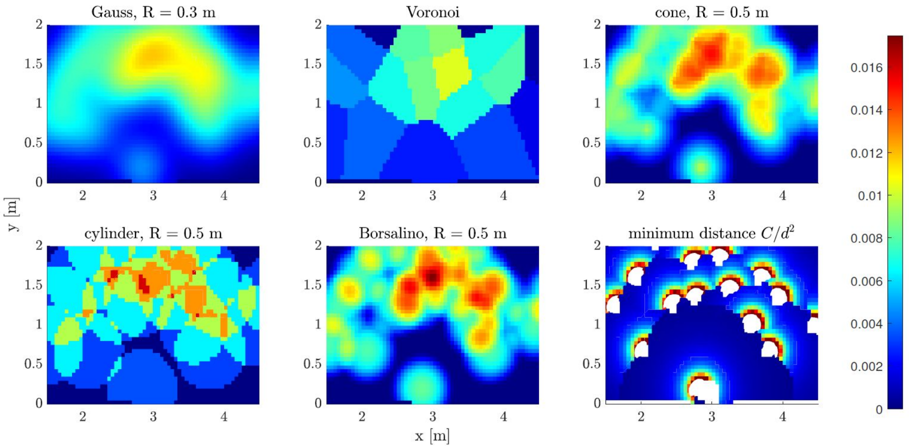
Density definition

Illustration of distributions



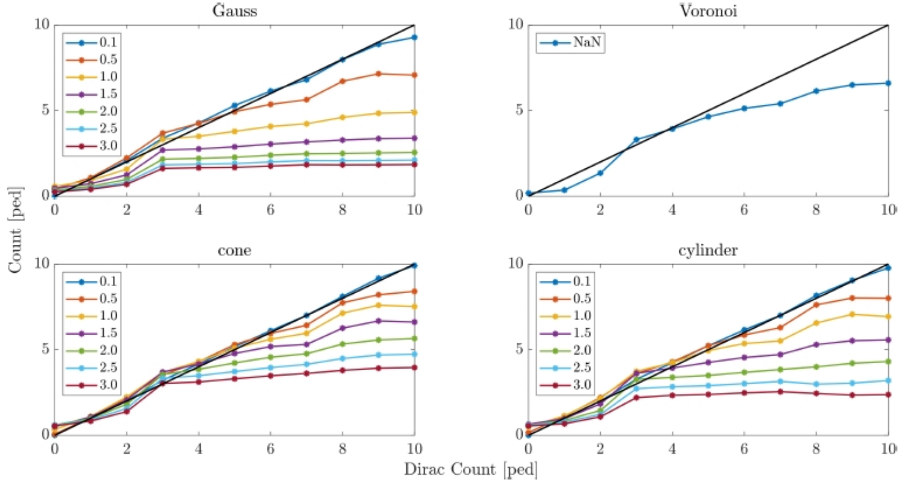
Density definition

Illustration of distributions



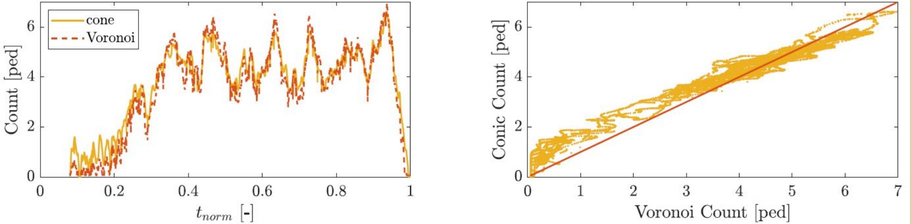
Kernel types - point approximation

Point approximation vs cone



Kernel types - Voronoi

Voronoi vs cone

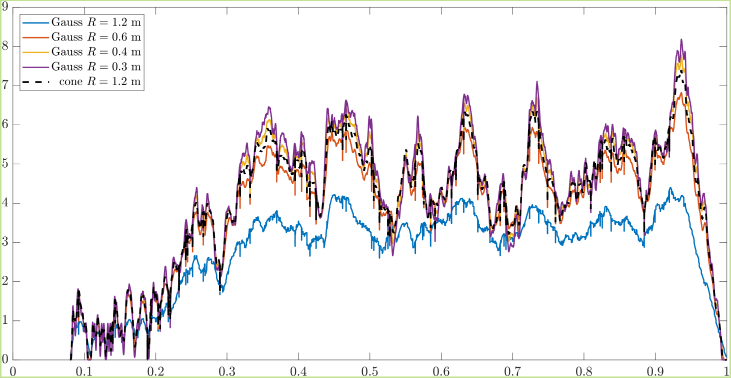


Kernel types - Gauss

Mass recalculation

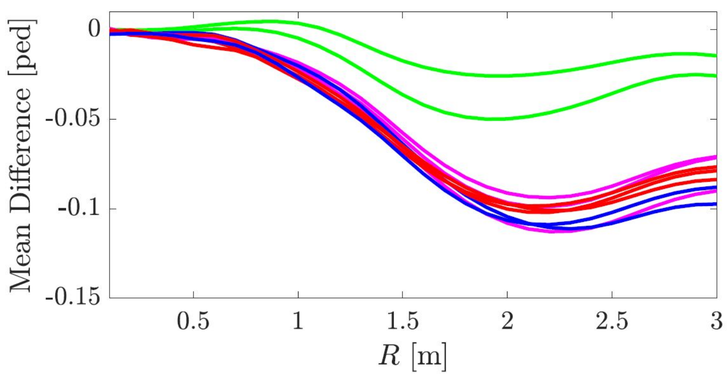
$$\int_{A_R} p(\boldsymbol{x},\sigma)\,\mathrm{d}\boldsymbol{x} = \int_{A_{k\sigma}} p(\boldsymbol{x},\sigma)\,\mathrm{d}\boldsymbol{x} = \int_0^{\frac{k^2}{2}} \mathrm{e}^{-t}\mathrm{d}t$$

Gauss vs cone



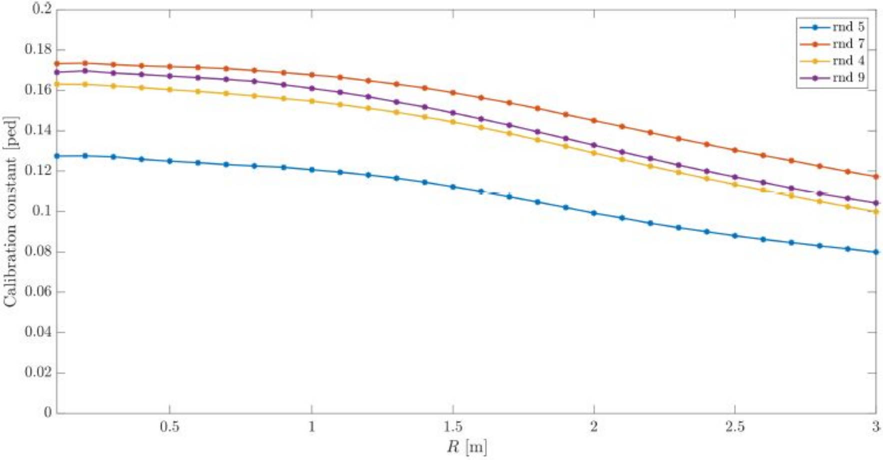
Kernel types - Borsalino

Borsalino vs cone



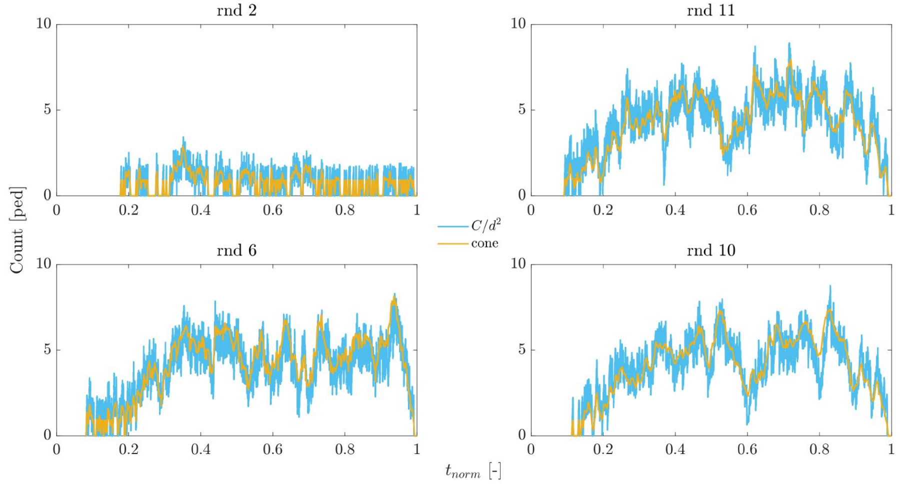
Kernel types - minimal distance

Minimal distance calibration



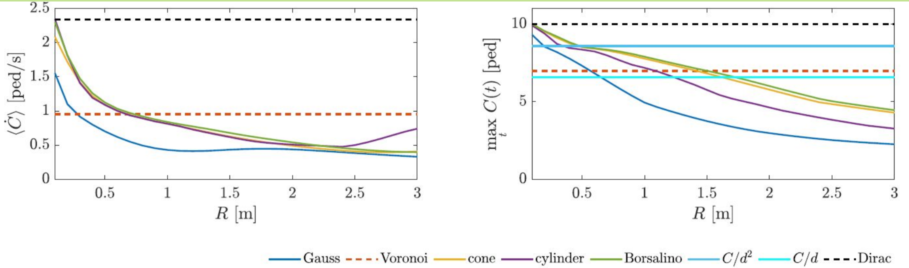
Kernel types - minimal distance

Minimal distance vs cone



Comparison

Roughness, featureless



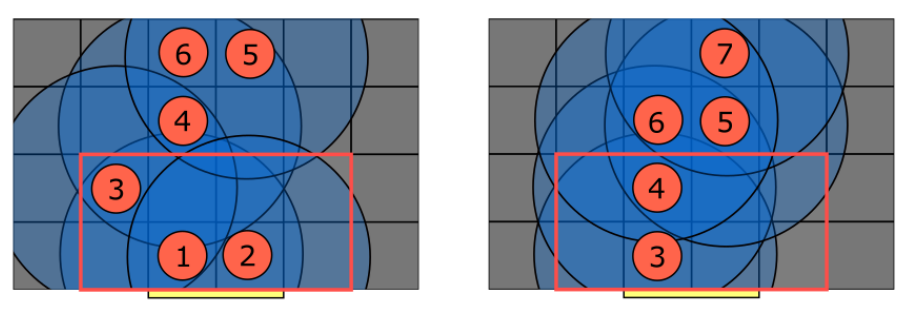
Comparison

Mean occupancy

	Mean ped. count [ped]		Training data for Min. distance				Testing data for Min. distance			
	Method	R [m]	Rnd 5	Rnd 7	Rnd 4	Rnd 9	Rnd 2	Rnd 11	Rnd 6	Rnd 10
NP	Voronoi	-	0.77	4.03	2.81	3.40	0.27	3.69	3.63	3.58
	Point approximation	-	1.51	4.29	3.34	3.80	0.68	4.18	4.05	4.01
	Minimum distance $1/d$	-	1.52	4.27	3.72	3.95	0.85	4.22	4.11	4.12
	Minimum distance $1/d^2$	-	1.46	4.21	3.49	3.82	0.73	4.12	4.04	4.00
P1	Conic kernel	0.3	1.59	4.49	3.48	3.99	0.73	4.35	4.26	4.21
	Cylindrical kernel	0.3	1.58	4.48	3.47	3.98	0.73	4.33	4.25	4.20
	Borsalino kernel	0.3	1.59	4.49	3.49	3.99	0.73	4.35	4.27	4.22
	Gaussian kernel	0.1	1.59	4.49	3.49	3.99	0.73	4.35	4.27	4.22
P2	Conic kernel	0.9	1.59	4.43	3.39	3.91	0.77	4.24	4.15	4.10
	Cylindrical kernel	0.9	1.60	4.37	3.33	3.84	0.81	4.15	4.07	4.00
	Borsalino kernel	0.9	1.59	4.44	3.40	3.93	0.76	4.26	4.16	4.12
	Gaussian kernel	0.3	1.58	4.44	3.41	3.93	0.75	4.27	4.18	4.13
P3	Conic kernel	1.5	1.52	4.19	3.17	3.65	0.78	3.94	3.89	3.81
	Cylindrical kernel	1.5	1.39	3.87	2.88	3.32	0.74	3.58	3.54	3.46
	Borsalino kernel	1.5	1.55	4.26	3.23	3.71	0.80	4.01	3.95	3.87
	Gaussian kernel	0.5	1.54	4.30	3.27	3.76	0.77	4.07	4.00	3.93

Discrete systems

Case study



- assuming inflow 1.5 ped/T

→ by standard def. alternating 2 and 3 peds in detector

→ using appropriate kernel, 2.5 ped ind detector

Conclusions

- generally used methods covered

$$\lim_{R\to 0^+} p(\boldsymbol{x}, R) = \lim_{R\to 0^+} \sum_{\alpha=1}^{N} p_\alpha(\boldsymbol{x}, R) = \sum_{\alpha=1}^{N} \delta_{\boldsymbol{x},\boldsymbol{x}_\alpha}$$

- rage based kernels converge to standard definition
- parameter based convergence of one method to the other
- important features identified
- useful even for discrete systems

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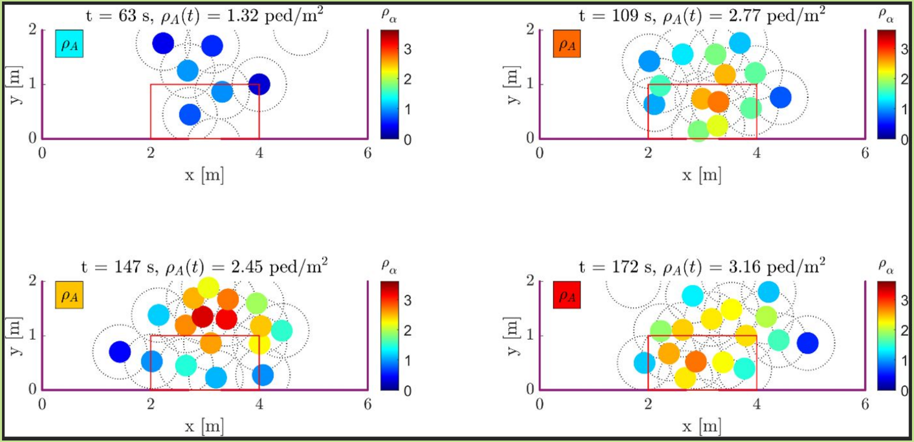
Thank you for your attention!

Katedra matematiky FJFI ČVUT v Praze

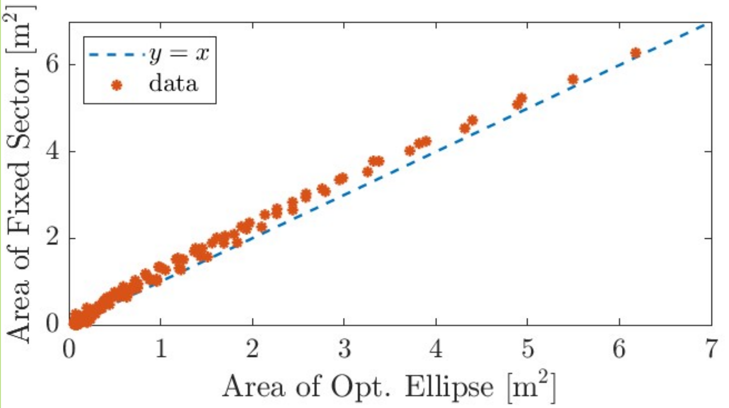
Local



t = 63 s, $\rho_A(t)$ = 1.32 ped/m^2
ρ_A
ρ_α
t = 109 s, $\rho_A(t)$ = 2.77 ped/m^2
ρ_A
ρ_α
t = 147 s, $\rho_A(t)$ = 2.45 ped/m^2
ρ_A
ρ_α
t = 172 s, $\rho_A(t)$ = 3.16 ped/m^2
ρ_A
ρ_α
x [m]
y [m]

Local

Area of an ellipse vs area of circular sector



Local

Width of an ellipse vs width of circular sector