Statistical models for estimation of unsignalized intersection capacity

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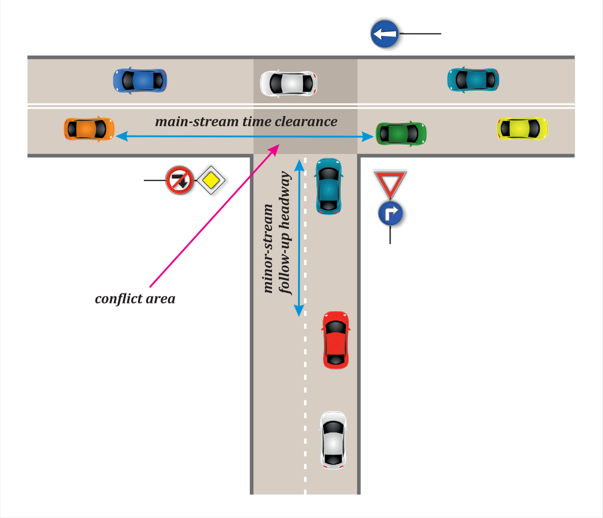
Considered model



Siegloch's framework

- Assumptions:
  - homogeneity and consistency of drivers
  - saturation condition for the minor traffic flow
- Siegloch's function:

$$s(t) = \sum_{k=0}^{+\infty} k\mathrm{P}[N_t = k]$$

- Intersection capacity equation (1973)

$$C = I \int_0^{+\infty} g(t)s(t)\,\mathrm{d}t$$

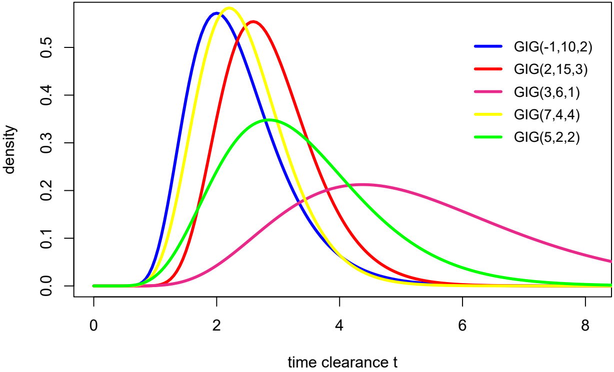
Intermediate step

- Analytical derivation of expression $\mathrm{P}[N_t = k]$

$$
\begin{aligned}
\mathrm{P}[N_t = k] &= \mathrm{P}[Y_1 + \cdots + Y_k < t < Y_1 + \cdots + Y_k + Y_{k+1}] = \\
&= \mathrm{P}\left[\sum_{i=1}^{k} Y_i < t < \sum_{i=1}^{k} Y_i + Y_{k+1}\right] = \\
&= \mathrm{P}[Z < t < Z + Y_{k+1}] = \\
&= \int_0^t \int_t^{+\infty} f_Z(q) f_{Y_{k+1}}(r - q) \, \mathrm{d}r \, \mathrm{d}q
\end{aligned}
$$

GIG(α,β,λ) distribution, $\alpha \in \mathrm{R},\ \beta, \lambda > 0$

$$f_X(x) = \Theta(x)\sqrt{\left(\frac{\lambda}{\beta}\right)^{\alpha+1}}\,\frac{1}{2\mathrm{K}_{\alpha+1}[2\sqrt{\beta\lambda}]}\; x^{\alpha}\,\mathrm{e}^{-\frac{\beta}{x}}\,\mathrm{e}^{-\lambda x}$$



Convolution of N identical GIG-distributed densities

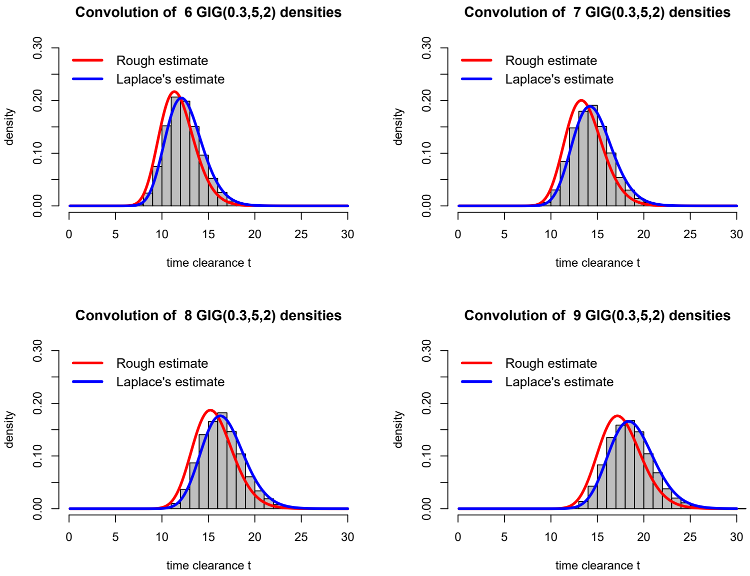
- The method of rough estimate

$$(\bigstar_{i=1}^{N} f_Y)(y) \sim \mathrm{GIG}(N\alpha + N - 1, N^2\beta, \lambda)$$

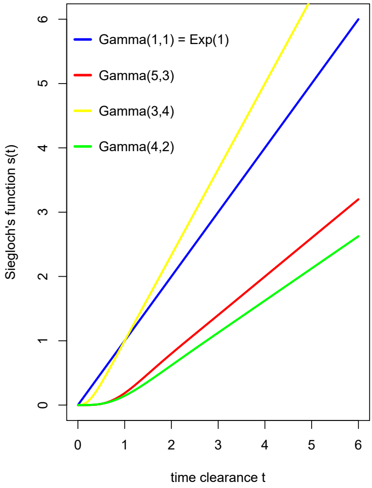
- The Laplace's method

$$(\bigstar_{i=1}^{N} f_Y)(y) \sim \mathrm{GIG}(N\alpha + (N-1)\frac{3}{2}, N^2\beta, \lambda)$$

Comparison of approximation accuracy



Siegloch's function for Gamma-distributed critical clearances



- For Gamma-distributed critical clearances, $\forall t > 0$

$$s(t) = \mathrm{e}^{-\lambda t} \sum_{l=0}^{\alpha-1} \sum_{k=0}^{+\infty} k \frac{(\lambda t)^{k\alpha+l}}{(k\alpha+l)!}$$

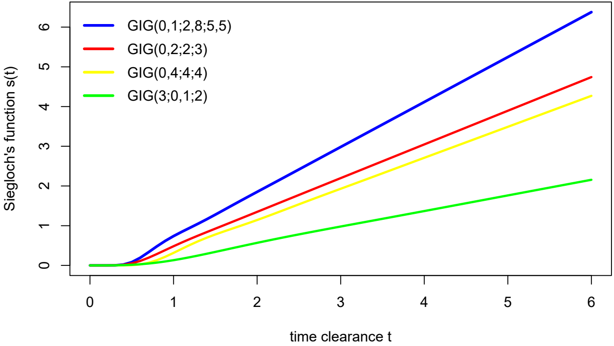
- For Exponentially-distributed critical clearances, $\forall t > 0$

$$s(t) = \lambda t$$

Siegloch's function for GIG-distributed critical clearances

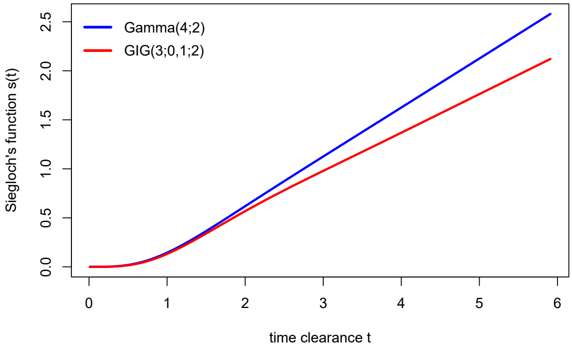
- For GIG-distributed critical clearances $\forall t > 0$

$$s(t) = \sum_{k=0}^{+\infty} kC_k \int_0^t q^{k\alpha + \frac{3}{2}(k-1)} \mathrm{e}^{-\frac{k^2\beta}{q}} \left(\int_t^{+\infty} (r-q)^{\alpha} \mathrm{e}^{-\frac{\beta}{r-q}} \mathrm{e}^{-\lambda r} \mathrm{d}r \right) \mathrm{d}q$$



Insufficient formula accuracy

- $\mathrm{Gamma}(4, 2) \approx \mathrm{GIG}(3, 0.1, 2)$, so where is the mistake?



Empirical derivation of expression $\mathrm{P}[N_t = k]$

- Siegloch's function:

$$s(t) = \sum_{k=0}^{+\infty} k \mathrm{P}[N_t = k]$$

- From The Law of Large Numbers follows

$$\frac{n_k}{N} \xrightarrow{\mathrm{p}} \mathrm{P}[N_t = k]$$

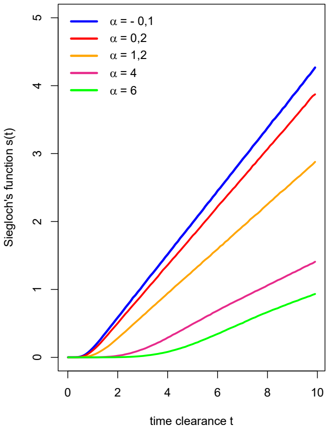
- Empirical Siegloch's function

$$s^{\star}(t) = \sum_{k=0}^{k_{\mathrm{max}}} k \frac{n_k}{N}$$

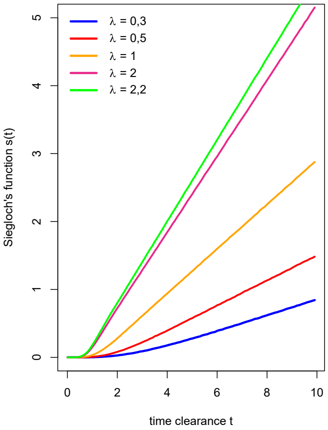
Behavior of empirical Siegloch's function

Considering the GIG(1.2,2,1) distribution as the default

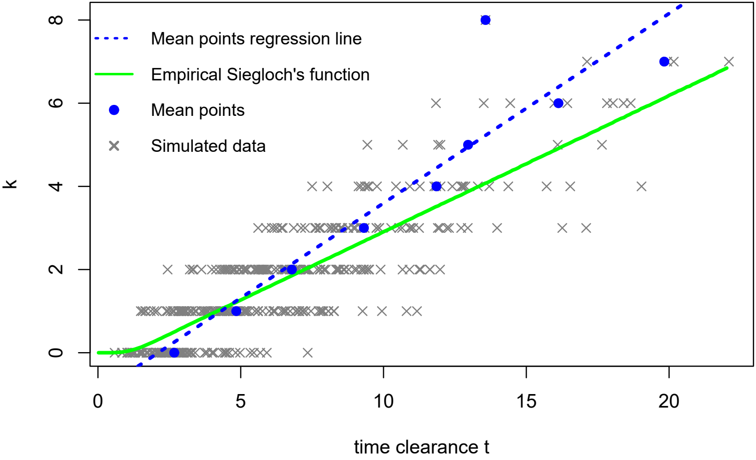
- α parameter change



- λ parameter change



Regression analysis for estimations of Siegloch's function



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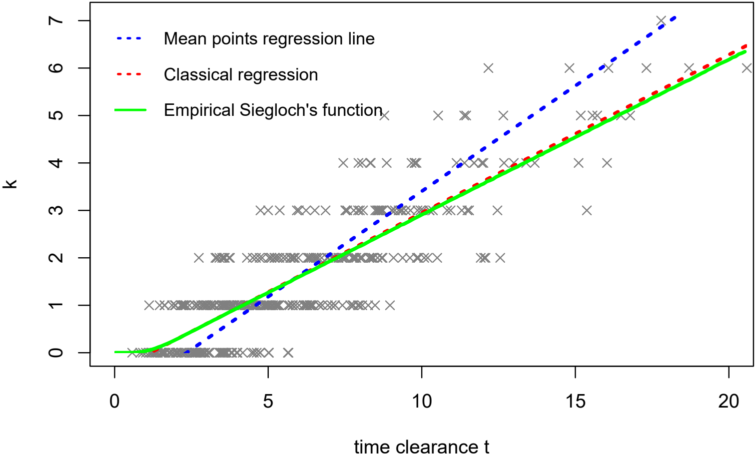

Conclusion

- Impact on the estimated capacity of the unsignalized intersections
- If we calculate numerically the integrals

$$J = \int_0^\infty g(t)s(t)\mathrm{d}t,$$

we get following values

J_{true}	J_{class}	J_{trad}
?	1.61	1.74

Thank you for your attention.