CRASH! BOOM! BANG!

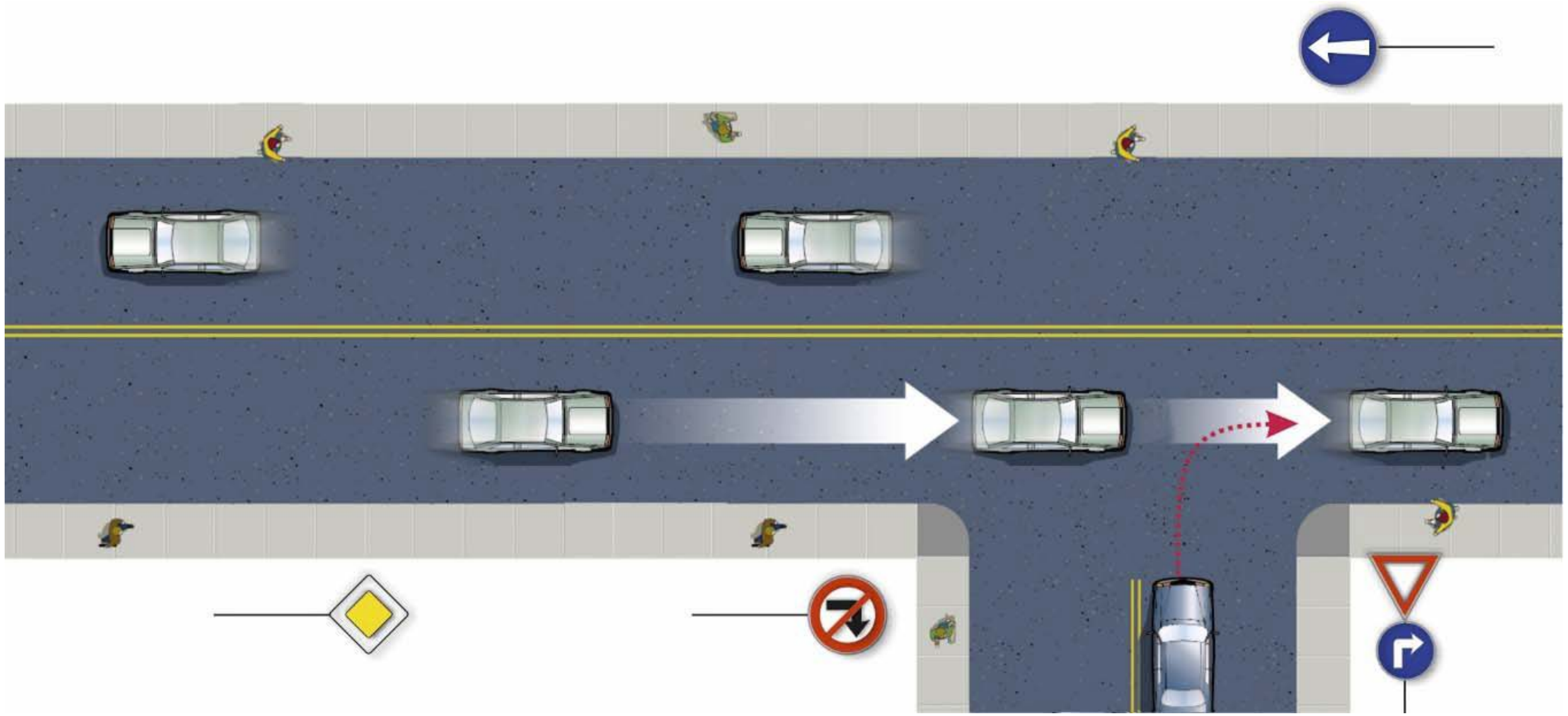
Mgr. Michaela Krbálková

Faculty of Science, University of Hradec Králové

Faculty of Transport Engineering, University of Pardubice

Investigated problem

- vehicular dynamics in the vicinity of an unsignalized intersection of the T-type



- **main stream**
  - vehicles not affected by events on the minor road
  - moving independent of vehicles in opposite direction
  - dynamic properties description well-developed (physics of vehicular traffic)

- **minor stream**
  - commanded direction of turn
  - implicit real-time decision-making procedure based on critical gap
  - acceptance or rejection of a given time gap

- => **Gap Acceptance theory** (GA)
- standardly-applied engineering method
- capacity models for optimal designing of urban intersections and roundabouts

Our goal

- **reformulation into a purely mathematical form** operating strictly in the language of random variables

- **in-depth revision of commonly accepted starting points**

- = essentials for capacity problem!

Fundamental terms

- **time headway** = time interval between two events:
  - 1) front bumper of the preceding vehicle intersects a detector line
  - 2) front bumper of the reference vehicle intersects a detector line

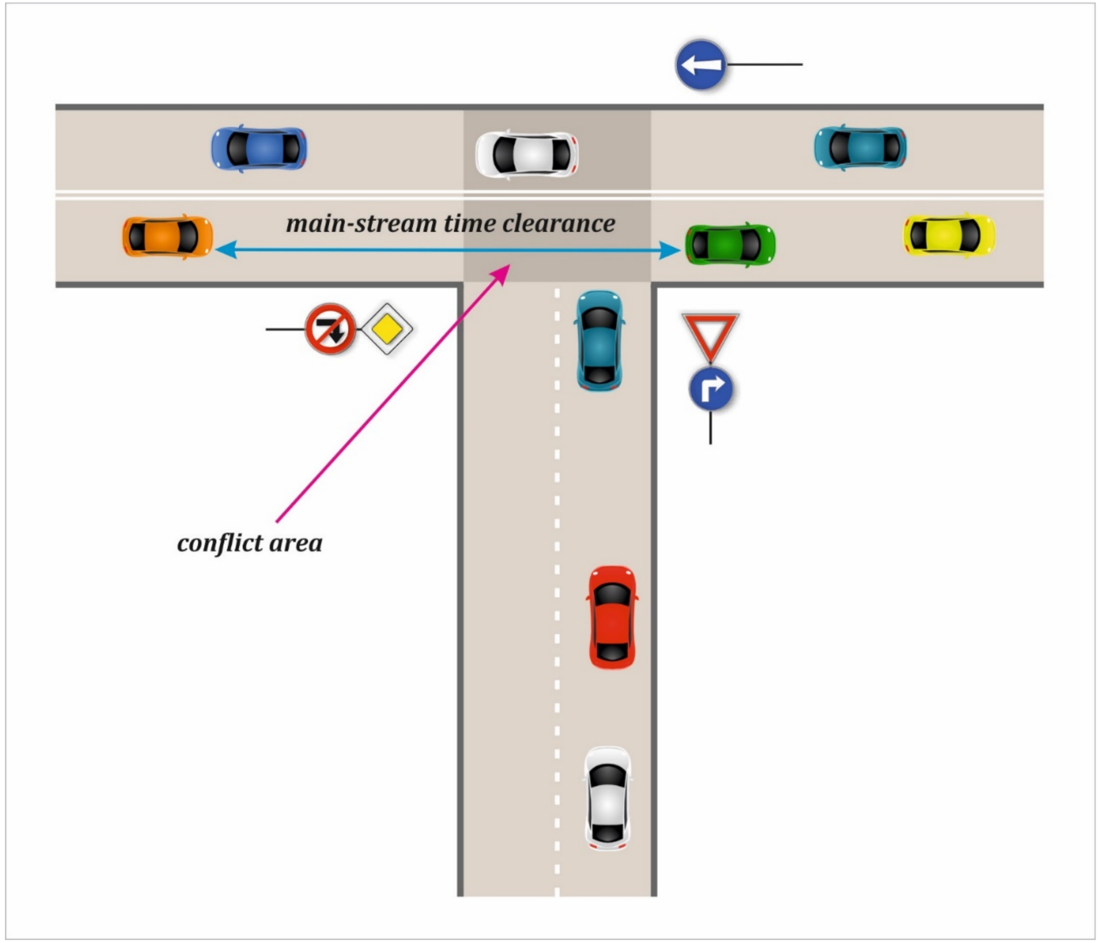
- **time clearance** = time interval between two events:
  - 1) back bumper of the preceding vehicle intersects a detector line
  - 2) front bumper of the reference vehicle intersects a detector line

- **traffic intensity** = the number of vehicles passing a fixed point located on the main street within a time interval

- **main/minor traffic stream**: right-of-way / no right-of-way

The standardly-applied engineering methods

- *inputs*
  - main-stream time-clearance distribution: $g(x)$
  - main-stream traffic intensity: I
  - number of minor-stream vehicles accepting the main-stream clearance of size x: $(x; n(x))$



- *assumptions*
  - saturated traffic flow for the minor-stream
  - $g(x)$ exponentially distributed
  - critical gap = single value

- based predominantly on study by Werner Siegloch (1973)

- Siegloch‘s function – expectation of N_x

$$s(x) = \sum_{k=0}^{\infty} k \, \mathrm{P}[N_x = k]$$

- capacity of the unsignalized intersection

$$C = I \int_0^{\infty} g(x)s(x) \, dx$$

- *data collection*: high-definition video recording

- *processing:* computer vision methods (particular vehicles and their parameters)

- *form of data records*
  - individual main-stream clearances
  - number of vehicles that have accepted the gap (**acceptance order** k)
  - individual velocity
  - current traffic intensity

- **accepted clearance of order k**
  - main-stream clearance accepted by k vehicles
  - including variant $k = 0$
  - set A of all clearances = disjoint subsets A_k of clearances of acceptance order k

- **sample-acceptance ratios δ_k**
  - proportional decomposition of main-stream clearances corresponding to acceptance order k

$$\delta_k = \frac{|A_k|}{|A|}$$

Standardly-used approach for the approximation of the Siegloch‘s function

- empirical data: *(individual main-stream clearance x; acceptance order k)*

- averages of clearances x accepted by exactly k minor-stream vehicles

- dependence between acceptance order k and average clearances x

- linear regression for calculated average clearances ➡ **critical gap**

Discrepancies

ASSUMPTIONS:

- saturated traffic flow for the minor-stream
- X is exponentially distributed, i.e. $\mathrm{g(x)} = \Theta(x)\lambda \mathrm{e}^{x}$
- minor-stream headways exponentially distributed
- critical gap = single value

REALITY:

- saturated traffic flow for the minor-stream
- ✗ **NO!** GIG distribution
- ✗ **NO!** saturated traffic flow ≠ exponential distribution
- ✗ **NO!** it differs from driver to driver, from time to time, etc.

Reformulation to random variables

- vast majority of contemporal approaches are „average-based“

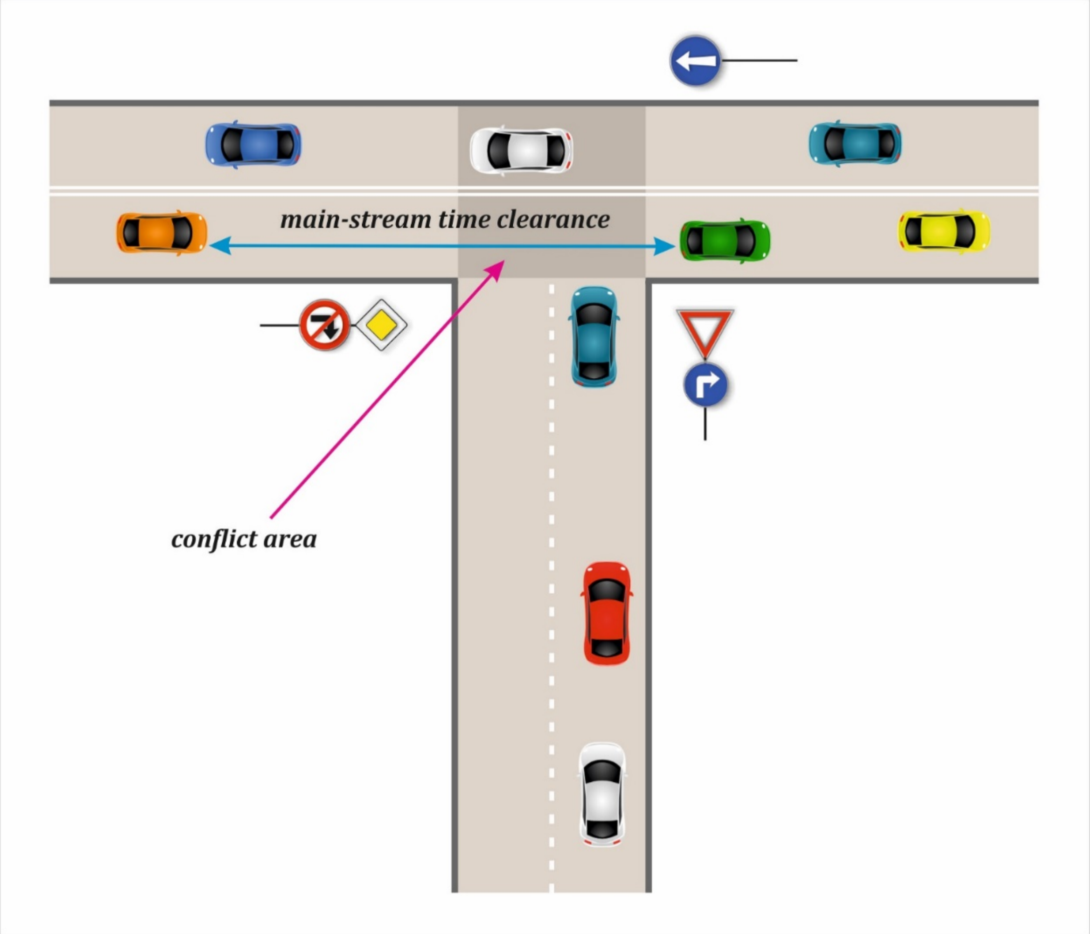
- the same numerical characteristic (i.e. critical gap)
  - ➡ various distributions
    - ➡ different Siegloch‘s function
      - ➡ (dramatically) different capacity

… CRASH!

- new assumption: **critical gap varies in each individual implementation**

- **main-stream time clearance**
  - random variable X
  - stochastic nature

- **critical gap**
  - random variable Y!
  - minimum time clearance between two succeeding vehicles in the priority stream that a minor-street driver is willing to accept for entering the main-stream



New probabilistic model

- statistical distributions of main-stream time-clearances $g(x)$ and critical gaps $h(y)$
- $g(x), h(y) \neq$ exponential probability density
- probability densities $g(x), h(y)$ = **comprehensive knowledge about intersection capacity**

… BOOM! 🙂

Gap Acceptance procedure

- simple decision routine
- critical gap $Y \sim h(y)$ compared to an offered main-stream clearance $X \sim g(x)$:

$X < Y \rightarrow k = 0$	$X \geq Y$ 2nd order critical gap $Y^{(2)} = Y_1 + Y_2, \; Y_1, Y_2 \sim h(y)$ i.i.d.
$X < Y^{(2)} \rightarrow k = 1$	$X \geq Y^{(2)}$ 3rd order critical gap $Y^{(3)} = Y_1 + Y_2 + Y_3, \; Y_1, Y_2, Y_3 \sim h(y)$ i.i.d.
$X < Y^{(3)} \rightarrow k = 2$	$X \geq Y^{(3)}$ 4th order critical gap $Y^{(4)}$ etc.
…	…
$X < Y^{(j)} \rightarrow k = j - 1$	where $Y^{(j)} = \sum_{i=1}^{j} Y_j$.

- repeat with new clearance X

Distributions of gaps accepted by k vehicles

- main-stream time clearances: random variable $X \sim g(x)$ with realization x

- individual critical gap of the driver kth minor-stream vehicle: random variable $Y_k \sim h(y)$ with realization y_k

- *modeled proces of inclusion*:
  - generate $x, y_1, y_2, \ldots$ and apply decision rule

$y_1 < x$	1st driver enters the main stream; remaining time $x - y_1$
$y_2 < x - y_1$	2nd driver accept the same gap; remaining time $x - y_1 - y_2$

etc. until the remaining time clearance is not accepted by the actual driver

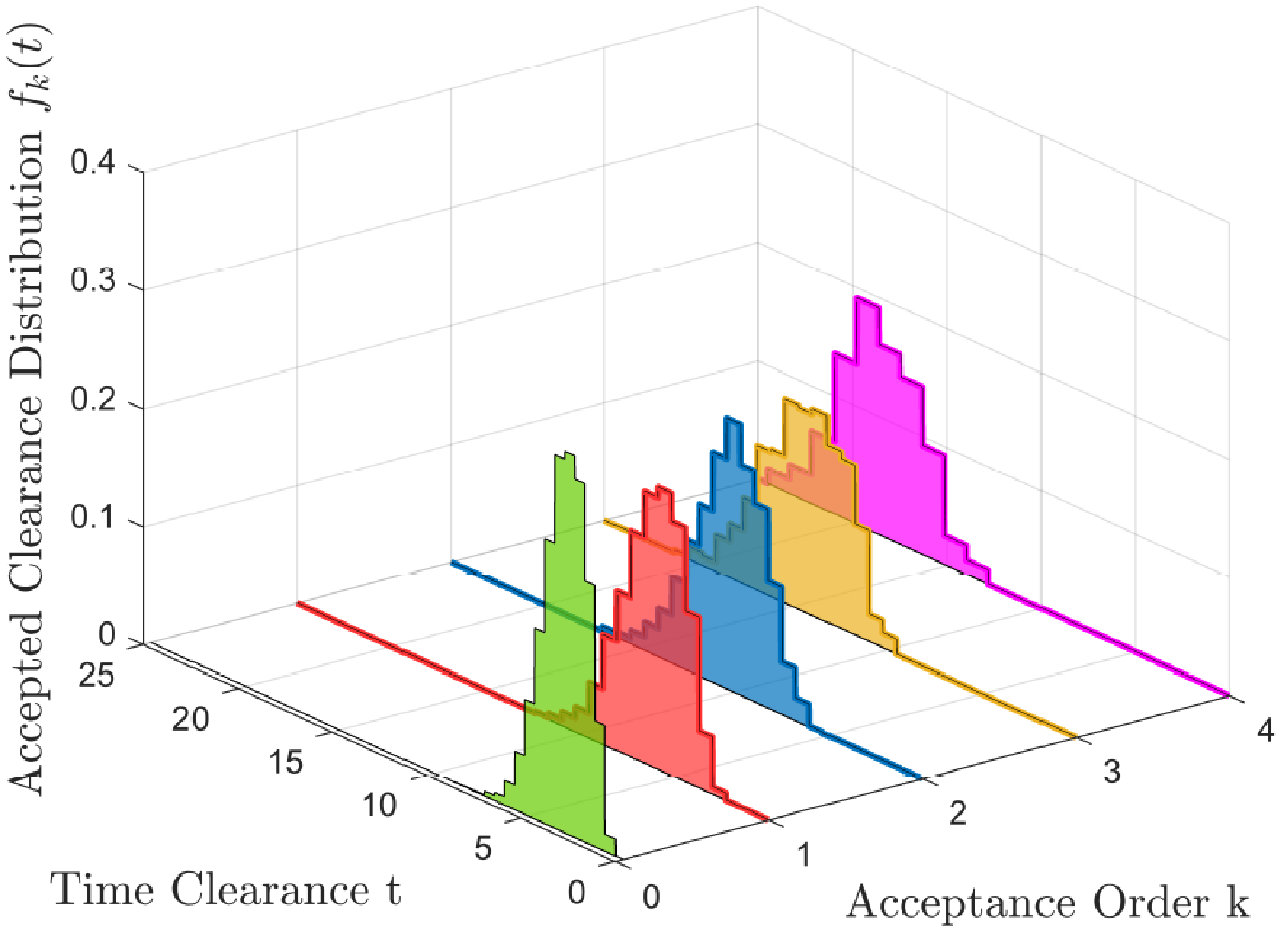
- $X \sim g(x)$ independent of $Y_1, \dots Y_{j+1}$ and $Y_1, \dots Y_{j+1}$ i.i.d. with density function $h(y)$

- event "time clearance is accepted by exactly k vehicles":

$$Y_1 + Y_2 + \cdots + Y_k < X < Y_1 + Y_2 + \cdots Y_j + Y_{k+1}$$
$$\mathrm{Z} = Y_1 + Y_2 + \cdots Y_k$$
$$\mathrm{Z} < X < Z + Y_{k+1}$$

- $Y_1, \dots Y_k$ i.i.d. → density function $h_k(z)$ of random variable Z: $h_k(z) = \left(\star_{i=1}^{k} h\right)(z)$

- outputs: 1) theoretical distributions of time clearances accepted by k vehicles $f_k(x)$
  2) Δ_k = theoretical counterparts of the sample acceptance-ratios δ_k



Accepted Clearance Distribution $f_k(t)$
0.4
0.3
0.2
0.1
0
25
20
15
10
5
0
Time Clearance t
0
1
2
3
4
Acceptance Order k

Goals vs. results

Our goal:

- decomposition of set of main-stream to several subsets
- distribution of clearances accepted by exactly k minor-stream vehicles

Results:

- decomposition tool = *implicit acceptance-rule* described mathematically by probability density $h(y)$
- $h(y)$ describes how individual critical gaps are distributed
- $Y \sim h(y)$ decides on acceptance/rejection of an offered priority-stream gap

Classic problem vs. real problem

- **„math-textbooks“ formulation of the problem**
  - distribution g(x) and h(y) are known
  - What is the decomposition of main-stream clearances into subsets corresponding to acceptance order k?
  - What are distributions of these subsets?

- **„real-life“ formulation of the problem ☺**
  - We can know $g(x)$, estimated from empirical data.
  - We have sample-acceptance ratios corresponding to acceptance order k.
  - We can estimate distributions of clearances accepted by k vehicles.
  - We don´t know the acceptance rule $h(y)$ …

Conclusion

➡ We can find $h(y)$ by consistent reformulation of this problem into language of stochastic variables …

… numerically so far …

… but we hope also for deriving by theoretical (mathematical) approaches …

… TO GO OVER WITH A BANG!