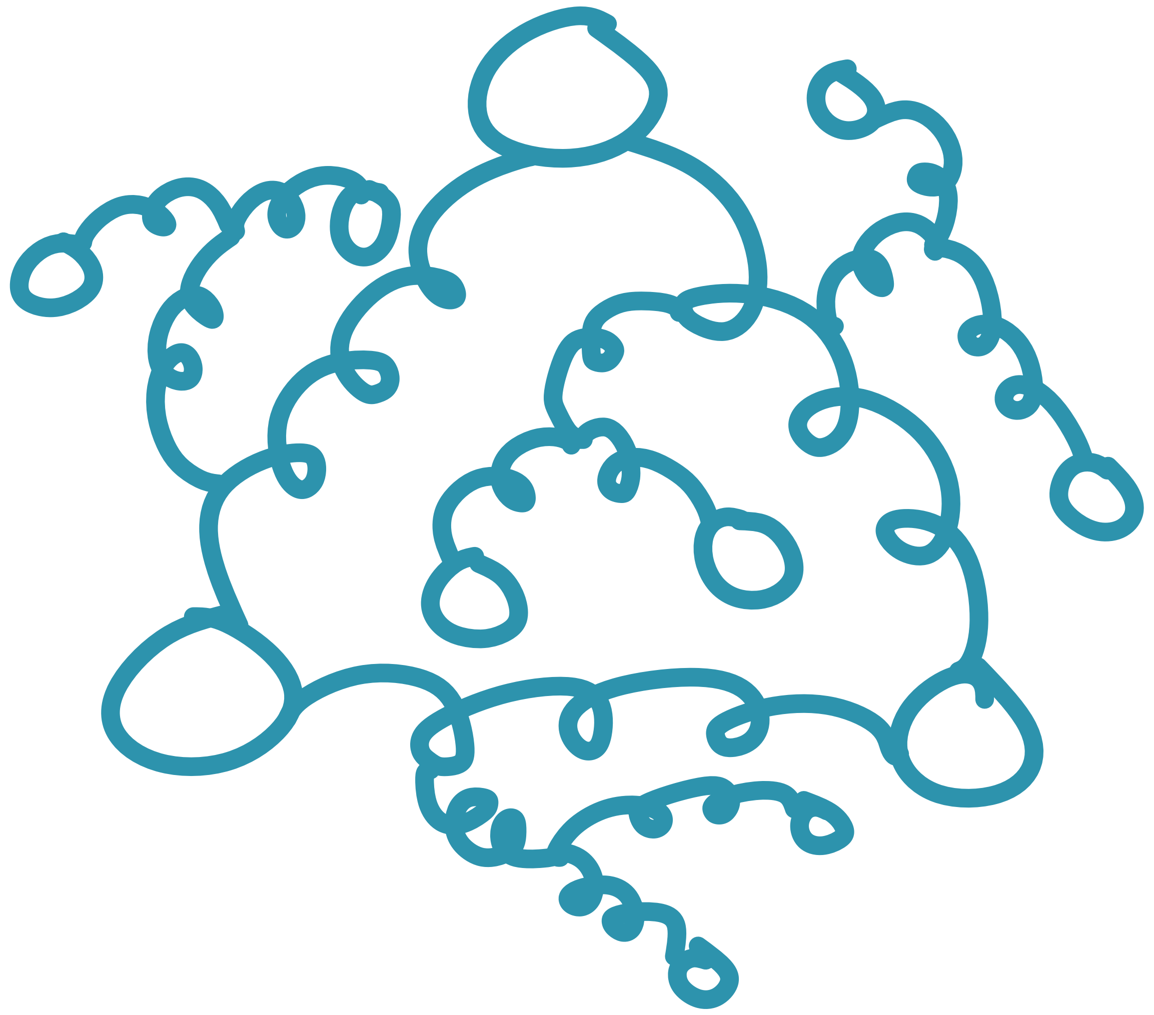
Studies of high-energy limit of QCD using Balitsky-Kovchegov equation

Matěj Vaculčiak, 14. 9. 2023

hadron structure & saturation

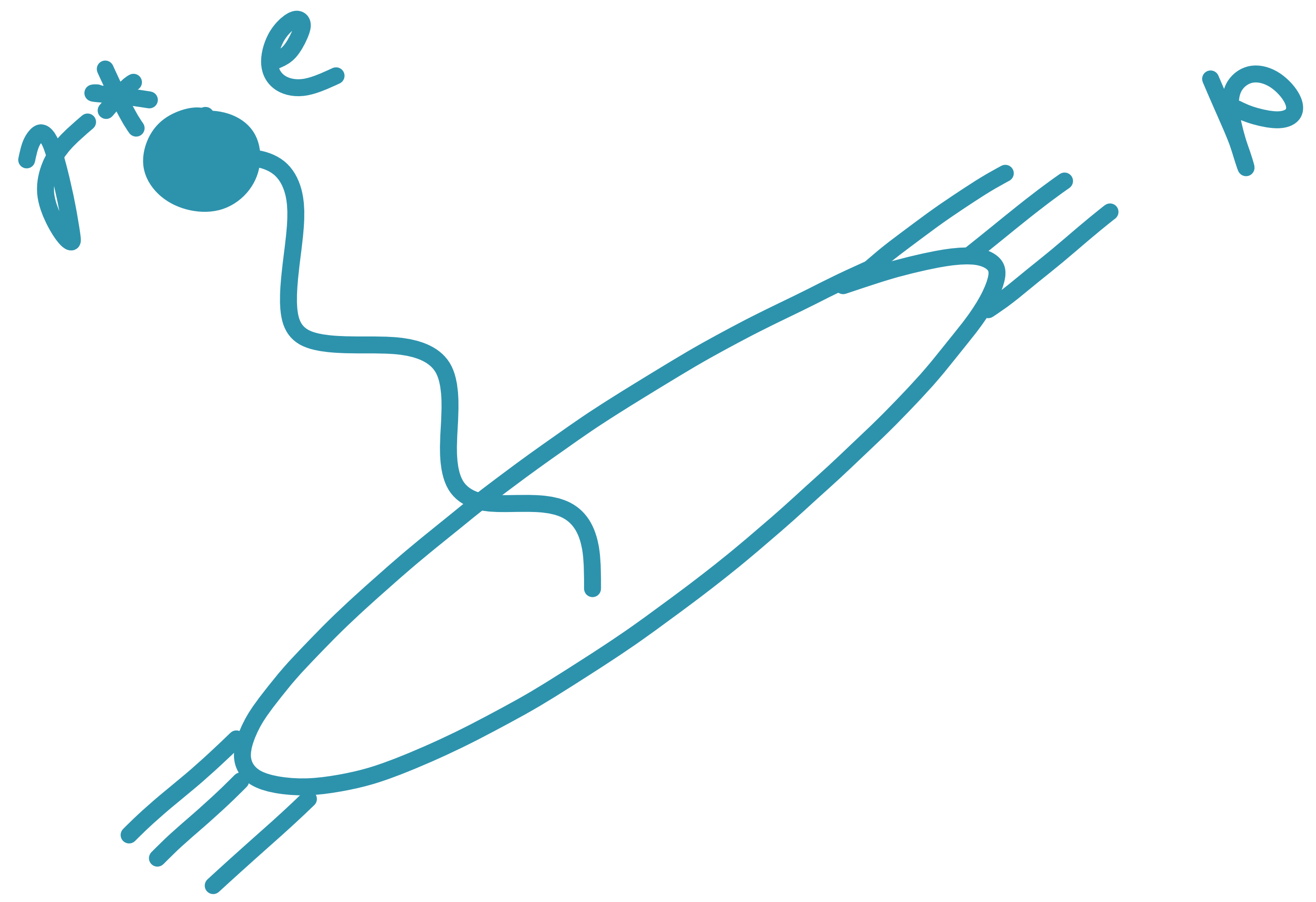
- open question of modern physics
- QCD
  - complex → effective theories
  - rich hadron structure evolution
  - saturation → Balitsky-Kovchegov equation

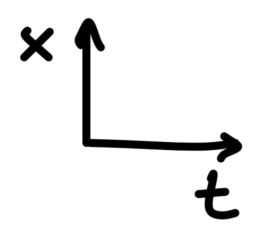


hadron structure & experiment

- colliding proton with a projectile at high E
    - typically electron-proton DIS
- interest in photon-proton cross section

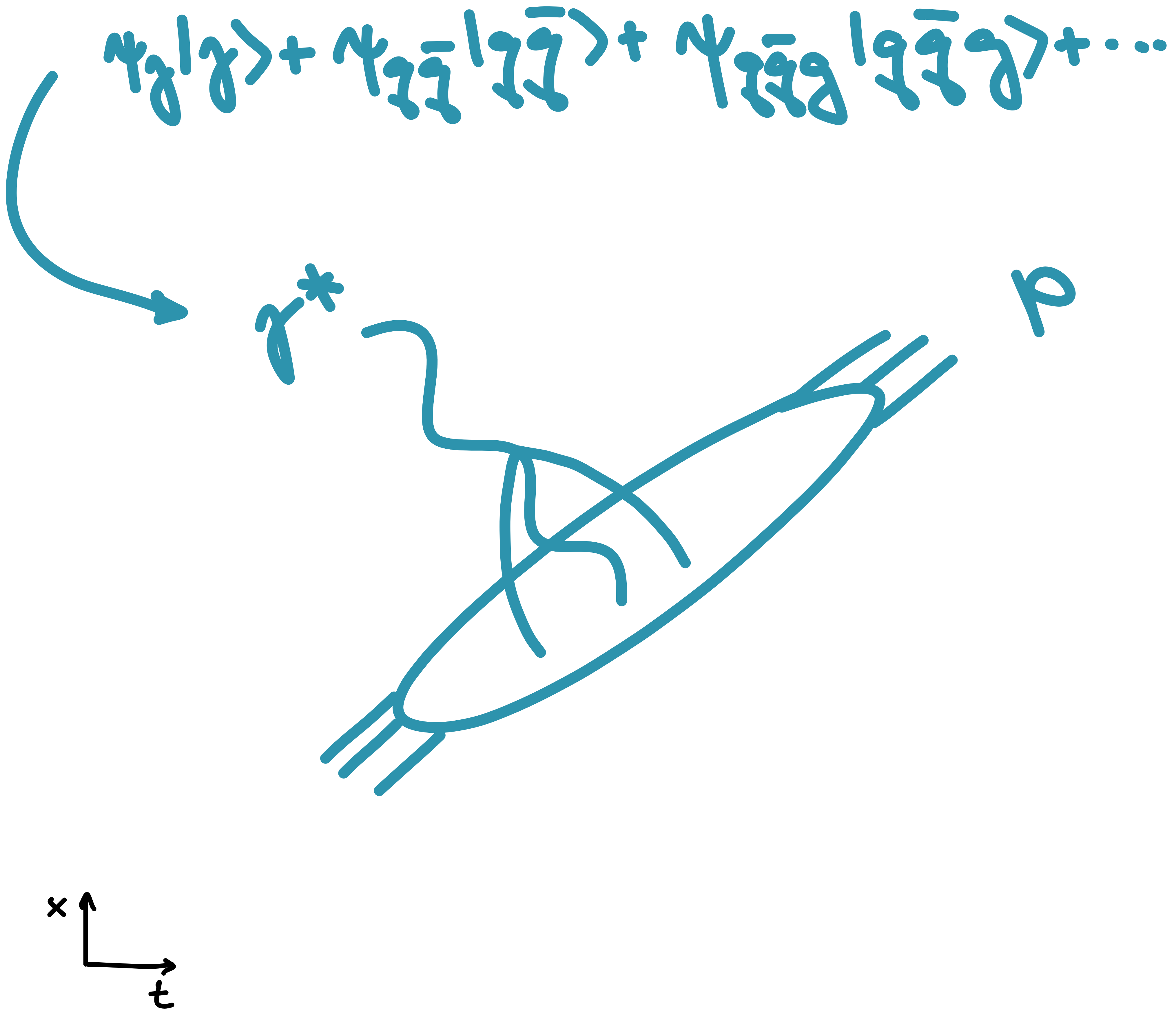
$$\sigma_{L,T}^{\gamma^* p}(x, Q^2)$$





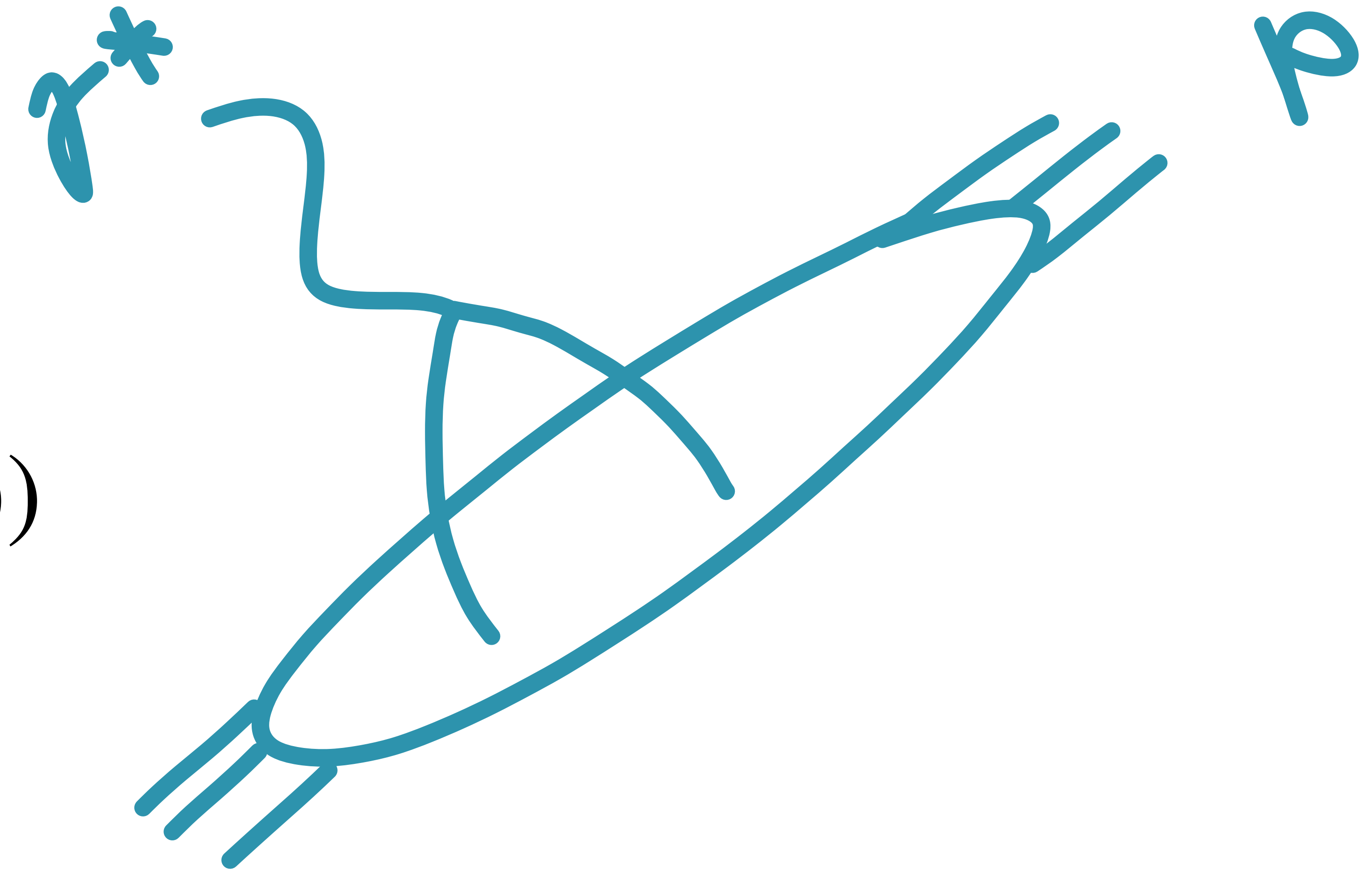
the colour dipole model

- replace γ* with the colour dipole
    - probability ~ light cone wave function $|\psi^{(f)}_{T,L}(\vec{r}, Q^2, z)|^2$
- shockwave approximation

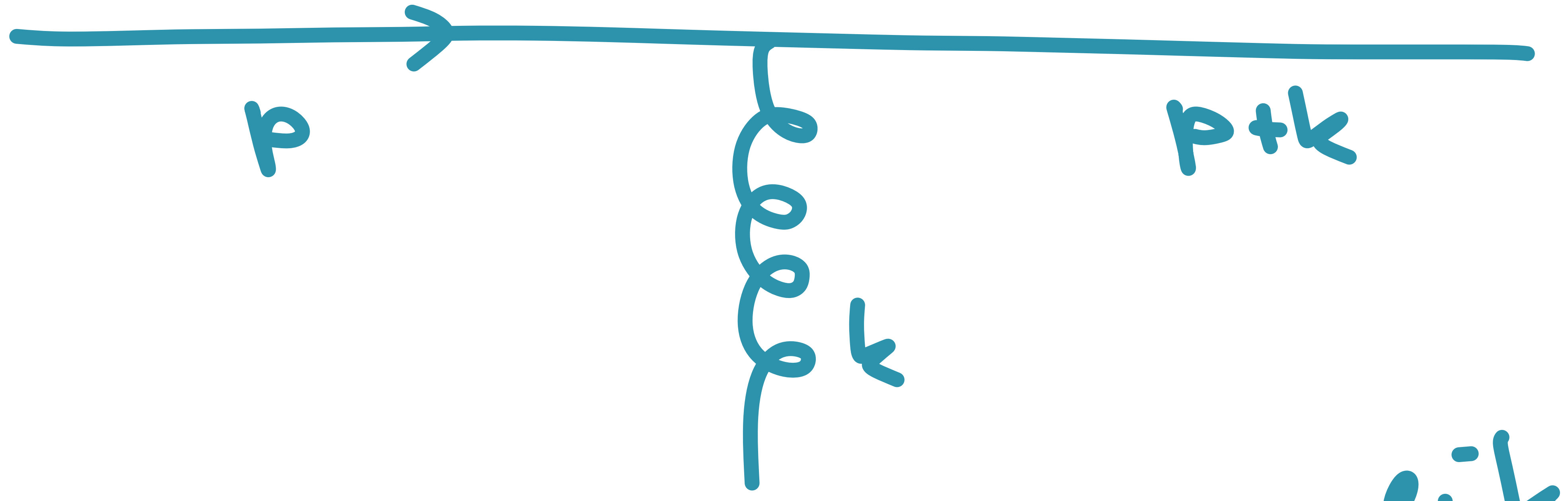


the colour dipole model

$$\sigma_{L,T}^{\gamma^* p}(x, Q^2) = \sum_f \int \mathrm{d}^2\vec{r} \int_0^1 \mathrm{d}z \, |\psi_{T,L}^{(f)}(\vec{r}, Q^2, z)|^2 \, 2 \int \mathrm{d}^2\vec{b} N(\vec{r}, \vec{b}, \tilde{x}_f(x))$$



dipole-hadron scattering

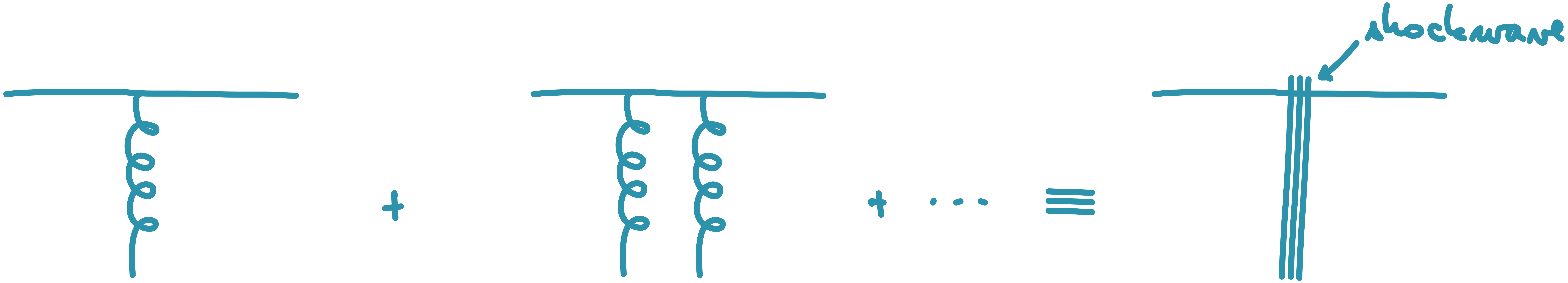


eikonal ~ $p^- \gg k^-$

$$\frac{1}{\sqrt{2p^-}}\frac{1}{\sqrt{2(p^-+k^-)}}\bar{u}(p+k)\; ig\gamma_\mu A^\mu(k)\; u(p) \;\approx igA^+(k)$$

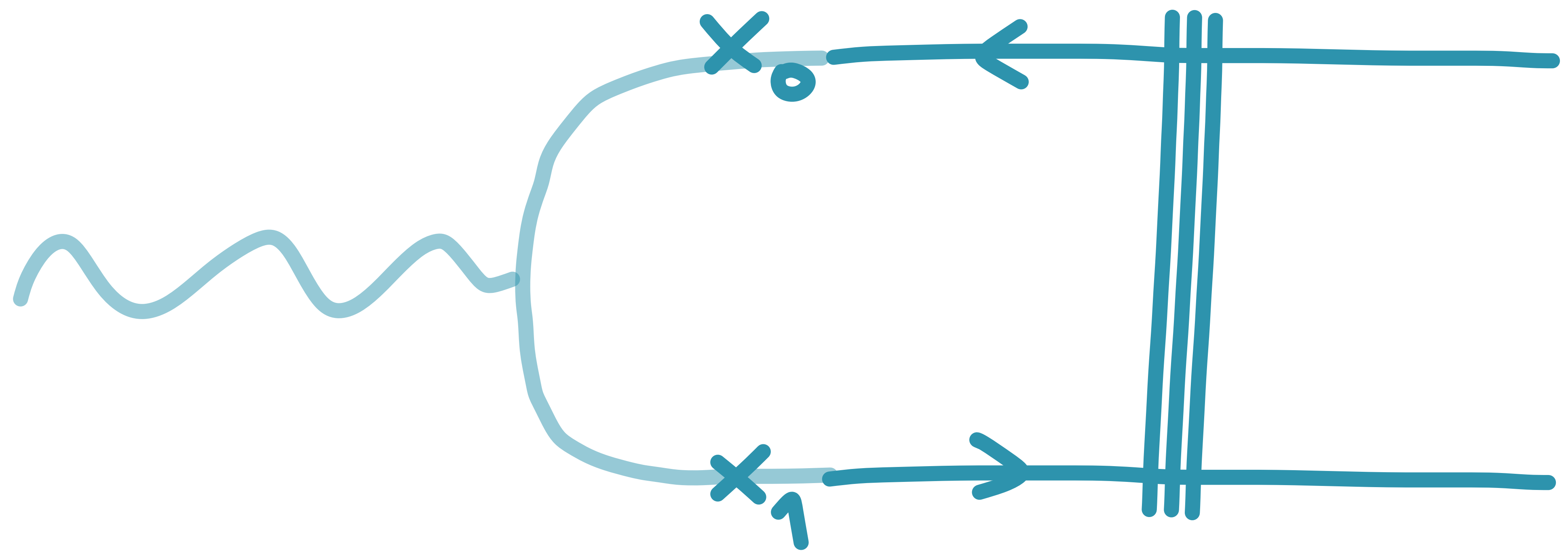
$$\rightarrow \int \mathrm{d}x^- igA^+(\vec{x})$$

dipole-hadron scattering



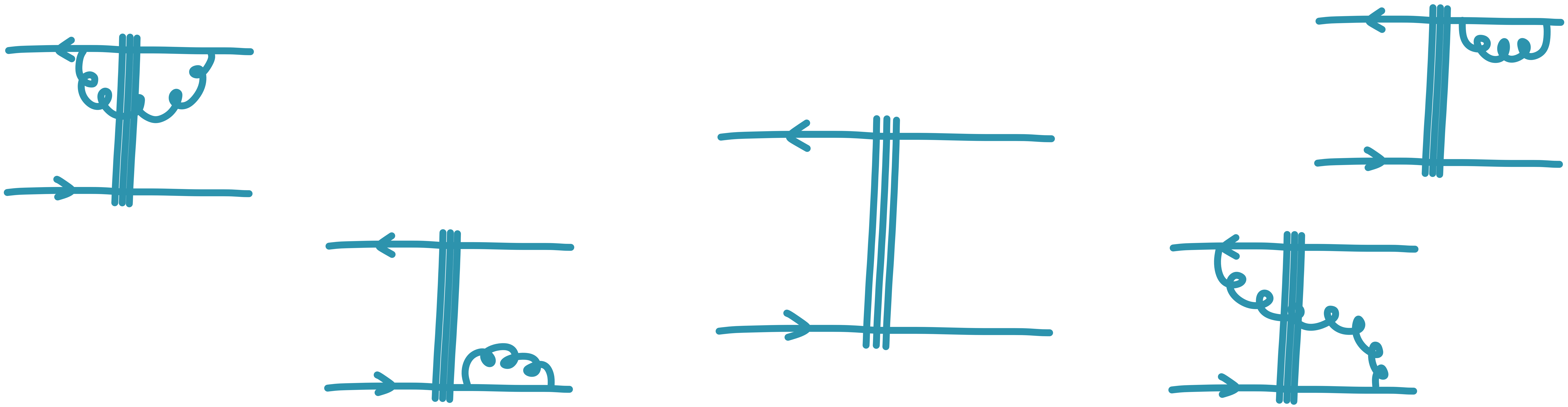
$$ig\int_{-\infty}^{+\infty} \mathrm{d}x_1^- A^+(\vec{x}_1) + (ig)^2 \int_{-\infty}^{+\infty} \mathrm{d}x_1^- \int_{x_1^-}^{+\infty} \mathrm{d}x_2^- A^+(\vec{x}_2) A^+(\vec{x}_1) \equiv \underbrace{P\exp\left\{ig\int_{-\infty}^{+\infty} \mathrm{d}x^- A^+(\vec{x})\right\}}_{V_{x_\perp}}$$

dipole-hadron scattering



$$N(\underline{x}_0, \underline{x}_1) = 1 - \left\langle \frac{1}{N_C} \mathrm{tr}\left[V_1 V_0^\dagger \right] \right\rangle$$

dipole-hadron scattering



$$\frac{\partial N(\underline{r}, \underline{b}, \eta)}{\partial \eta} = \int \mathrm{d}^2\underline{r}_1 K(\underline{r}, \underline{r}_1, \underline{r}_2) \left[N(\underline{r}_1, \underline{b}_1, \eta_1) + N(\underline{r}_2, \underline{b}_2, \eta_2) - N(\underline{r}, b, \eta) - N(\underline{r}_1, \underline{b}_1, \eta_1) N(\underline{r}_2, \underline{b}_2, \eta_2) \right]$$

$\eta = \ln \frac{x}{x_0}$

dipole-hadron scattering

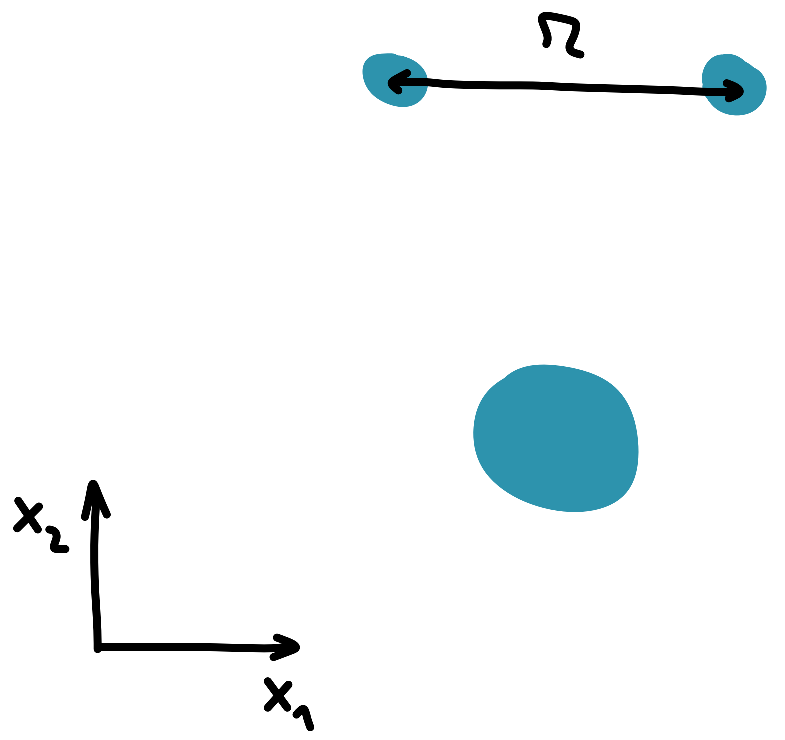
$$K_{lo} = \frac{\alpha_s N_C}{2\pi} \frac{r^2}{r_1^2 r_2^2}$$

$$K_{ci} = \frac{\bar{\alpha}_s}{2\pi} \frac{r^2}{r_1^2 r_2^2} \left[\frac{r^2}{\min\{r_1^2, r_2^2\}} \right]^{\pm \bar{\alpha}_s A_1}$$

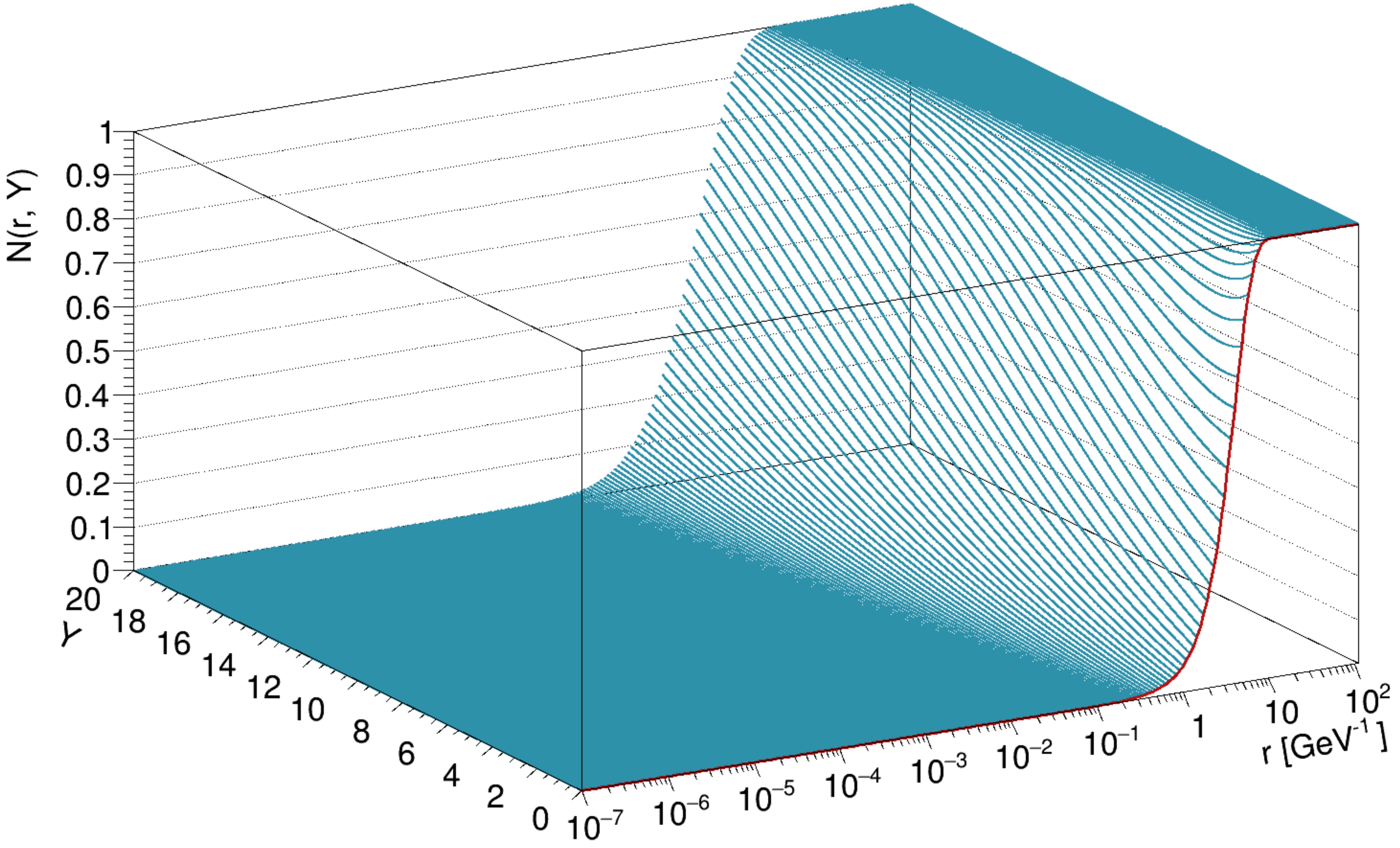
$$\frac{\partial N(\underline{r}, \underline{b}, \eta)}{\partial \eta} = \int \mathrm{d}^2 \underline{r}_1 K(\underline{r}, \underline{r}_1, \underline{r}_2) \, \Big[N(\underline{r}_1, \underline{b}_1, \eta_1) + N(\underline{r}_2, \underline{b}_2, \eta_2) - N(\underline{r}, b, \eta) - N(\underline{r}_1, \underline{b}_1, \eta_1) N(\underline{r}_2, \underline{b}_2, \eta_2) \Big]$$

numerical solutions - 1D

$$2\int \mathrm{d}\underline{b}\, N(\underline{r}, \underline{b}, Y) \approx \sigma_0\, N(r, Y)$$

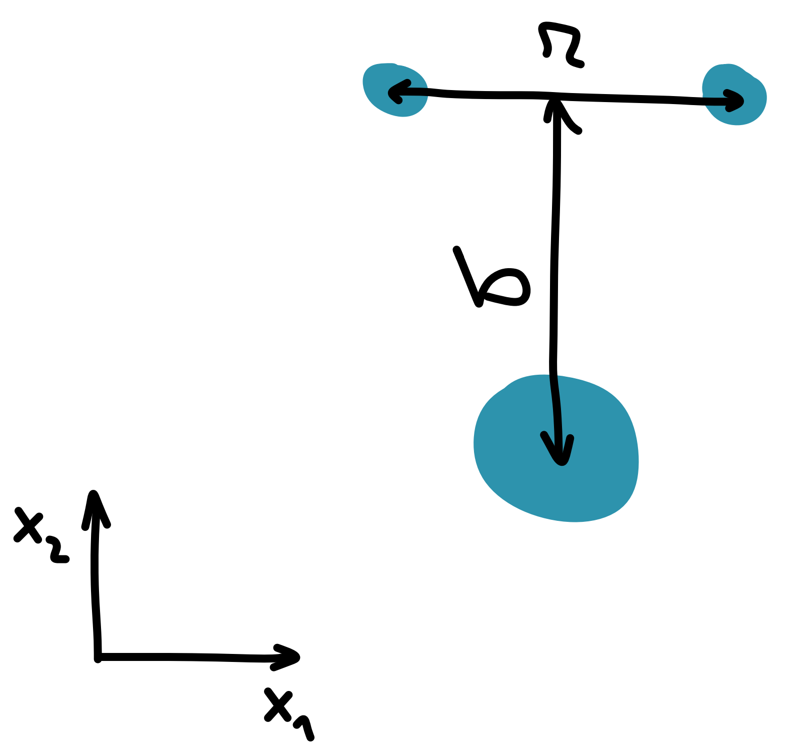


numerical solutions - 1D



numerical solutions - 2D

$$2\int \mathrm{d}\underline{b}\, N(\underline{r}, \underline{b}, Y) \approx 4\pi \int \mathrm{d}b\, N(r, b, Y)$$



numerical solutions - 2D

detour: Y vs η

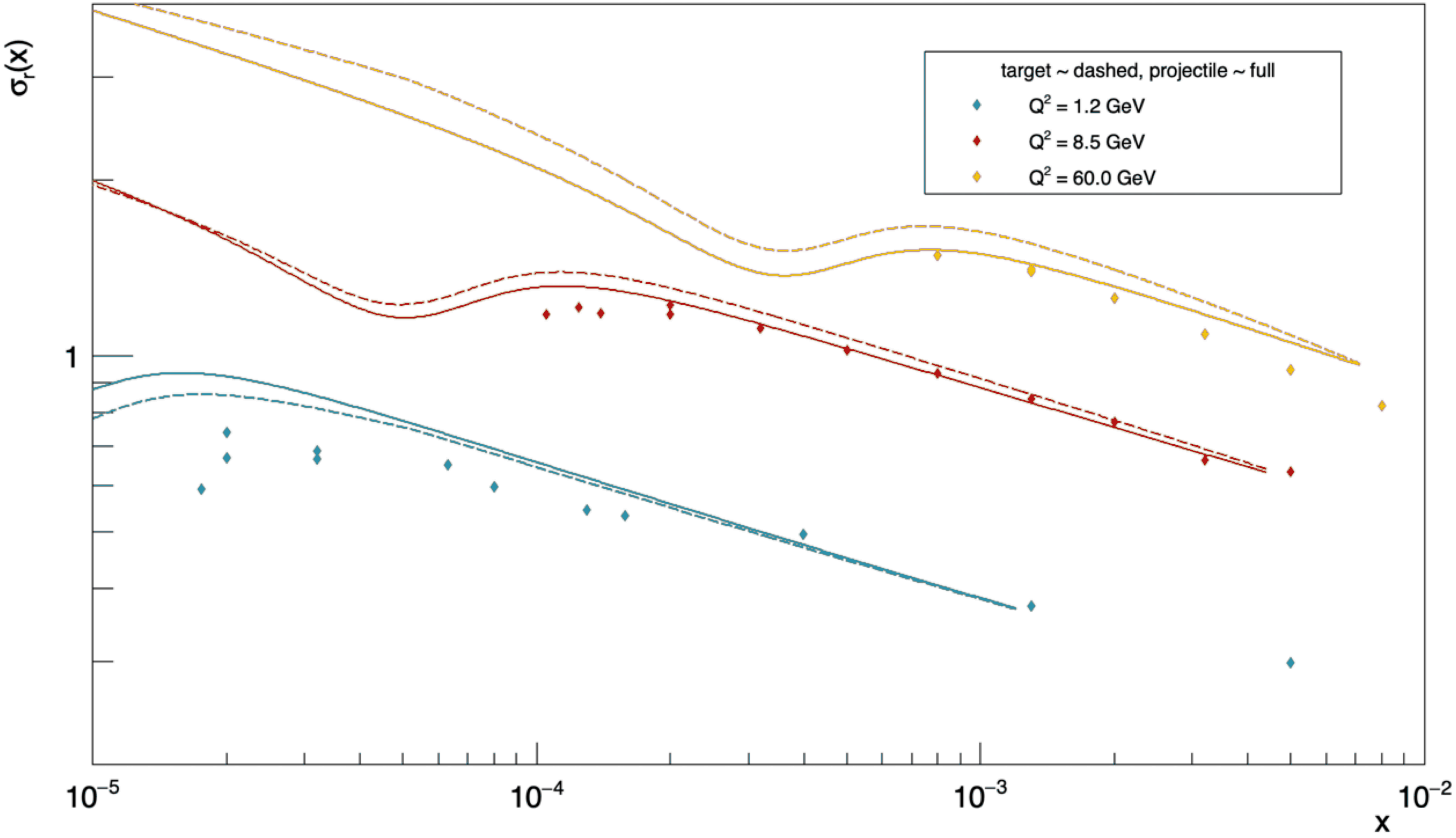
$Y = \eta + \ln \frac{Q^2}{Q_s^2}$

$$\frac{\partial N(\underline{r}, \underline{b}, Y)}{\partial Y} = \int \mathrm{d}^2 \underline{r}_1 K(\underline{r}, \underline{r}_1, \underline{r}_2) \left[N(\underline{r}_1, \underline{b}_1, Y) + N(\underline{r}_2, \underline{b}_2, Y) - N(\underline{r}, b, Y) - N(\underline{r}_1, \underline{b}_1, Y) N(\underline{r}_2, \underline{b}_2, Y) \right]$$

$$\frac{\partial N(\underline{r}, \underline{b}, \eta)}{\partial \eta} = \int \mathrm{d}^2 \underline{r}_1 K(\underline{r}, \underline{r}_1, \underline{r}_2) \left[N(\underline{r}_1, \underline{b}_1, \underline{\eta_1}) + N(\underline{r}_2, \underline{b}_2, \underline{\eta_2}) - N(\underline{r}, b, \eta) - N(\underline{r}_1, \underline{b}_1, \underline{\eta_1}) N(\underline{r}_2, \underline{b}_2, \underline{\eta_2}) \right]$$

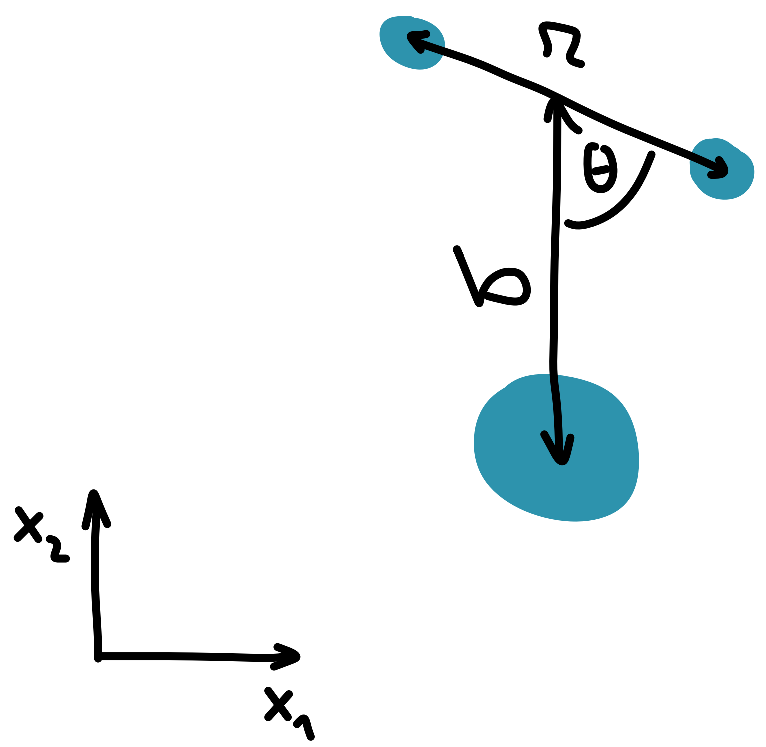
$\eta_j = \eta - \max\left\{0, 2 \ln \frac{r}{r_j}\right\}$

numerical solutions - 2D

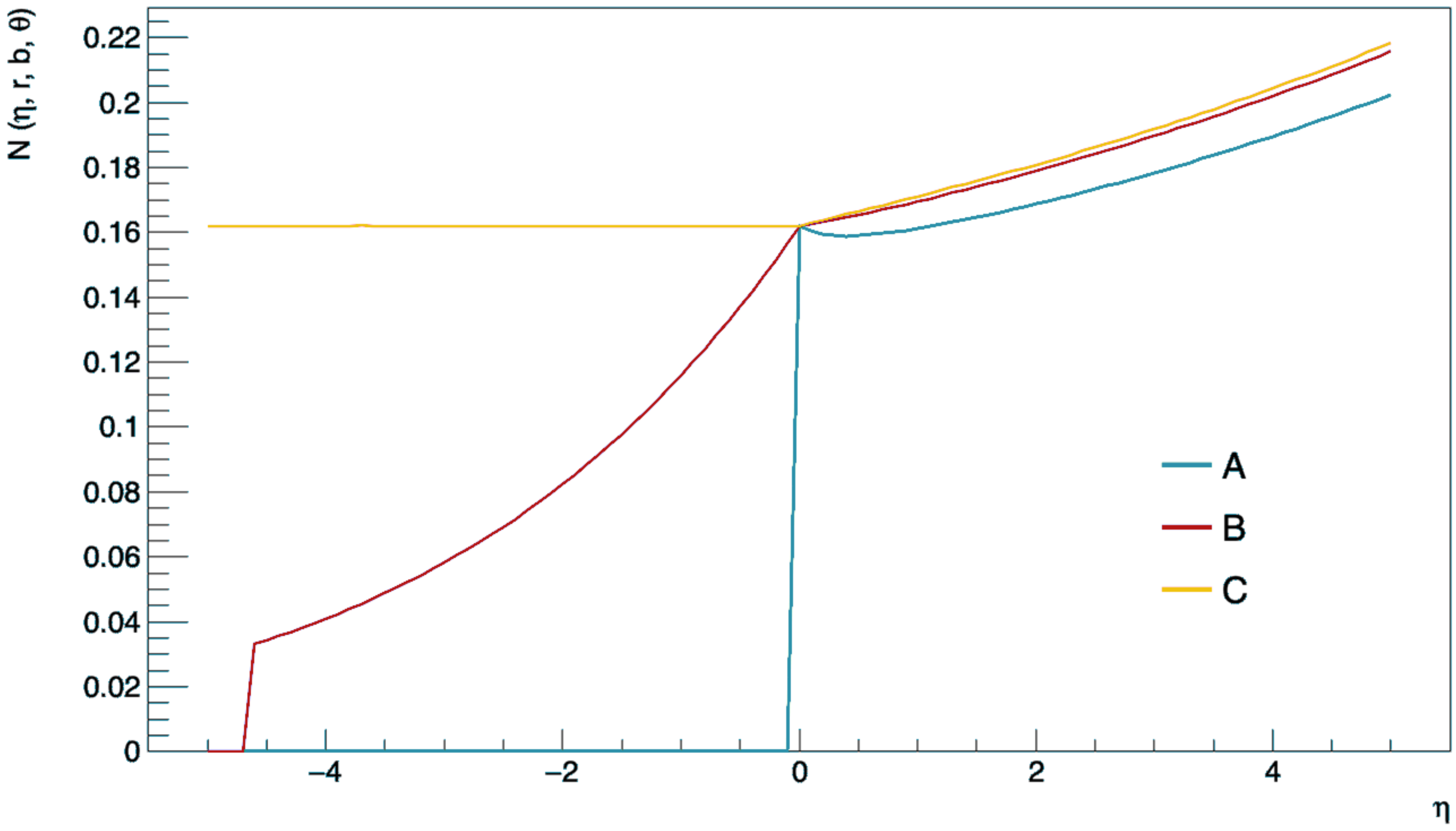


numerical solutions - 3D

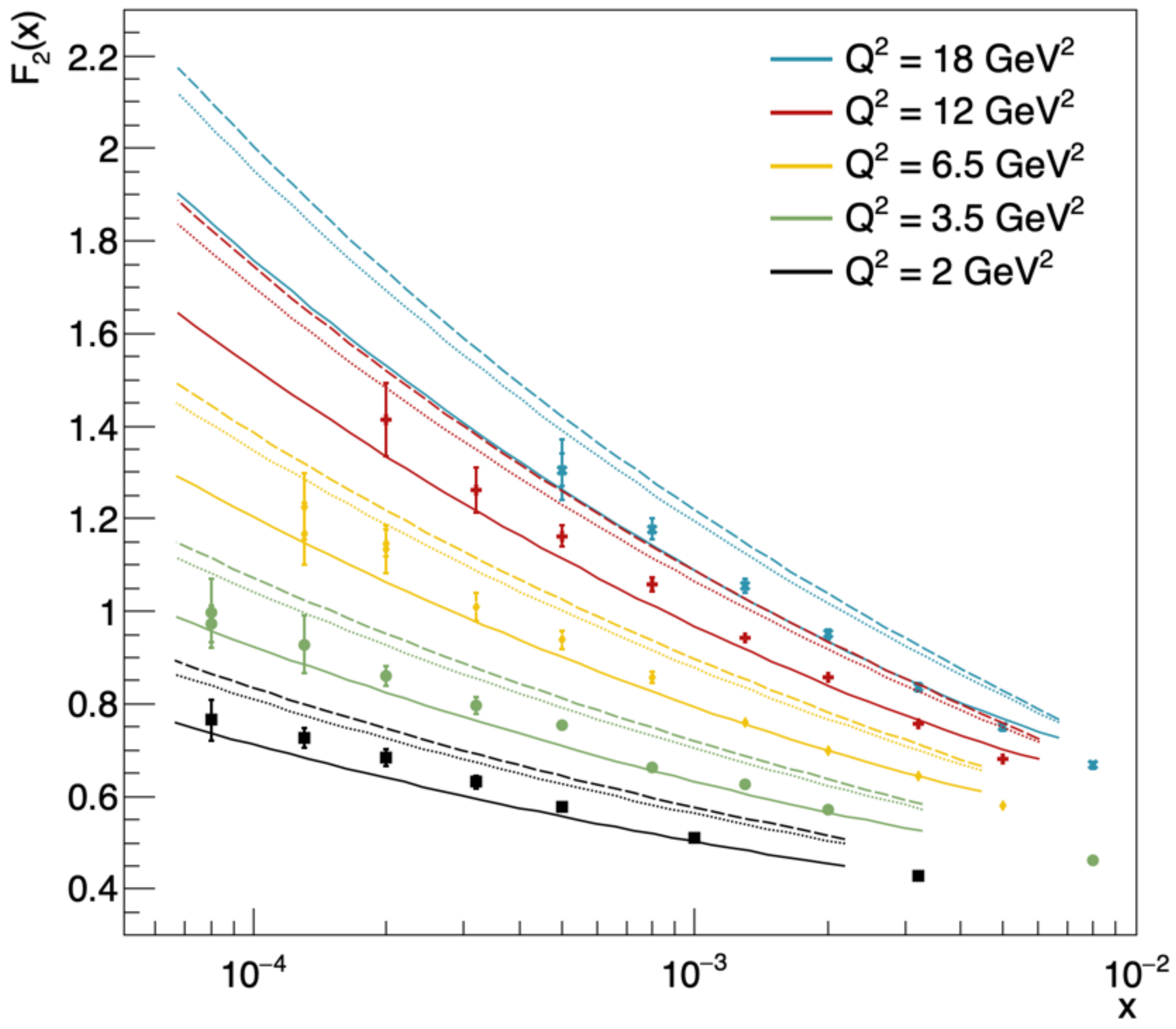
$$2\int \mathrm{d}\underline{b}N(\underline{r},\underline{b},\eta) \approx 2\int \mathrm{d}bN(r,b,\theta,\eta)$$

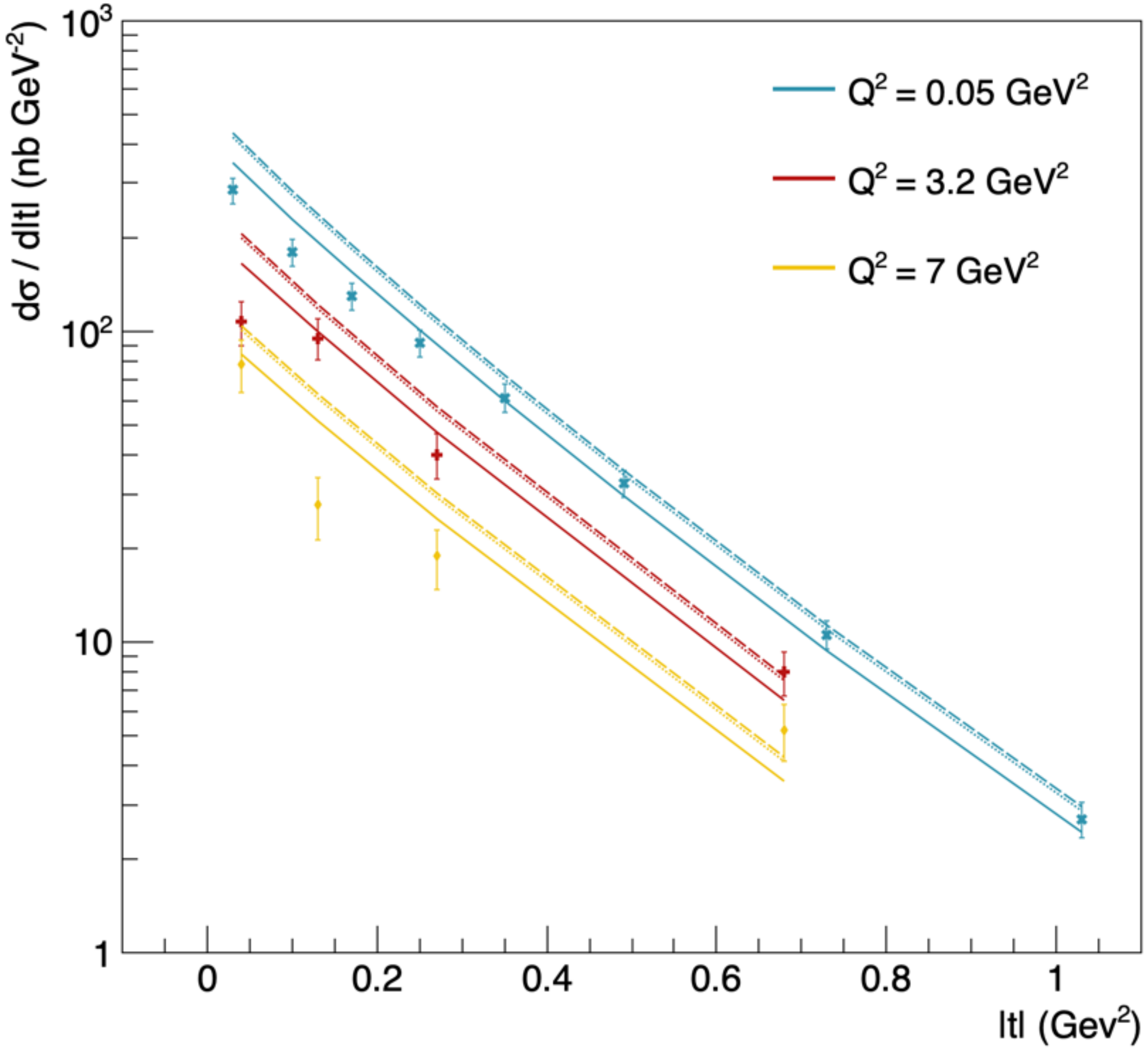


numerical solutions - 3D



numerical solutions - 3D





further steps

- 4D solution
- observables sensitive to 3D and 4D
    - dijet production
- nuclear target calculations
- higher order corrections

thank you

backup

$$N(r,b,\theta,x) = 1 - e^{-(\frac{x_0}{x})^{\lambda} Q_{s0}^2 \frac{r^2}{4} T(b,r)\left(1 + c\cos(2\theta)\right)}$$

$$N(r,b,\theta) = \left(1 - e^{-Q_{s0}^2 \frac{r^2}{4}}\right) T(b,r) \left(1 + (1 - e^{-\left(\frac{rb}{2B}\right)^2})\cos(2\theta)\right)$$

$$N(r,b,\theta) = 1 - e^{-Q_{s0}^2 \frac{r^2}{4} T(b,r)\left(1 + \frac{c}{2}\cos(2\theta)\right)}$$

$$N(r,b,\theta) = 1 - e^{-Q_{s0}^2 \frac{r^2}{4}\left[\ln\left(\frac{1}{r^2 m^2} + e\right) + \frac{b^2}{6m^2R^4}\cos(2\theta)\right] T(b,r)}$$

$$N(r,b,\theta) = 1 - e^{-Q_{s0}^2 \frac{r^2}{4} T(b,r)\left(1 + c\cos(2\theta)\right)}$$

$$T(r,b) = e^{-\frac{b^2 + (r/2)^2}{2B}}$$