Can DBSCAN Be Improved by Robust Preprocessing?

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Overview

1. Data whitening
2. Geometric median
3. Pursuit method
4. DBSCAN
5. Results

Data whitening

- data normalization
- $\mathrm{E}X = \mathbf{0}$ and $\mathrm{var}X = \mathbf{I}$

$$\mathbf{W} = \mathbf{X}_{\mathrm{S}}^{\mathrm{T}} \left(\frac{\mathbf{u}_{(1)}}{\lambda_{(1)}}, \frac{\mathbf{u}_{(2)}}{\lambda_{(2)}}, \dots \frac{\mathbf{u}_{(D)}}{\lambda_{(D)}} \right) \in \mathbf{R}^{n \times D}.$$

Geometric median

1. statistically robust alternative to center of mass
2. $\mathbf{y} \in \mathbb{R}^n$,where

$$\sum_{i=1}^{m} ||\mathbf{x}_i - \mathbf{y}||_2 = \min .$$

3. Weiszfeld algorithm [4]

Pursuit method

1. data decorrelation
2. optimal linear combination of base vectors
3. robust variance

$$||\mathbf{w}_1^\star|| = \min,$$

$$\mathrm{var}^\star(\mathbf{z}) = 1.$$

$$\frac{||\mathbf{w}_1^\star||^2}{\mathrm{var}^\star(\mathbf{z})} = \min_{\mathbf{w}_1^\star \neq \mathbf{0}},$$

Robust Standard Deviation Estimates

- S_n [1]

$$S_n = 1.1926 \cdot \operatorname*{lomed}_{i=1,\ldots,n} \cdot \operatorname*{himed}_{j=1,\ldots,n} |x_i - x_j|$$

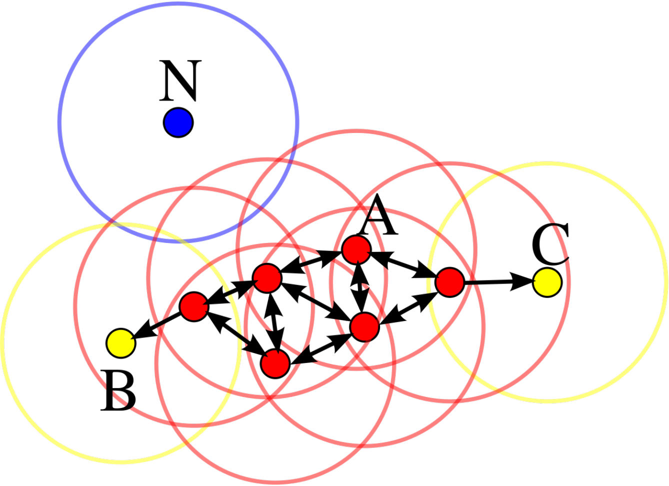
- Q_n [1]

$$Q_n = 2.2219 \cdot \{|x_i - x_j|; i < j\}_{(\lfloor \frac{n}{4} \rceil)}$$

- MAD_n [3]

$$MAD_n = 1.4826 \cdot \mathrm{med}_i |x_i - \mathrm{med}_j x_j|$$

DBSCAN



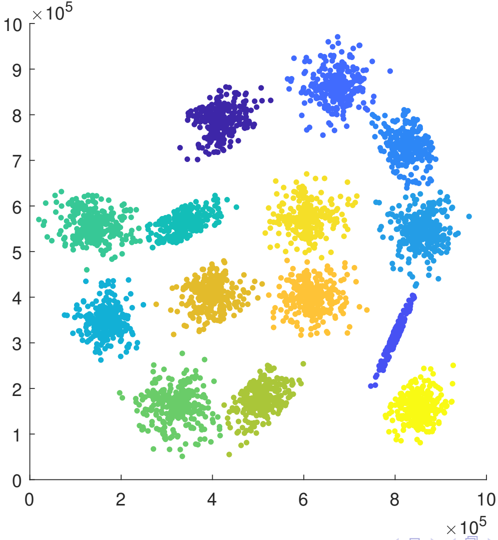
1

[1]By Chire - Own work, CC BY-SA 3.0,
https://commons.wikimedia.org/w/index.php?curid=17045963

- minpts - minimal number of neighbours
- ε - maximal distance to neighbours
- core points
- border points
- noise points (outliers)

Results

Dataset [2]



Non-robust approach

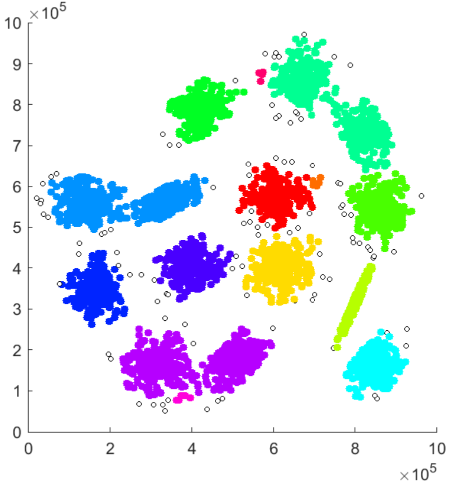


Figure: minpts = 4, $\varepsilon = 0.95$, 2.2 % outliers

Robust approach

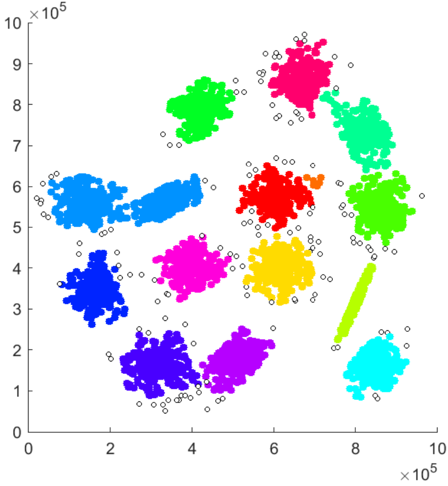


Figure: minpts = 5, $\varepsilon = 0.8$, 1.8 % outliers

References

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Computational Statistics, Vol. 1, 1:411–428, 01 1992.

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P. J. Rousseeuw and C. Croux.

Alternatives to the median absolute deviation.

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On the point for which the sum of the distances to n given points is minimum.

Annals of Operations Research, 167(1):7–41, Mar 2009.