Heterogeneous Variants of Balanced Particle Systems

Jiří Nábělek

Supervisor: Doc. Mgr. Milan Krbálek, Ph.D.

Faculty of Nuclear Sciences and Physical Engineering

Czech Technical University in Prague

Contents

1 Introduction

2 Motivational Task

3 Quasi-Homogeneous BPS

4 Heterogeneous BPS with Periodic Generators

5 Conclusion

Introduction

Role of Traffic System Analysis

Objectives:

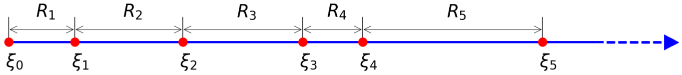
1. Describe and quantify traffic flow.
2. Obtain information about the capacity of a given road.

Applications:

1. In the design of new highways and intersections.
2. Identifying problematic sections of the road.

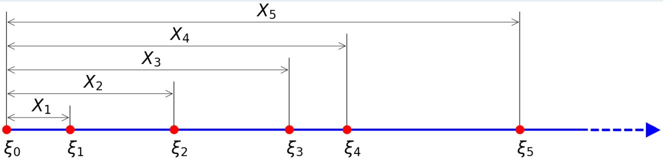
Description Using Gaps

Distance between individual agents (vehicles) $\xi_0, \xi_1, \xi_2, \ldots$.



Description Using Multi-Gaps

Distance of agents from the reference particle ξ_0.

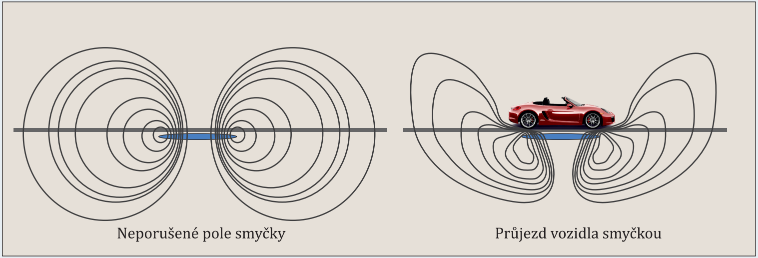


Description Using Interval Frequencies

Number of agents in an interval of fixed length L.

Measurement Methods

Double-loop induction detectors



Doc. Mgr. Milan Krbálek, Ph.D. Passage of a vehicle through a double-loop detector. In: M. Krbálek, J. Vacková, Mathematical Modeling of Traffic, Czech Technical University Publishing, Prague 2022

Macroscopic Characteristics

1. Traffic density
2. Traffic flow rate
3. Average speed of traffic flow

Standard Data Processing Methods

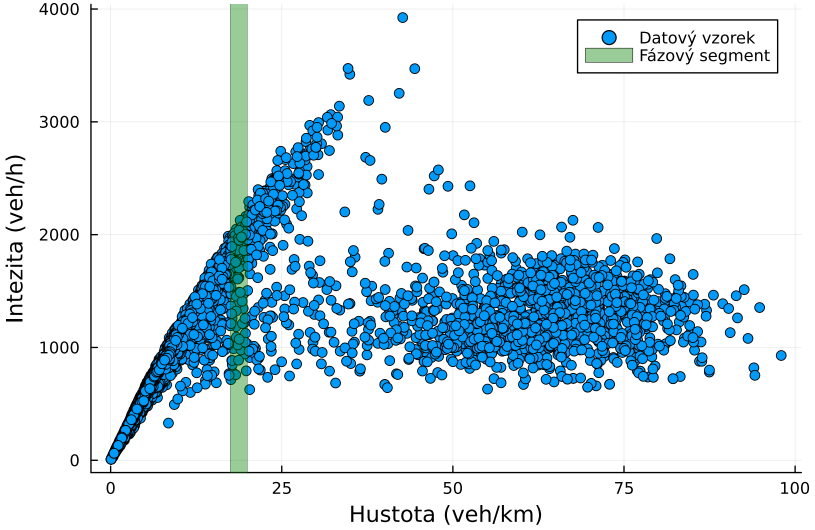
Data Sample

Variables:

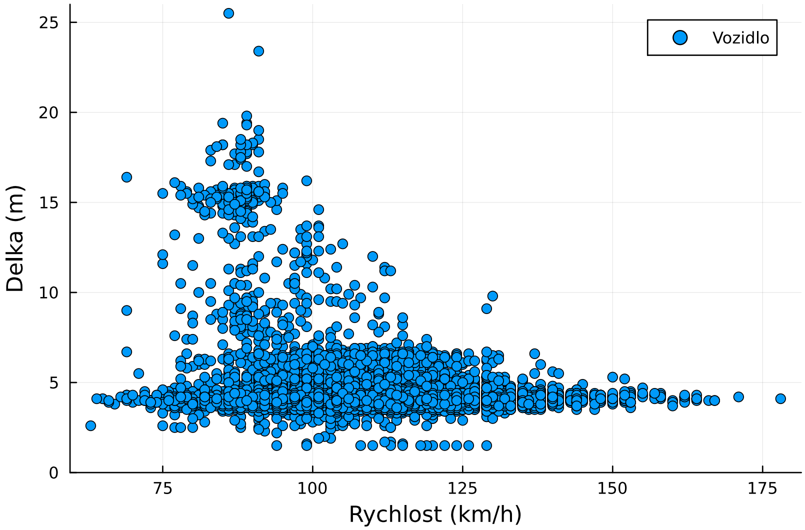
1. Vehicle length $[m]$
2. Passage time $[s]$
3. Average speed in section $[km/h]$
4. Lane
5. Spatial gaps $[m]$
6. Time gaps $[s]$

length	time	speed	lane	spatial gaps	time gaps
3.3	47.7	88	1	642.0	26.4
3.7	74.1	88	2	382.5	15.8
3.5	89.9	98	1	739.7	27.3
3.7	117.2	119	2	135.1	4.2
3.1	121.4	106	2	1198.2	40.8
4.1	162.2	118	1	422.0	13.0
3.2	175.2	144	1	200.8	5.1
3.4	180.3	99	1	403.6	14.8
3.6	195.1	115	1	676.8	21.3
3.9	216.4	121	1	3851.3	114.7
3.5	331.1	82	2	30.7	1.5
3.8	332.6	83	2	408.9	17.9
3.9	350.5	115	1	98.3	3.2
3.8	353.7	108	1	530.2	17.8
3.9	371.5	110	1	992.2	32.6
3.2	404.1	84	2	90.1	4.0
3.2	408.1	110	1	1396.2	45.8
3.6	453.9	115	1	478.8	15.1
3.5	469.0	90	1	211.5	8.6
3.5	477.6	137	1	3752.6	98.7
⋮	⋮	⋮	⋮	⋮	⋮

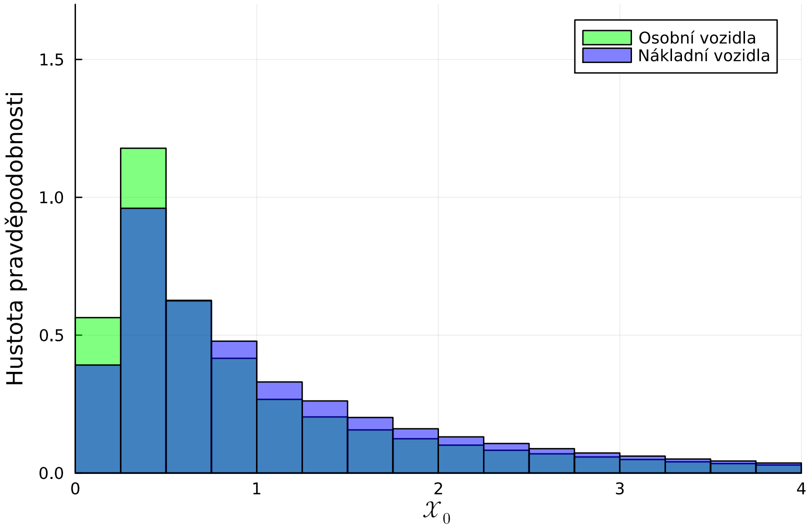
Dependence of Flow Rate on Traffic Density



Vehicle Lengths in Selected Data Sample



Probability Density of Gaps



Definition of Quasi-Homogeneous BPS

Quasi-Homogeneous BPS

A quasi-homogeneous balanced particle system is understood as a sequence of gaps $(\mathscr{R}_k)_{k=0}^{\infty}$, where $\mathscr{R}_0, \mathscr{R}_1, \mathscr{R}_2, \ldots$ is any sequence of non-negative, absolutely continuous, and independent random variables, whereas $\mathscr{R}_N, \mathscr{R}_{N+1}, \mathscr{R}_{N+2}, \ldots$, where $N \in \mathbb{N}_0$, are identically distributed random variables with the probability density function $h(x) \in \mathscr{B}$. The probability density functions of the random variables $\mathscr{R}_0, \mathscr{R}_1, \ldots, \mathscr{R}_{N-1}$ are denoted as $h_0(x), h_1(x), \ldots, h_{N-1}(x) \in \mathscr{B}$ and the probability density functions $h(x)$ and $h_0(x), h_1(x), \ldots, h_{N-1}(x)$ will be called generators of the quasi-homogeneous balanced particle system.

First-Order Statistics

Definitions for Quasi-Homogeneous BPS

1. Trend function: $\omega(L) = \mathbb{E}(\mathscr{N}_L)$,
2. Cluster function: $r(x) = \sum_{k=0}^{+\infty} g_k(x)$,
3. Laplace transform: $\mathbb{H}(s) := \mathscr{L}[h(x)]$, $\mathbb{R}(s) := \mathscr{L}[r(x)]$.

Selected Theorems for First-Order Statistics

1. $$\mathbb{R}(s) = \sum_{k=0}^{N-1} \prod_{i=0}^{k} \mathbb{H}_i(s) + \prod_{i=0}^{N-1} \mathbb{H}_i(s) \frac{\mathbb{H}(s)}{1 - \mathbb{H}(s)},$$
2. $$\mathscr{L}[\omega(x)](s) = \frac{1}{s} \sum_{k=0}^{N-1} \prod_{i=0}^{k} \mathbb{H}_i(s) + \frac{1}{s} \prod_{i=0}^{N-1} \mathbb{H}_i(s) \frac{\mathbb{H}(s)}{1 - \mathbb{H}(s)}.$$

Example for $N = 1$

Consider that $h_0(x)$ has an Exponential and $h(x)$ an Erlang distribution, then:

$$g_k(x) = \left(\frac{n+1}{n}\right)^{k(n+1)} \mathrm{e}^{-(n+1)x} \sum_{j=k(n+1)}^{+\infty} \frac{(nx)^j}{j!}.$$

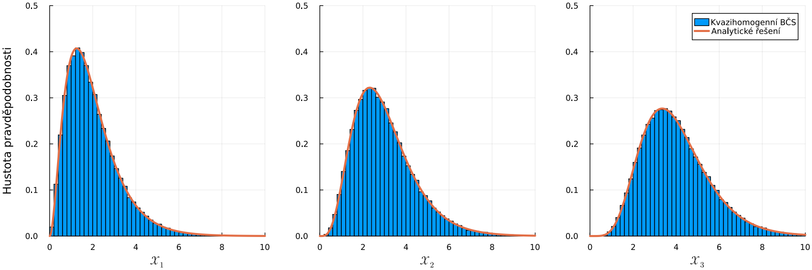
First-Order Statistics for $n = 1$

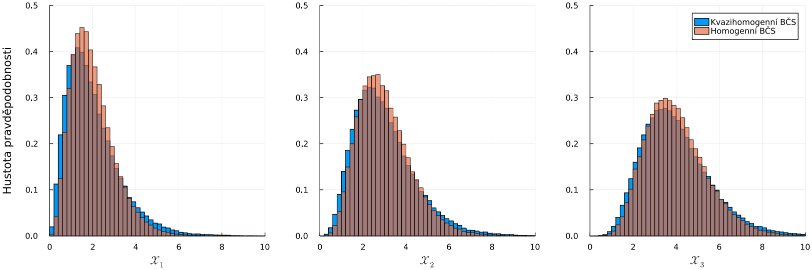
1. $\omega(x) = x - \frac{\mathrm{e}^{-4x}}{12} + \frac{\mathrm{e}^{-x}}{3} - \frac{1}{4}$,
2. $r(x) = \frac{\mathrm{e}^{-4x}}{3} - \frac{\mathrm{e}^{-x}}{3} + 1$.

Second-Order Statistics

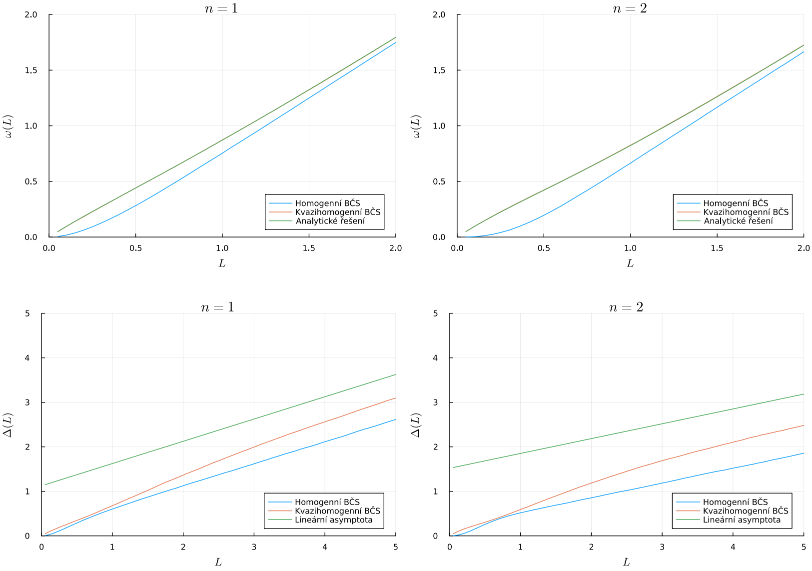
1. $\chi = B^{'}(0) = \frac{1}{n+1}$,
2. $\delta = \frac{1}{2} B^{''}(0) = \frac{1}{6} \frac{n(14n+13)}{(n+1)^2}$.

Probability Density of Gaps





Selected First- and Second-Order Statistics



Definition of Heterogeneous BPS

Heterogeneous BPS with Periodic Generators

A heterogeneous balanced particle system with periodic generators is understood as a sequence of gaps $(\mathscr{R}_k)_{k=0}^{\infty}$, where $\mathscr{R}_0, \mathscr{R}_1, \mathscr{R}_2, \ldots$ is any sequence of non-negative, absolutely continuous, and independent random variables, whereas:

$$\forall i \in \{0, 1, \ldots N-1\} : \mathscr{R}_i, \mathscr{R}_{N+i}, \mathscr{R}_{2N+i}, \ldots \sim h_i(x),$$

where $N \in \mathbb{N}_0$ and $\forall i \in \{0, 1, \ldots N-1\}$, $\mathscr{R}_i, \mathscr{R}_{N+i}, \mathscr{R}_{2N+i}, \ldots$ are identically distributed random variables with the probability density function $h_i(x) \in \mathscr{B}$. The probability density functions $h_0(x), h_1(x), \ldots, h_{N-1}(x)$ will be called generators of the heterogeneous balanced particle system with periodic generators.

Example for $N = 2$

Consider that $h_0(x)$ has a Poisson and $h_1(x)$ an Erlang distribution. Further, let in front of particle k:

1. k_0 particles have generator $h_0(x)$,
2. k_1 particles have generator $h_1(x)$.

The probability density function of multi-gaps:

$$g_k(x) = \frac{(n+1)^{(k_1+1)(n+1)}}{(k_1+n+k_1 n)!\,k_0!}\mathrm{e}^{-x}\cdot \sum_{i=0}^{k_0} \binom{k_0}{i} \frac{x^{k_0-i}(-1)^i}{n^{(k_1+1)(n+1)+i}} \gamma(nk_1+n+k_1+i+1, nx).$$

Conclusion

Main contributions:

- Formal definition of special variants of heterogeneous BPS.
- Analytical derivation of description and statistics for first- and second-order characteristics for specific cases of special variants of heterogeneous BPS.
- Application of the developed theory and properties of special variants of heterogeneous BPS in traffic data processing.

Thank you for your attention.