Parameter Estimation in Cyclic Plastic Loading

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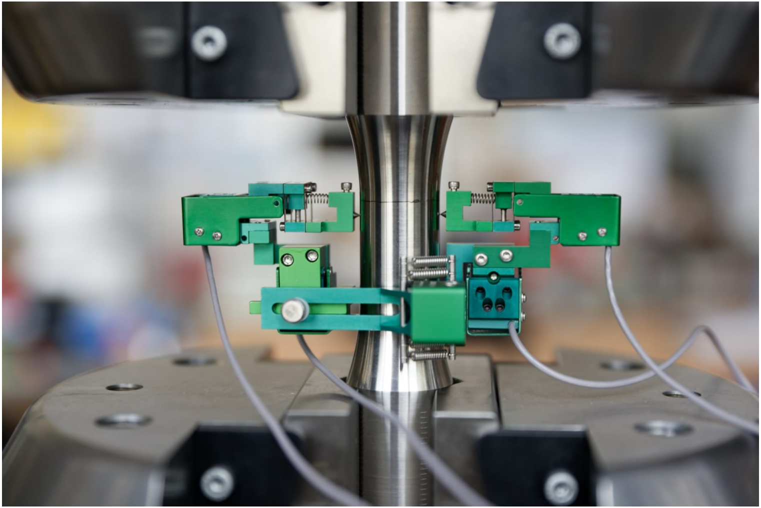
Content

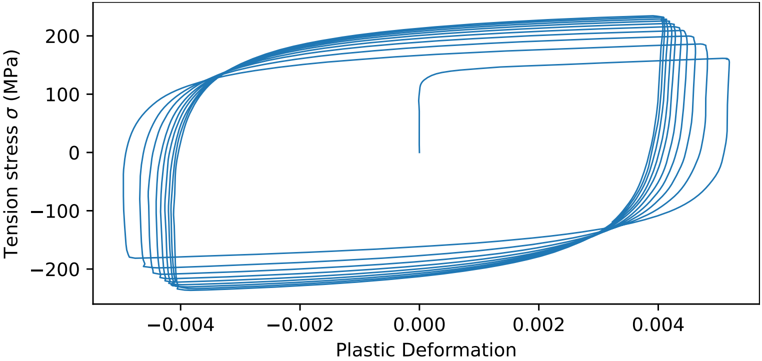
Cyclic Loading Measurement



The axial-torsional extensometer Epsilon Tech 3550.

Cyclic Loading and the Bauschinger Effect

- Bauschinger effect
- Permanent changes in the metallic specimen under cyclic loading



Denoised measured data from cyclic plastic loading.

Hardening Model MAFTr

- Developed by Marek et al. in 2022 [1]
- Needs to be calibrated on cyclic loading experimental data.

Parameter	**Unit**	**Description**
k_0	MPa	Initial yield strength
κ_1	MPa	Adjustment of the rate of isotropic hardening
κ_2	MPa^{-1}	Inverted asymptotic limit of isotropic hardening
c_i	-	Adjustment of the evolution rates of the backstress components
a_i	MPa	Asymptotic limits of the backstress components

Parameters of analytical model developed by Marek et al.

[1] R. Marek et al. "A quick calibration tool for cyclic plasticity using analytical solution". In: Engineering Mechanics 27/28 (May 2022), pp. 249 –252. DOI: 10.21495/512249.

Synthetic datasets for Plastic Deformation

- $\mathbf{D}_1$ dataset
  - consists of pairs $(\boldsymbol{\theta}_i, S(\boldsymbol{\theta}_i))$, where $\boldsymbol{\theta}_i$ is generated from the a priori distribution.
  - Plastic deformation setup ϵ_{p} remains constant across the dataset.
  - Dataset length is $L = 10^6$.

- Extended $\mathbf{D}_2$ Dataset
  - consists of triplets $(\boldsymbol{\theta}_i, S(\boldsymbol{\theta}_i), \epsilon_{\mathrm{p}})$.
  - Dataset length is $L = 10^6$.

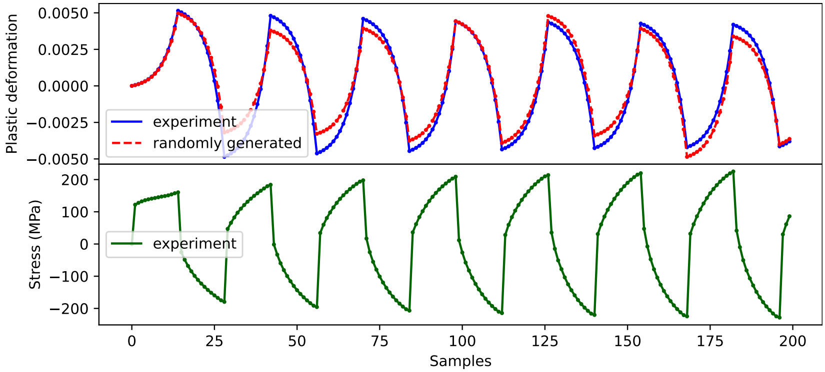
A Priori Distribution for parameters

- A uniform distribution is selected for all 11 parameters.
- Conditions:
  - Sum of a_i parameters in the range of $[150, 350]$.
  - Ordering of c_i parameters to ensure uniqueness of the training objective.

	k_0	κ_1	κ_2^{-1}	$\log(c_1)$	$\log(c_{2,3,4})$	$a_{1,2,3,4}$
min	15	100	30	$\log(1000)$	$\log(50)$	0
max	250	10000	150	$\log(10000)$	$\log(2000)$	350

Range of the a priori uniform distribution for each parameter.

Data Downsampling Strategy



Top: Interpolated plastic deformation covering each load segment by 15 points.
Bottom: Interpolated stress from the measured data.

Approaches

- Neural Networks
  - Feed Forward Networks
  - LSTM [1]
  - GRU [2]
- Non-neural methods
  - TTOpt [3]
- Refinement using the Nelder-Mead method

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[1] Sepp Hochreiter and Jürgen Schmidhuber. "Long Short-term Memory". In: Neural computation 9 (Dec. 1997), pp. 1735–80. DOI: 10.1162/neco.1997.9.8.1735.

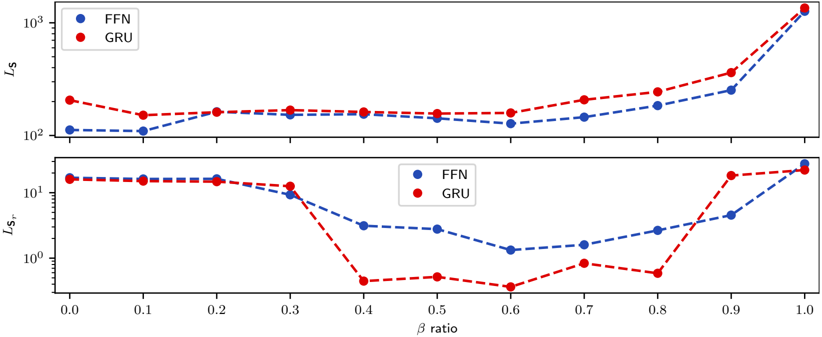
[2] Kyunghyun Cho et al. Learning Phrase Representations using RNN Encoder-Decoder for Statistical Machine Translation. 2014. arXiv: 1406.1078 [cs.CL].

[3] Konstantin Sozykin et al. TTOpt: A Maximum Volume Quantized Tensor Train-based Optimization and its Application to Reinforcement Learning. arXiv:2205.00293 [cs, math].

Novel loss function

- Calculating $\widehat{\mathbf{S}} := M_{\widehat{\boldsymbol{\theta}}}(\epsilon_{\mathrm{p}})$ from NN-predicted $\widehat{\boldsymbol{\theta}}$, where $M_{\boldsymbol{\theta}}$ represents the MAFTr model.

$$L(\widehat{\boldsymbol{\theta}}_{\mathcal{N}}, \boldsymbol{\theta}_{\mathcal{N}}, \widehat{\mathbf{S}}, \mathbf{S}) := k\|\widehat{\boldsymbol{\theta}}-\boldsymbol{\theta}\|_2^2+\alpha(1-k)\|\widehat{\mathbf{S}}-\mathbf{S}\|_2^2,\ k \in [0,1],\ \alpha \in \mathbb{R}_+.$$



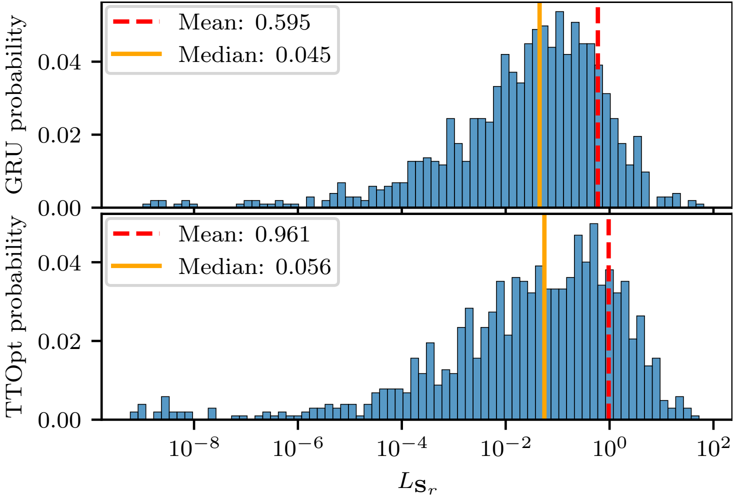
Comparison of $L_{\boldsymbol{\theta}}$ and $L_{\mathbf{S}}$ metrics for FFN and GRU trained with $L(\widehat{\boldsymbol{\theta}}_{\mathcal{N}}, \boldsymbol{\theta}_{\mathcal{N}}, \widehat{\mathbf{S}}, \mathbf{S})$ for different k values.

Numerical results on synthetic data

architecture	dataset	$L_{\mathbf{S}}$	$L_{\mathbf{S}}^{(r)}$
GRU	$\mathbf{D}_1^T$	111.64	**0.137**
LSTM	$\mathbf{D}_1^T$	149.91	0.510
FFN	$\mathbf{D}_1^T$	**106.90**	2.314
GRU	$\mathbf{D}_2^T$	242.43	**0.595**
LSTM	$\mathbf{D}_2^T$	528.03	7.523
FFN	$\mathbf{D}_2^T$	350.02	7.058
TTOpt	$\mathbf{D}_2^T$	**63.30**	0.961

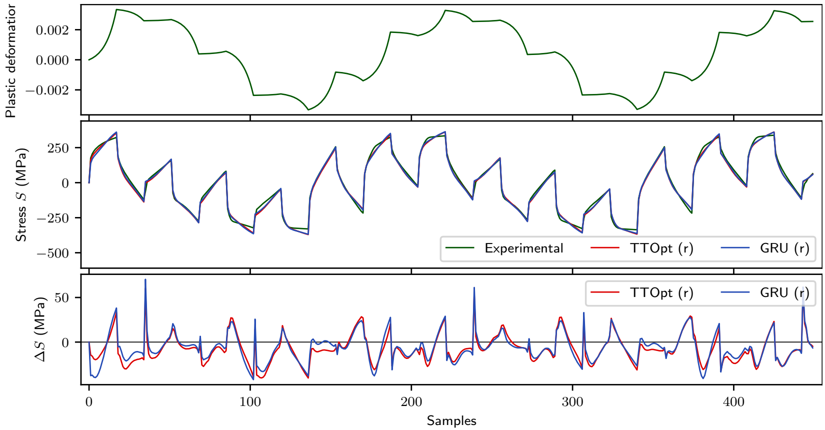
Metrics and their refined values of selected GRU and FFN networks compared to TTOpt on test datasets $\boldsymbol{D}_1^T$ and $\boldsymbol{D}_2^T$.

Prediction analysis



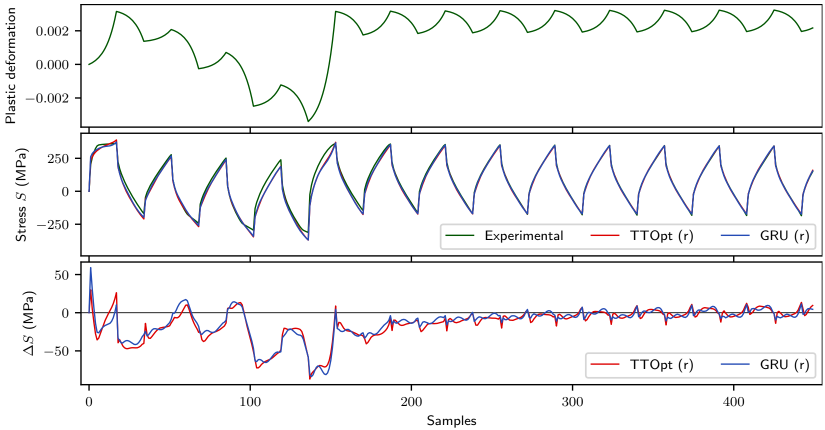
Histogram of refined predictions of both GRU and TTOpt on test dataset $\boldsymbol{D}^T$.

Validation 1



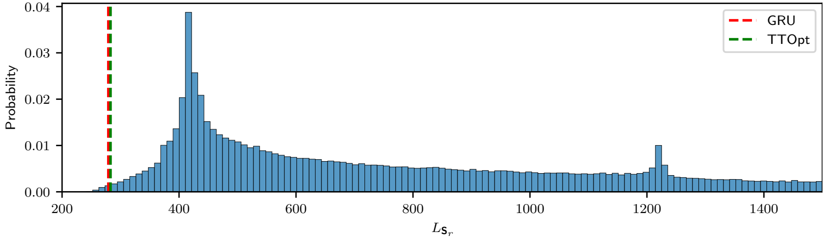
Top: Plastic deformation in the measured experiment #1. Middle: Predicted stress using the refined estimated parameters $\boldsymbol{\theta}$ of both GRU and TTOpt. Bottom: Stress prediction error.

Validation 2



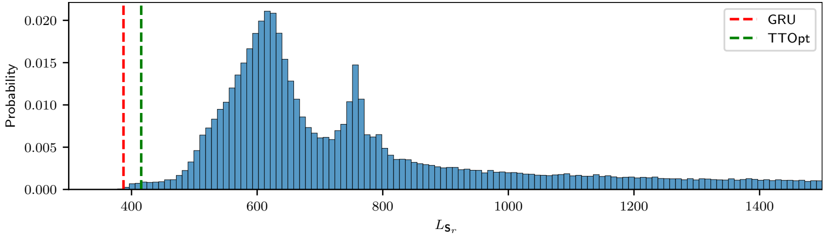
Top: Plastic deformation in the measured experiment #2. Middle: Predicted stress using the refined estimated parameters $\boldsymbol{\theta}$ of both GRU and TTOpt. Bottom: Stress prediction error.

Validation 1 - Nelder Mead



Histogram of randomly generated $\boldsymbol{\theta}$ using the a priori distribution after its refinement using the Nelder-Mead simplex optimization on experiment #1. For comparison, the refined predictions of GRU and TTOpt are depicted by dashed lines.

Validation 2 - Nelder Mead



Histogram of randomly generated $\boldsymbol{\theta}$ using the a priori distribution after its refinement using the Nelder-Mead simplex optimization on experiment #2. For comparison, the refined predictions of GRU and TTOpt are depicted by dashed lines.

Conclusion

- The novel loss function combining L_θ and $L_\mathbf{S}$ enhances the training effectiveness.
- FFNs and CNNs appeared to be unstable on $\mathbf{D}_2$ dataset.
- Both GRU and TTOpt exhibit comparable performance on both synthetic and real-world data.
- The entire approach is not model specific and can be easily used with other material models.

Bibliography

[1] R. Marek et al. "A quick calibration tool for cyclic plasticity using analytical solution". In: *Engineering Mechanics* 27/28 (May 2022), pp. 249 –252. DOI: 10.21495/512249.

[2] Konstantin Sozykin et al. *TTOpt: A Maximum Volume Quantized Tensor Train-based Optimization and its Application to Reinforcement Learning*. arXiv:2205.00293 [cs, math]. Sept. 2022. URL: http://arxiv.org/abs/2205.00293 (visited on 09/21/2023).

[3] Kyunghyun Cho et al. *Learning Phrase Representations using RNN Encoder-Decoder for Statistical Machine Translation*. 2014. arXiv: 1406.1078 [cs.CL].

[4] Sepp Hochreiter and Jürgen Schmidhuber. "Long Short-term Memory". In: *Neural computation* 9 (Dec. 1997), pp. 1735–80. DOI: 10.1162/neco.1997.9.8.1735.

[5] Ilya Loshchilov and Frank Hutter. *Decoupled Weight Decay Regularization*. 2019. arXiv: 1711.05101 [cs.LG].

[6] John A. Nelder and Roger Mead. "A simplex method for function minimization". In: *Computer Journal* 7 (1965), pp. 308–313.

Data Downsampling Strategy

- Stress response recorded at 10Hz for 4 hours.
- Needs to preserve points of reversals.
- Each segment is downsampled to 15 points.

$$\epsilon_i^{(j)} := \epsilon_{i-1}^{(\mathrm{r})} + \sum_{k=1}^{j} \delta_k, \quad \forall i \in \{1, \dots, K\},\ \forall j \in \{1, \dots, N-1\},$$

- The increments follow a geometrical sequence:

$$\delta_{k+1} = \sqrt[N-1]{R}\,\delta_k,\ \forall k \in \{1, \dots, N-1\}, \quad R = 20.$$