Acoustic Emission, Time Reversal Signal Processing and ϕ-divergences

Classification of signals based on ϕ-divergences

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Acoustic Emission

- localized source → rapid release of energy → transient elastic waves
- AE application:
  - Structure Health Monitoring (SHM) & Non-Destructive Testing (NDT)
  - various industries - aerospace, automotive, civil engineering, etc.
- AE monitoring: detection → localization →source identification - signal classification
- AE signals classification
  - signal parameters
  - classification methods - supervised & unsupervised

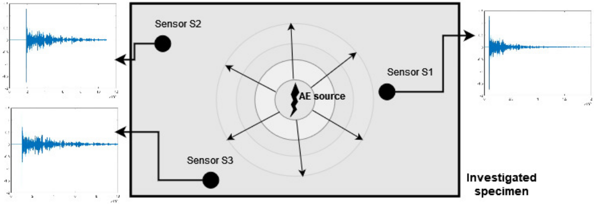
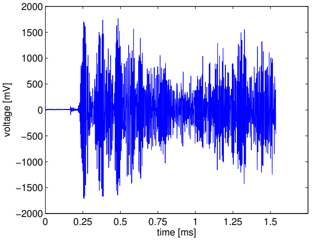
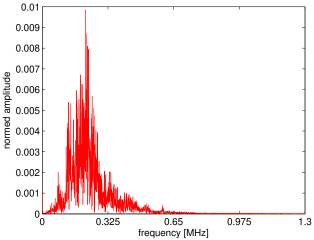


Figure: *Scheme for measuring AE using three acoustic sensors.*

Signal Parameters





Normalized spectrum of the signal

$$\widetilde{S}(f) = \frac{|X_f|}{\sum_{f=0}^{F_s/2} |X_f|}, \qquad f = 0, \dots, F_s/2\,.$$

Parameter W_α:

$$W_\alpha = \underset{l \in [0, F_s/2]}{\operatorname{argmin}} \sum_{f=0}^{F_s/2} |l - f| \left| \widetilde{S}(f) - \frac{1}{T} \right|^\alpha, \qquad \alpha \in \mathbb{N}$$

Parameter Q_β:

$$Q_\beta = \min\{F \in [0, F_s/2] : \sum_{f=0}^{F_s/2} \widetilde{S}(f) \geq \beta\}, \qquad \beta \in (0,1).$$

Parameter Z_c:

$$Z_c = \sum_{t=\widetilde{t}}^{T} \delta(x(t))\,, \qquad \widetilde{t} = \min J\,,$$

where

$$J = \{j \in [1, T] : x(j) \geq c \cdot \max_{t \in [1,T]} |x(t)|\}, \qquad c \in (0,1).$$

ϕ-divergences as a parameter

Definition of ϕ-divergence

For generating divergence function $\phi : (0, \infty) \to \mathbb{R}$, convex on $(0, \infty)$ and strictly convex at $t = 1$ with $\phi(1) = 0$ we define ϕ-*divergence* of P and Q by the relation

$$D_\phi(P, Q) = \int_{\mathcal{X}} q \, \phi \left(\frac{p}{q} \right) \mathrm{d}\mu \,, \qquad P,\, Q \in \mathcal{P}(\mathcal{X}, \mathcal{A}) \,.$$

ϕ-divergence based signal attributes

$$D_\phi(\widetilde{S}, \widetilde{S}^{ref}(f)) = \sum_{f=0}^{F_s/2} \widetilde{S}^{ref(f)} \phi \left(\frac{\widetilde{S}(f)}{\widetilde{S}^{ref(f)}} \right) , \qquad \widetilde{S}^{ref}(f) = \frac{1}{\nu} \sum_{a=1}^{\nu} \widetilde{S}_a(f), \; f = 0, \ldots, F_s/2 \,,$$

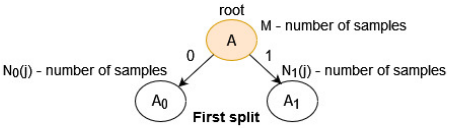
Divergence Decsion Tree (DDT)

Classification of data set $\{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$, $\mathbf{x}_i \in \mathcal{X} \subset \mathbb{R}^d$, $i \in \hat{n}$, $d \in \mathbb{N}$,

The goal of DDT

maximisation of **criterion function**

$$\sum_{internal\,node\,j} \frac{N_0(j)}{M} D_\phi(P_0(j), P(j)) + \frac{N_1(j)}{M} D_\phi(P_1(j), P(j))$$



$P_0(j)$,$P_1(j)$,$P(j)$ - empirical distributions

The resulting DDT algorithm

1. perform the *Principal Component Analysis* (PCA) $\rightarrow$ selection of attributes,
2. combinations of PCA attributes $\rightarrow$ k-means $\rightarrow$ division into two clusters,
3. maximal value of the criterion function over the subset of all the divisions into two clusters $\rightarrow$ selection the resulting variant of k-means clustering
4. the node with the largest contribution to the criterion function $\rightarrow$ next splitting

Experiments - AE classification

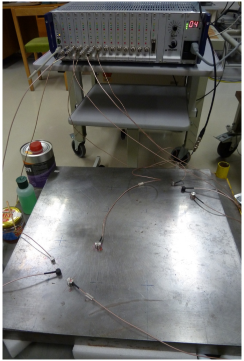


Figure: *Acoustic emission experimental setup.*

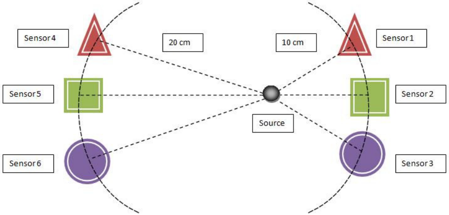


Figure: *Scheme of AE experiment.*

Experiments - AE classification

Importance of parameters vs. importance of method

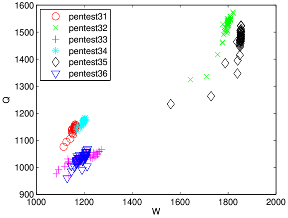


Figure: *Feature space W vs.* $Q_{0.33}$.

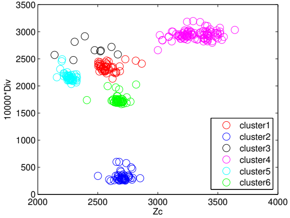


Figure: *Fuzzy classification.*

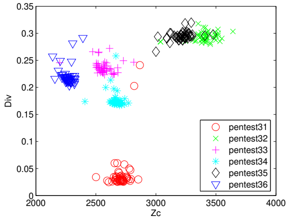


Figure: *Feature space Zc vs. Div.*

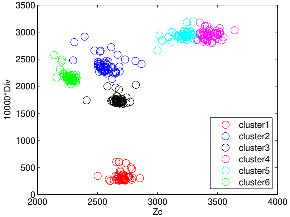


Figure: *DDT classification.*

Time Reversal Technique - Basic Principle

- a point source $s(t)$ at a position r_0 and a receiver at a position r_i
- measured signal

$$s_G(t) = s(t) * G(r_i, r_0; t), \quad \text{for } t \in [0, T]$$

- time reversation of measured signal

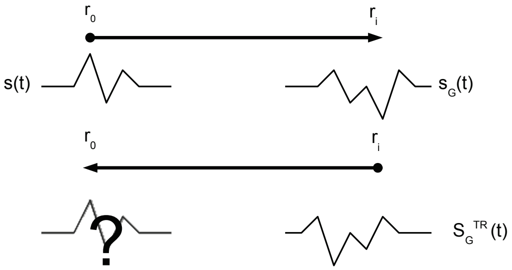
$$s(T-t) * G(r_i, r_0; T-t),$$

- at the position r_0

$$\tilde{s}(t) = s(T-t) * G(r_i, r_0; T-t) * G(r_0, r_i; t),$$

- after calculation

$$\tilde{\mathbf{s}}(\mathbf{t}) = \frac{1}{16\pi \|r_i - r_0\|^2} s(t) = \mathbf{a} \cdot \mathbf{s}(\mathbf{t})$$



Classification using TR signals

The experiment with bubbles classification

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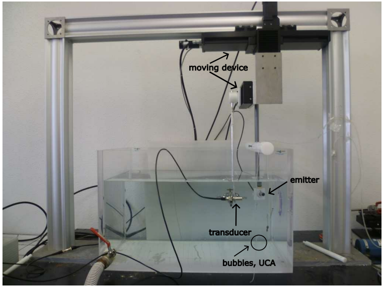
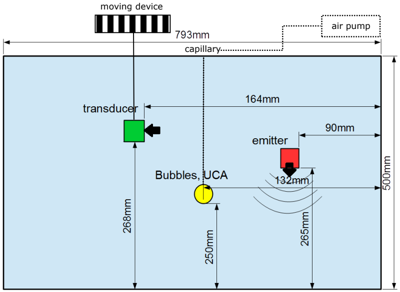




(a) The experimental water tank (b) Scheme of the experimental water tank

Figure: *The experimental water tank with the emitter, the transducer (the receiver), and scatterers (the frontal view). The directions of signal emitting (from the emitter) and signal receiving (by the transducer) are indicated by arrows.*

Classification + TR

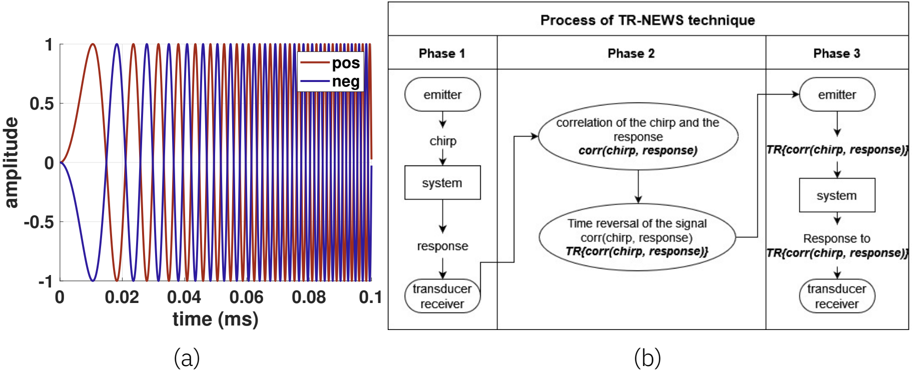
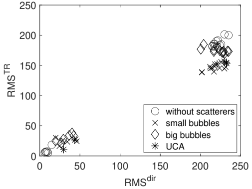


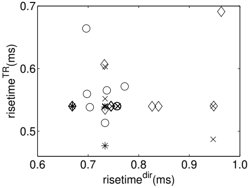
Figure: *Chirp excitations; with initial amplitudes increasing to positive values (pos) and with initial amplitudes decreasing to negative values (neg). Diagram of the TR NEWS process used for signal processing optimization using correlation.*

Classification + TR

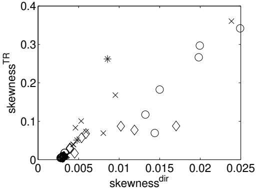
The separation of different scatterers (CDS experiment)



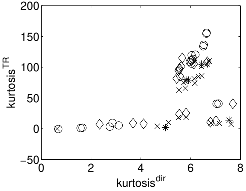
(a) RMS



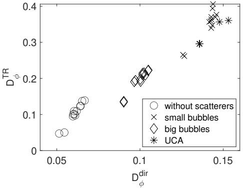
(b) Risetime



(c) Skewness



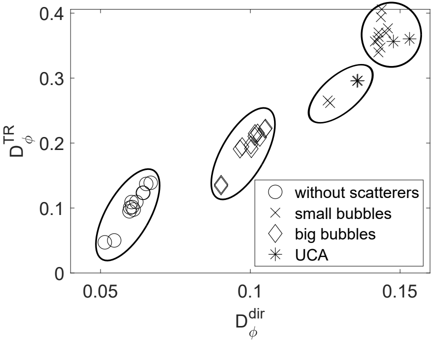
(d) Kurtosis



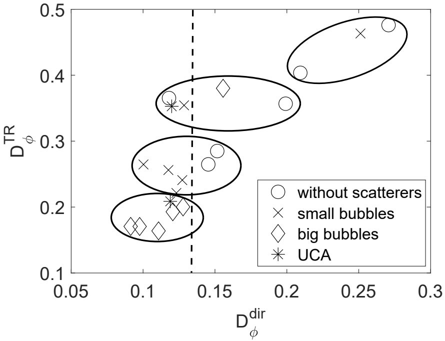
(e) $D_\phi = H^S_{1/2}$

Figure: *Signal parameters: (a) RMS, (b) Risetime, (c) skewness, (d) kurtosis (e) ϕ-divergence.*

Classification + TR



(a) Fuzzy classification for pos + neg chirps.

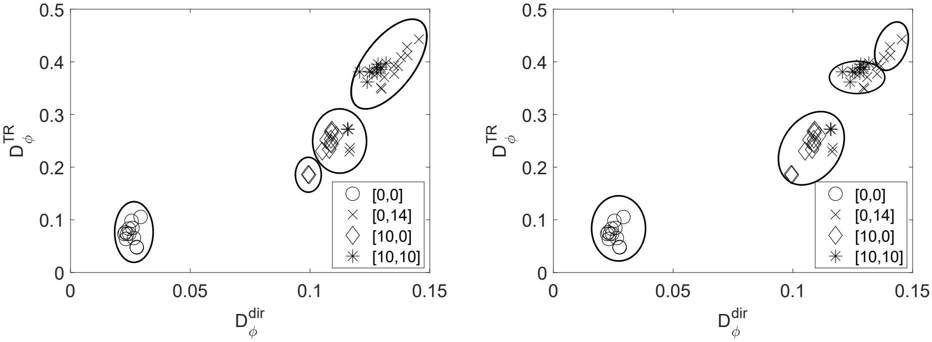


(b) Fuzzy classification for PI signals solely.

Figure: *Fuzzy classification of different scatterers (CDS) by means of* D_ϕ^{TR} *versus* D_ϕ^{dir} *attributes (resulting clusters are indicated by ellipses): (a) for positive + negative chirps, (b) PI signals solely.*

Classification + TR

Different positions of one scatterer (CDP experiment)



(a) Fuzzy classification (b) DDT classification

Figure: *Classification of different positions (CDP) of small bubbles by means of D_ϕ^{TR} versus D_ϕ^{dir} for positive + negative chirps (resulting clusters indicated).*

Echodentography experiment

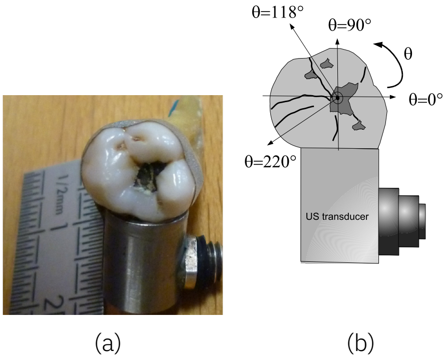


Figure: *Scheme of the human tooth with attached emitter.*

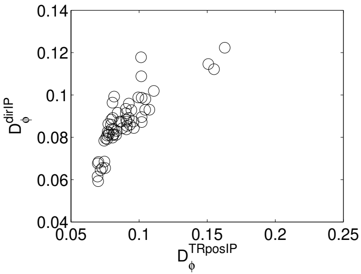
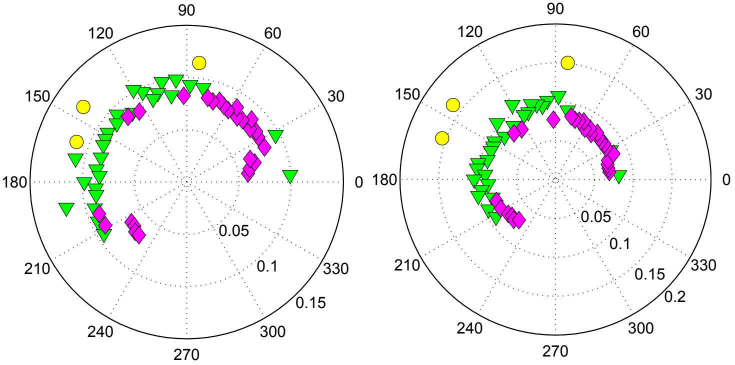


Figure: *ϕ-divergence attributes D_ϕ^{dirIP} and $D_\phi^{TRposIP}$ for the 'tooth' measured data*

Echodentography experiment

Fuzzy classification on the divergence parameters D_{ϕ}^{dirIP} and $D_{\phi}^{TRposIP}$



(a) The ϕ divergence parameter D_{ϕ}^{dirIP} (b) The ϕ divergence parameter $D_{\phi}^{TRposIP}$

Conclusions

- ϕ-divergence between normalized spectra can be successfully used as signal parameter
- ϕ-divergence is used also as a part of a classification method
- time reversal technique enables to minimize the influence of the medium through which the signal passes on signal classification
- our parameters and method was successfully applied on several experiments

Thank you for your attention.

Many thanks to my supervisors and GAMS community.