Introduction to generalized parton distributions

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8. Miniworkshop difrakce a ultraperiferních srážek

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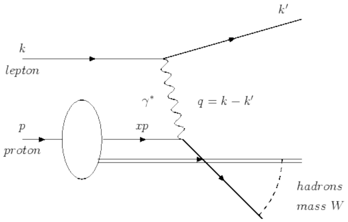
Outline

Introduction

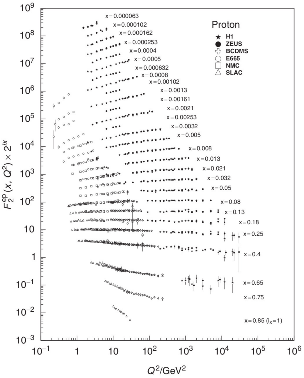
- What we want to know: partonic picture of nucleons in terms of quarks and gluons : joint position and momentum space information about constituents of a nucleon
- In classical physics, this corresponds to the phase space of a many-body system with fixed number of species
- But nucleon is a quantum mechanical object!
- Problem no. 1 - operator of the number of particles in quantum system do not commute with the Hamiltonian. So info about the energy of solo nucleon means no info about nucleon composition.
- Problem no. 2 - uncertainty principle means if we know position of a constituent, we cannot get info about the momentum of the same constituent and vice versa

- Solution of Feynman/Bjorken - what we can address is the composition during the interaction, since added momentum transfer Q^2 disrupts the coherence
- In parton model, a simplistic approach was taken (correspondence between DIS and parton view)
- $F_2(x, Q^2)$ from DIS in the Bjorken limit $Q^2 \to \infty$, $\nu \to \infty$, $x = \frac{Q^2}{2M\nu}$ finite
- $\sum\limits_{partons} Z_p^2 f_p(x)$ in infinite momentum frame (fun fact - equal to light-front frame)



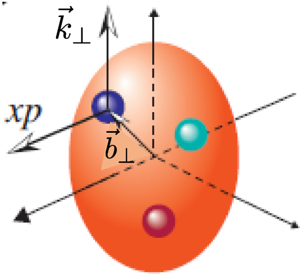
Parton distributions

- $f_p(x)$ is a parton distribution (well, only for valence quarks)
- The only variable there is x (from DIS correspondence comes that it is a fraction of parton 4-momenta wrt the nucleon 4-momenta)
- Later on, it was confirmed that the correspondence holds even outside the Bjorken limit (gluon exchange)
- Second variable Q^2 is the incoming energy into the interaction (or equivalently Lorentz boost)
- Good news - universality of the distribution, bad news - not invariant to Lorentz boost, defined only in momentum phase space



Generalized parton distributions

- Very well, but we have a very simple distribution valid in a very specific frame
- Can we get something more general?
- What are the kinematic variables in momentum and position phase space?
- $k^+ = xp^+$ is a longitudinal momentum of a parton
- $\vec{b}_T$ is a distance from the center of nucleon in transverse plane to the motion
- $\vec{k}_T$ is a transverse momentum of the parton



PDF : $f(x)$

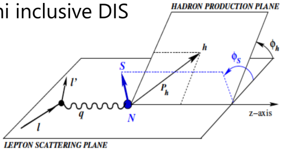
x : momentum fraction wrt P

$\int d^2\mathbf{k}_\perp$

Transverse momentum dependent PDF (TMD) :

$f(x, \mathbf{k}_\perp)$

Semi inclusive DIS

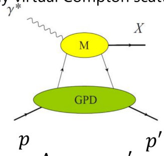


$\Delta_\perp \to 0$

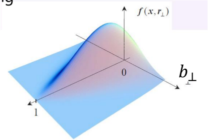
Generalized PDF (GPD) :

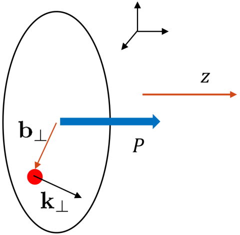
$G(x, \Delta_\perp) \quad \mathbf{b}_\perp \overset{FT}{\leftrightarrow} \Delta_\perp \quad \tilde{G}(x, b_\perp)$

Deeply virtual Compton scattering



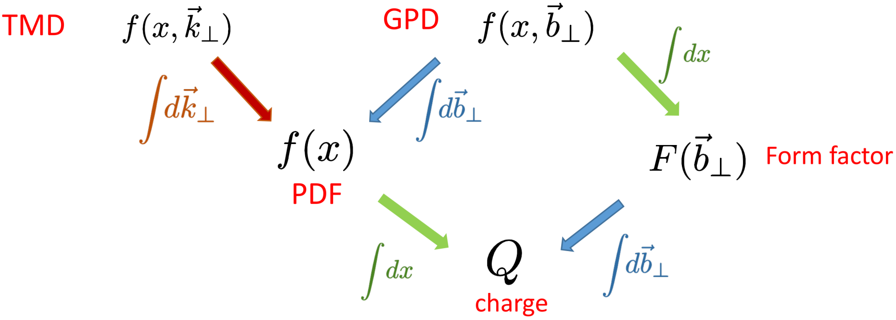
$\Delta = p - p'$





Y. Hagiwara, Y. Hatta, R. Pasechnik, M. Tasevsky, O. Teryaev, arXiv:1706.01765 [hep-ph]

Generalized parton distributions



Wigner distribution

GTMD Meissner, Metz, Schlegel (2009)

$$G(x, \vec{k}_\perp, \vec{\Delta}_\perp)$$

$\vec{b}_\perp \leftrightarrow \vec{\Delta}_\perp$

Husimi Hagiwara, YH (2015)

$$H(x, \vec{k}_\perp, \vec{b}_\perp)$$

Gaussian smearing in k, b

Wigner

$$W(x, \vec{k}_\perp, \vec{b}_\perp)$$

$\int d\vec{b}_\perp$ $\int d\vec{k}_\perp$

TMD $f(x, \vec{k}_\perp)$

GPD $f(x, \vec{b}_\perp)$

$\int d\vec{k}_\perp$ $\int d\vec{b}_\perp$ $\int dx$

$f(x)$ PDF

$F(\vec{b}_\perp)$ Form factor

$\int dx$ $\int d\vec{b}_\perp$

Q charge

E. Wigner. *Phys. Rev.* 40:749 (1932)

Wigner distribution

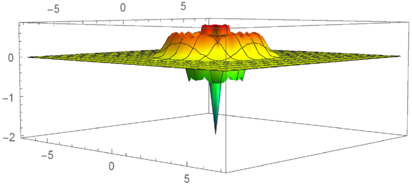
$$W(x,p) = \int d\xi e^{ip\xi}\psi^*(x-\xi/2)\psi(x+\xi/2)$$

$\psi(x)$: wave function

Ex. Harmonic Oscillator in 1D

$$W(q,p)^{(n)} = 2(-1)^n e^{-\frac{2H}{\hbar\omega}} L_n\left(\frac{4H_O}{\hbar\omega}\right)$$

$n = 3$



$$H_O = \frac{p^2}{2m} + \frac{m\omega^2 q^2}{2}$$

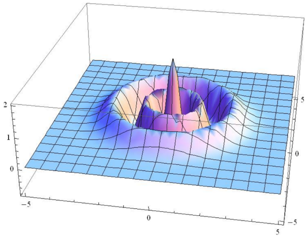
Wigner distribution

- Wigner distribution is an analogy to the probability distribution in classical physics (many-body system in a box evolving to certain configuration)
- However, in QM not positively definite, no probabilistic interpretation
- If we integrate over $p(x)$ we get $x(p)$ distribution

$$\int \frac{dp}{2\pi\hbar} W(x,p) = |\psi(x)|^2$$

- Expectation value of any operator is

$$\langle \mathcal{O} \rangle = \int dxdp W(x,p) \mathcal{O}(x,p)$$



Husimi distribution

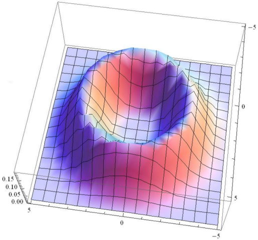
- Gaussian smearing of Wigner distribution

$$H(x,p) = \int \frac{dx'dp'}{\pi\hbar} e^{\frac{m\omega}{\hbar}(x-x')^2} e^{\frac{(p-p')^2}{m\omega\hbar}} W(x,p)$$

- valid within region of minimum uncertainty $\Delta x = \sqrt{\hbar/2m\omega}$, $\Delta p = \sqrt{\hbar m\omega/2}$,

$$\Delta x \Delta p = \hbar/2$$

- positively definite, probabilistic interpretation
- If we integrate over $p(x)$ we DO NOT get $x(p)$ distribution



Wigner distribution in QCD

Quark Wigner distribution

Belistky, Ji, Yuan (2004) , Ji (2003)

$$W_\Gamma(\vec{r},k) = \frac{1}{2}\int \frac{d^3\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} \langle \vec{q}/2|\hat{\mathcal{W}}_\Gamma(0,k)| - \vec{q}/2\rangle$$

$$\hat{\mathcal{W}}_\Gamma(\vec{r},k) = \int d^4\xi e^{ik\cdot\xi}\bar{\Psi}(\vec{r}-\xi/2)\Gamma\Psi(\vec{r}+\xi/2)\delta(\xi^+)2\pi$$

Gluon : $\bar{\Psi}(\vec{r}-\xi/2)\Gamma\Psi(\vec{r}+\xi/2) \to F^{+\nu}(\vec{r}-\xi/2)F^+_\nu(\vec{r}+\xi/2)$

Wigner distribution at high energy

Lorce, Pasquini (2011)

Using infinite Momentum Frame

$$W_\Gamma(\boldsymbol{b}_\perp,k) = \frac{1}{2}\int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp\cdot\boldsymbol{b}_\perp} \langle \Delta_\perp/2|\hat{\mathcal{W}}_\Gamma(0,k)| - \Delta_\perp/2\rangle$$

Wigner distribution in QCD

- Small x approximation

$$x \ll 1 \rightarrow e^{-ixP^+\xi^-} \approx 1$$

The gluon Wigner distribution at small x

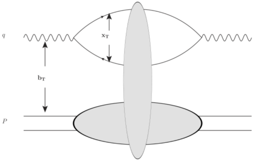
$$xW_g(x, \mathbf{k}, \mathbf{b}_\perp) = \frac{2N_c}{\alpha_S} \int \frac{d^2\mathbf{r}}{(2\pi)^2} e^{i\mathbf{k}\cdot\mathbf{r}} \left(\frac{1}{4}\nabla^2_{\mathbf{b}_\perp} - \nabla^2_{\mathbf{r}} \right) S_Y(\mathbf{r}, \mathbf{b}_\perp)$$

Y. Hatta, B. W. Xiao, F. Yuan Phys. Rev. Lett. 116, 202301 (2016)

$$S_Y(\mathbf{r}, \mathbf{b}_\perp) = \frac{1}{N_c} tr\left(U(\mathbf{b}_\perp + \mathbf{r}/2) U^\dagger(\mathbf{b}_\perp - \mathbf{r}/2) \right)$$

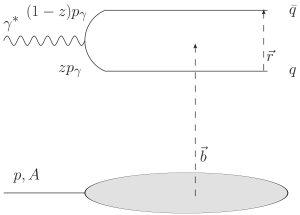
: S-matrix of the dipole-nucleon scattering

$Y = \ln(1/x)$:rapidity



Balitsky-Kovchegov equation

- BK equation describes the dressing of a color-dipole under the evolution towards higher energies (Lorentz boost)
- $\vec{r}$ is a transverse width of a dipole, $\vec{b}$ is a transverse distance of a dipole from a target



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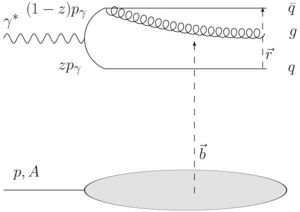

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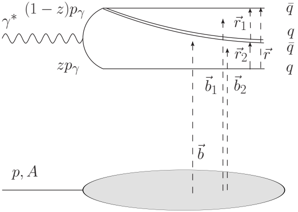

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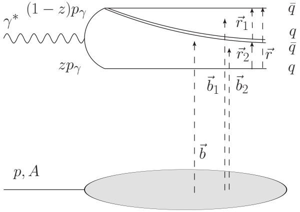
- In large-N_c limit gluon can be interpreted as a new dipole
- Effectively parent dipole splits into two daughter dipoles $(\vec{r}_1, \vec{b}_1)$ and $(\vec{r}_2, \vec{b}_2)$

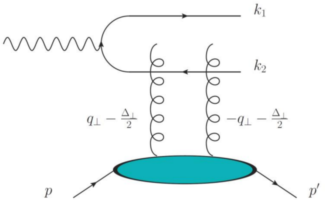


Balitsky-Kovchegov equation

$$\frac{\partial N(\vec{r},\vec{b},Y)}{\partial Y} = \int \mathrm{d}\vec{r}_1 K(\vec{r}_1,\vec{r}_2,\vec{r}) \times$$
$$\times \Big(N(\vec{r}_1,\vec{b}_1,Y) + N(\vec{r}_2,\vec{b}_2,Y) - N(\vec{r},\vec{b},Y) - N(\vec{r}_1,\vec{b}_1,Y)N(\vec{r}_2,\vec{b}_2,Y)\Big)$$

- Evolution in rapidity of the interaction between a colour dipole and a hadronic target
- Since $N(\vec{r},\vec{b},Y) = 1 - S_Y(\vec{r},\vec{b}_T)$ it can be directly used to calculate gluon Wigner distribution

Diffractive dijet production in DIS



Y. Hatta, B. W. Xiao, F. Yuan Phys. Rev. Lett. 116, 202301 (2016)

$$\vec{P}_\perp = \frac{1}{2}(\vec{k}_{2\perp} - \vec{k}_{1\perp})$$
$$\vec{k}_{1\perp} + \vec{k}_{2\perp} = -\vec{\Delta}_\perp$$

$$\frac{d\sigma^{\gamma_T^* A \to q\bar{q}X}}{dy_1 d^2k_{1\perp} dy_2 d^2k_{2\perp}} = 2N_c \alpha_{em} e_q^2 \delta(x_{\gamma^*} - 1) z(1-z)[z^2 + (1-z)^2] \int d^2\vec{q}_\perp d^2\vec{q}'_\perp S(\vec{q}_\perp, \vec{\Delta}_\perp) S(\vec{q}'_\perp, \vec{\Delta}_\perp)$$
$$\times \left[\frac{P_\perp}{P_\perp^2 + \epsilon_f^2} - \frac{P_\perp - q_\perp}{(P_\perp - q_\perp)^2 + \epsilon_f^2}\right] \cdot \left[\frac{P_\perp}{P_\perp^2 + \epsilon_f^2} - \frac{P_\perp - q'_\perp}{(P_\perp - q'_\perp)^2 + \epsilon_f^2}\right]$$

$$\epsilon_f^2 \equiv z(1-z)Q^2 \qquad q^+ = \sqrt{2}\omega \qquad z = \frac{k_{1\perp} e^{y_1}}{k_{1\perp} e^{y_1} + k_{2\perp} e^{y_2}}$$

Summary

- We can do it with our recent results on the solution of BK equation!

Belistky, Ji, Yuan (2004)
Ji (2003)
Y. Hagiwara, Y. Hatta, (2015)
Y. Hatta, B. W. Xiao, F. Yuan (2016)
Bomhof, Mulders (2008)
Dominguez, Marquet, Xiao, Yuan (2011)
Echevarria, et al. (2016)
Lorce, Pasquini, (2011)
Hagiwara, Hatta, Ueda (2016)
Altinoluk, Armesto, Beuf, Rezaeian (2015)
Mantysaari, Mueller, Schenke (2019)
Mantysaari, Mueller, Salazar, Schenke (2019)
Hagiwara, Hatta, Pasechnik, Tasevsky, Teryaev (2017)