Three-body Interactions in Mean-Field Model of Nuclei and Hypernuclei

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Motivation

- in my bachelor thesis, the *NN* potential was **corrected by a phenomenological term** – simulation of *NNN* interactions
- the *NNN* interactions have great influence on nuclear ground-state energies and charge radii [A. Günther et al. PRC 82, 024319 (2010).]
- aim of the research project – **study of the effect of the *NNN* interactions in the mean-field model** of nuclei and hypernuclei

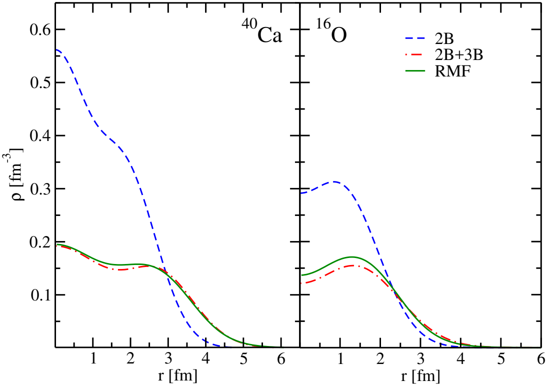

Figure: Document of the effect of the *NNN* interactions on the radial density distributions.

Three-body Interactions

- baryon-baryon potentials in the low-energy scale can be described by **Chiral Perturbation Theory** (ChPT)
- ChPT – effective field theory with consistent hierarchy of two-, three-, four-, ..., and many-baryon forces
- three-body interactions emerge at the **next-to-next-to leading order** of perturbation

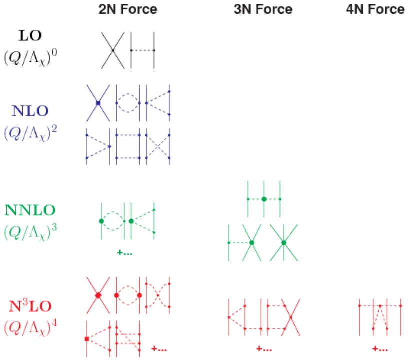

Figure: Hierarchy of nuclear forces in ChPT [R. Machleidt, arXiv:1308.0103].

Mean-Field Model

- mean field – (hyper)nuclear potential **self-consistently generated** from interactions between all constituents (nucleons or nucleons + Λ)
- protons, neutrons, and Λ are considered to be different particles placed in different potential wells
- **Hartree-Fock method**
- traditionally based on **phenomenological** two-body NN interactions
- our approach – formalism with $NN, NNN, \Lambda N, \Lambda NN$ interactions
- employed chiral potential $\mathrm{NNLO_{sat}}$ [A. Ekström et al., PRC 91, 051301(R) (2015)] – **realistic** NN and NNN interactions
- computations only with NN, NNN interactions (nuclei); $NN, NNN, \Lambda N$ interactions (hypernuclei)

The Hartree-Fock Method

- the starting point of this method is the hypernuclear Hamiltonian

$$\widehat{\mathbf{H}} = \widehat{\mathbf{T}}_{\mathbf{N}} + \widehat{\mathbf{T}}_{\boldsymbol{\Lambda}} + \widehat{\mathbf{V}}^{\mathbf{NN}} + \widehat{\mathbf{V}}^{\boldsymbol{\Lambda}\mathbf{N}} + \widehat{\mathbf{V}}^{\mathbf{NNN}} + \widehat{V}^{\Lambda NN} - \widehat{\mathbf{T}}_{\mathbf{CM}}, \tag{1}$$

- $\widehat{T}_N, \widehat{T}_\Lambda$ ... kinetic terms
- $\widehat{V}^{NN}, \widehat{V}^{\Lambda N}$ ... two-body terms
- $\widehat{V}^{NNN}, \widehat{V}^{\Lambda NN}$ ... three-body terms
- $\widehat{T}_{CM}$ ... center-of-mass kinetic term

$$\widehat{T}_{CM} = \frac{1}{2M(A+0.19)} \left(\sum_{a=1}^{A} \widehat{\vec{P}}_a^2 + 2 \sum_{a<b} \widehat{\vec{P}}_a \cdot \widehat{\vec{P}}_b \right) \tag{2}$$

- $M \approx 938$ MeV ... nucleon mass, A ... baryon number, $\widehat{\vec{P}}_a$ ... a-th particle momentum
- **we derive formalism of the HF method for Hamiltonian (1)**
- **we implement only $\widehat{\mathbf{T}}_{\mathbf{N}}, \widehat{\mathbf{T}}_{\boldsymbol{\Lambda}}, \widehat{\mathbf{V}}^{\mathbf{NN}}, \widehat{\mathbf{V}}^{\mathbf{NNN}}, \widehat{\mathbf{V}}^{\boldsymbol{\Lambda}\mathbf{N}}$ – tests on ^{40}Ca, ^{16}O, $^{41}_{\Lambda}$Ca, $^{17}_{\Lambda}$O**

The Wave Function of the Ground State

- the ground-state wave function is a product of proton and neutron **Slater determinants** (fermions, antisymmetrization) and one single-particle wave function of the Λ particle

$$|\Psi_0\rangle = |\Psi_0\rangle_{\mathrm{p}} \otimes |\Psi_0\rangle_{\mathrm{n}} \otimes |\Psi_0\rangle_{\Lambda} \tag{1}$$

- we introduce the HF method in the formalism of the **second quantization**, i.e. in terms of **creation, annihilation operators** ($a^\dagger, a$ for protons, $b^\dagger, b$ for neutrons, $c^\dagger, c$ for Λ)

$$|\Psi_0\rangle_{\mathrm{p}} = \prod_{i=1}^{Z} a_i^\dagger |0\rangle, \quad |\Psi_0\rangle_{\mathrm{n}} = \prod_{i=1}^{N} b_i^\dagger |0\rangle, \quad |\Psi_0\rangle_{\Lambda} = c_1^\dagger |0\rangle \tag{2}$$

- Z ... proton number, N ... neutron number, products run over states lowest in energy

The HF Equations for Protons, Neutrons, and the Λ Hyperon

$$
\begin{aligned}
t^{\mathrm{p}}_{ij} + \sum_{kl} V^{\mathrm{pp}}_{ikjl}\rho^{\mathrm{p}}_{lk} + \sum_{kl} V^{\mathrm{pn}}_{ikjl}\rho^{\mathrm{n}}_{lk} + \sum_{kl} V^{\mathrm{p}\Lambda}_{ikjl}\rho^{\Lambda}_{lk} + \frac{1}{2}\sum_{klmn} V^{\mathrm{ppp}}_{ikljmn}\rho^{\mathrm{p}}_{mk}\rho^{\mathrm{p}}_{nl} \\
+\frac{1}{2}\sum_{klmn} V^{\mathrm{pnn}}_{ikljmn}\rho^{\mathrm{n}}_{mk}\rho^{\mathrm{n}}_{nl} + \sum_{klmn} V^{\mathrm{ppn}}_{ikljmn}\rho^{\mathrm{p}}_{mk}\rho^{\mathrm{n}}_{nl} + \sum_{klmn} V^{\mathrm{pp}\Lambda}_{ikljmn}\rho^{\mathrm{p}}_{mk}\rho^{\Lambda}_{nl} \\
+\sum_{klmn} V^{\mathrm{pn}\Lambda}_{ijklmn}\rho^{\mathrm{n}}_{mk}\rho^{\Lambda}_{nl} = \underline{\underline{\varepsilon^{\mathrm{p}}_{i}}}\delta_{ij}.
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
t^{\mathrm{n}}_{ij} + \sum_{kl} V^{\mathrm{nn}}_{ikjl}\rho^{\mathrm{n}}_{lk} + \sum_{kl} V^{\mathrm{pn}}_{kilj}\rho^{\mathrm{p}}_{lk} + \sum_{kl} V^{\mathrm{n}\Lambda}_{ikjl}\rho^{\Lambda}_{lk} + \frac{1}{2}\sum_{klmn} V^{\mathrm{nnn}}_{ikljmn}\rho^{\mathrm{n}}_{mk}\rho^{\mathrm{n}}_{nl} \\
+\frac{1}{2}\sum_{klmn} V^{\mathrm{ppn}}_{klimnj}\rho^{\mathrm{p}}_{mk}\rho^{\mathrm{p}}_{nl} + \sum_{klmn} V^{\mathrm{pnn}}_{klimnj}\rho^{\mathrm{p}}_{mk}\rho^{\mathrm{n}}_{nl} + \sum_{klmn} V^{\mathrm{nn}\Lambda}_{ikljmn}\rho^{\mathrm{n}}_{mk}\rho^{\Lambda}_{nl} \\
+\sum_{klmn} V^{\mathrm{pn}\Lambda}_{klimnj}\rho^{\mathrm{p}}_{mk}\rho^{\Lambda}_{nl} = \underline{\underline{\varepsilon^{\mathrm{n}}_{i}}}\delta_{ij}.
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
t^{\Lambda}_{ij} + \sum_{kl} V^{\mathrm{p}\Lambda}_{kilj}\rho^{\mathrm{p}}_{lk} + \sum_{kl} V^{\mathrm{n}\Lambda}_{kilj}\rho^{\mathrm{n}}_{lk} + \frac{1}{2}\sum_{klmn} V^{\mathrm{pp}\Lambda}_{klimnj}\rho^{\mathrm{p}}_{mk}\rho^{\mathrm{p}}_{nl} + \frac{1}{2}\sum_{klmn} V^{\mathrm{nn}\Lambda}_{klimnj}\rho^{\mathrm{n}}_{mk}\rho^{\mathrm{n}}_{nl} \\
+\sum_{klmn} V^{\mathrm{pn}\Lambda}_{klimnj}\rho^{\mathrm{p}}_{mk}\rho^{\mathrm{n}}_{nl} = \underline{\underline{\varepsilon^{\Lambda}_{i}}}\delta_{ij}.
\end{aligned} \tag{3}
$$

Numerical Implementation

- we implement the **extension of the HF code** which has been used in the study of multipole response in neutron-rich nuclei [D. Bianco et al., NPP 41, 025109 (2014).]
- we employ **chiral potential NNLO**$_{\rm sat}$ [A. Ekström et al., PRC 91, 051301(R) (2015)]
  - includes NN and NNN interactions
- **chiral LO ΛN interaction** [H. Polinder et al., NPA 779, 244 (2006).]
  - explicit linear dependence on the cut-off parameter
  - this interaction was derived without counter terms
  - we implement cut-off 550 MeV
- the terms with ΛNN **interactions are not taken into account**

Results

- studied nuclei: ^{16}O, ^{40}Ca – doubly magic and spherically symmetric
- studied hypernuclei: $^{17}_{\Lambda}$O, $^{41}_{\Lambda}$Ca
- small configuration space – truncation: $N_{\mathrm{max}} = 4$, $N^{(12)}_{\mathrm{max}} = 8$, $N^{(123)}_{\mathrm{max}} = 12$
- documentation of the **qualitative effect of the *NNN* interactions**
    - **radial density distributions**
    - **charge radii**
    - **neutron single particle spectra** (the same effect applies to proton s.p. spectra)
    - Λ **s.p. spectra**
- comparison of calculations done purely with the *NN* interactions to the ones done with the *NN* and the *NNN* interactions
    - **(2B)** vs. **(2B + 3B)**

Radial nuclear density distributions of the ^{40}Ca and the ^{16}O

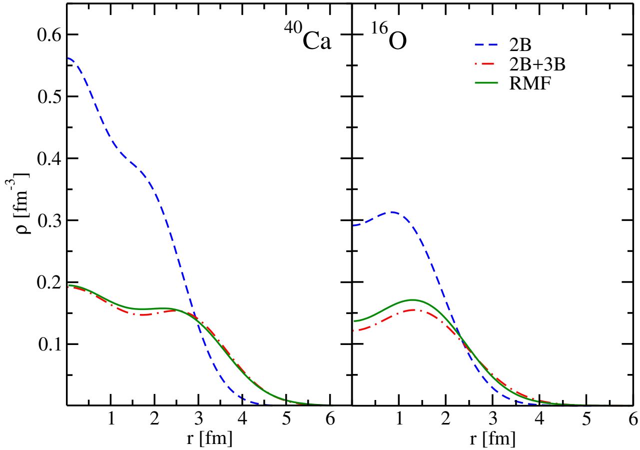

Figure: Radial density distribution of ^{40}Ca and ^{16}O calculated only with *NN* interactions (2B – dashed blue line), radial density distribution of ^{40}Ca and ^{16}O calculated with *NN* and *NNN* interactions (2B + 3B – dash-dotted red line), realistic radial density distribution of ^{40}Ca and ^{16}O calculated with RMF model (RMF – full green line).

- **calculations with *NNN* interactions (2B+3B) are closer to the realistic density distributions**

The Charge Radii of the ^{40}Ca and the ^{16}O

Table: The charge radii r_{ch} of the ^{40}Ca and the ^{16}O calculated with the *NN* interactions (2B) and the charge radii of the ^{40}Ca and the ^{16}O calculated with the $NN + NNN$ interactions (2B+3B) compared to the experimental data (exp) taken from [I. Angeli, ADNDT 87, 185 (2004)].

	r_{ch} [fm]		
^{A}X	2B	2B+3B	exp
^{40}Ca	2.58	3.18	3.48
^{16}O	2.23	2.67	2.70

- **charge radii calculated with the *NNN* interactions (2B+3B) are realistic**
- configuration space too small for ^{40}Ca

The Neutron Single-Particle Spectra in the ^{40}Ca and in the ^{16}O

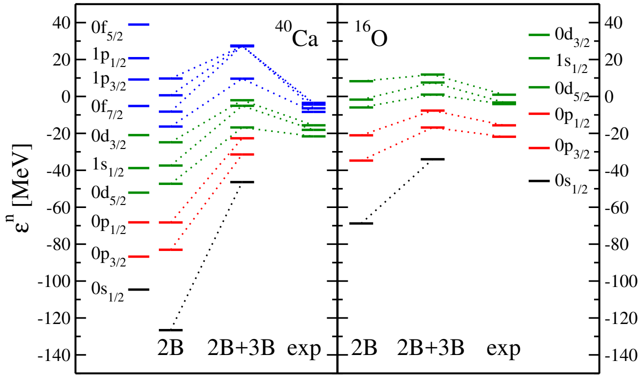

Figure: The neutron single-particle energies ε^n in ^{40}Ca and ^{16}O calculated with only two-body *NN* interactions (2B) and with two-body *NN* plus three-body *NNN* interactions (2B+3B) compared to the empirical values (exp) [V.I. Isakov et al., PJA 14, 29-36 (2002)].

- **the *NNN* interactions (2B+3B) quench the gaps between major shells and the gaps within the shells**
- the results for the sd- shells in ^{40}Ca are not realistic – small config. space

The Λ Single-Particle Spectra in the $^{41}_{\Lambda}$Ca and in the $^{17}_{\Lambda}$O

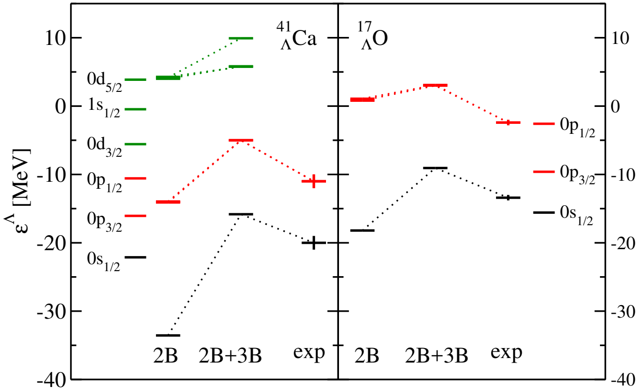

Figure: The Λ single-particle energies ε^{Λ} in $^{41}_{\Lambda}$Ca and $^{17}_{\Lambda}$O calculated with only two-body *NN* interactions (2B) and with two-body *NN* plus three-body *NNN* interactions (2B+3B) compared to the experimental data (exp) [M. Agnello et al., PLB 698, 219 (2011), R. E. Chrien, NPA 478, 705c (1998)].

- sd- shells in $^{41}_{\Lambda}$Ca not realistic – not taken into account
- **the *NNN* interactions shrink the gaps between the s- and p- shells**
- spectra shifted upwards in energy – **cut-off dependence of the Λ*N* interaction**
- spin-orbit splitting in p- levels

The Λ single-particle spectra in the $^{41}_{\Lambda}$Ca and in the $^{17}_{\Lambda}$O II

Table: Single-particle energies of the Λ hyperon in $^{41}_{\Lambda}$Ca and $^{17}_{\Lambda}$O calculated with NN interactions (2B) and with $NN + NNN$ interactions (2B+3B) compared to the experimental data (exp) – $^{41}_{\Lambda}$Ca [R.E. Chrien, NPA 478, 705c (1998)], $^{17}_{\Lambda}$O [M. Agnello et al., PLB 698, 219 (2011)].

	ε^{Λ} [MeV]					
		$^{41}_{\Lambda}$Ca			$^{17}_{\Lambda}$O	
s.-p. level	2B	2B+3B	exp	2B	2B+3B	exp
$0s_{1/2}$	-33.561	-15.820	-20.0 $\pm$ 1.0	-18.203	-9.055	-13.5 $\pm$ 0.4
$0p_{3/2}$	-14.095	-5.016	-11.0 $\pm$ 1.0	1.076	3.090	-2.4 $\pm$ 0.4
$0p_{1/2}$	-13.958	-4.987	-11.0 $\pm$ 1.0	0.805	3.005	-2.4 $\pm$ 0.4

- **the NNN interactions give realistic gaps between s- and p- shells**
- standard ordering of levels in the p- shell: $0p_{3/2}$, $0p_{1/2}$
- **good results for $^{41}_{\Lambda}$Ca, opposite ordering in the p- shell in the $^{17}_{\Lambda}$O**
- **property of the employed ΛN interaction, not the model!!**
  - weak tensor term

Conclusions

- **derivation of the formalism of the HF method** which includes NNN and ΛNN potentials
- **implementation of the NN and NNN interactions** in the chiral potential $\mathrm{NNLO_{sat}}$ **in the HF code**
- test calculations – the NNN interactions **flatten the density distributions, extend the charge radii, shrink gaps between the major shells in neutron s.p. spectra, shrink gaps between the major shells in Λ s.p. spectra**
- **employed ΛN interaction is strongly cut-off dependent**
  - opposite ordering of p- levels in the $^{17}_{\Lambda}\mathrm{O}$ – possibly weak tensor term

Outlook

- **systematics** – dependence of the spectra (proton, neutron, Λ) on the size of the configuration space
- **derivation of the tensor part of the ΛN interaction** – phenomenological tensor term, study of spin-orbit splitting of the p- shell
- **implementation of the ΛNN interaction**
- beyond mean-field calculations (TDA)