

# Generating general 3D Gaussian distributions

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- You can listen and use the attached script



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$$G(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right) \quad (1)$$

Is implemented in ROOT as function Gaus.

```
gRandom->Gaus(mean, sigma);
```



Can be still found online explicitly.

$$G(x, y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp \left[ -\frac{1}{2(1-\rho^2)} \left( \frac{(x-\mu_x)^2}{\sigma_x^2} + \frac{(y-\mu_y)^2}{\sigma_y^2} - \frac{2\rho(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y} \right) \right]$$



## 2D case

Can be still found online explicitly.

$$G(x, y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp \left[ -\frac{1}{2(1-\rho^2)} \left( \frac{(x-\mu_x)^2}{\sigma_x^2} + \frac{(y-\mu_y)^2}{\sigma_y^2} - \frac{2\rho(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y} \right) \right]$$

Note: From 2 parameters in 1D case now we have 5 free parameters in 2D.

Quick implementation as TF2.



# Many D's case

For dimension  $k$ :

$$G_k = \frac{1}{\sqrt{(2\pi)^k |\Sigma|}} \exp\left(-\frac{1}{2}(\vec{x} - \vec{\mu})^T \Sigma^{-1}(\vec{x} - \vec{\mu})\right) \quad (2)$$

Where:  $\Sigma_{ij} = \langle x_i x_j \rangle - \mu_i \mu_j$ ,

so for 3D case:

$$\Sigma = \begin{bmatrix} \sigma_x^2 & \rho_{xy}\sigma_x\sigma_y & \rho_{xz}\sigma_x\sigma_z \\ \rho_{xy}\sigma_x\sigma_y & \sigma_y^2 & \rho_{yz}\sigma_y\sigma_z \\ \rho_{xz}\sigma_x\sigma_z & \rho_{yz}\sigma_y\sigma_z & \sigma_z^2 \end{bmatrix}$$



# How it's generated



Thank you for your attention!

