# Geometric scaling for the total $\gamma^* p$ cross secton in the low $x$ region

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# Saturation model

- The *ep* deep inelastic scattering (DIS) data at low $x$ described with help of saturation model
- Established QCD dipole picture of the interaction between the transversely (T) or longitudinally (L) polarised photon $\gamma^*$ and the proton $p$ which are collinear
- Total $\gamma^* p$ cross section:

$$\sigma_{T,L}(x, Q^2) = \int d^2r \int_0^1 dz |\Psi_{T,L}(r, z, Q^2)|^2 \hat{\sigma}(r, x) \qquad (1)$$

# The total $\gamma^* p$ cross section

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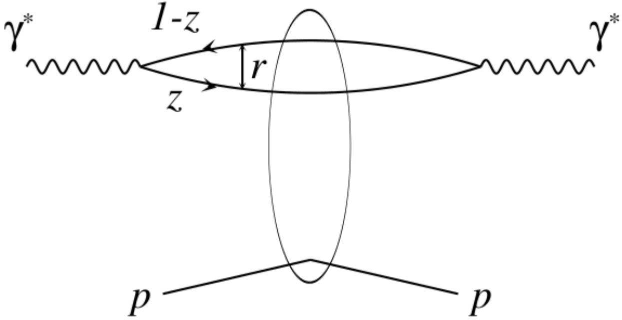


- Phys. Rev. D59 (1999) 014017

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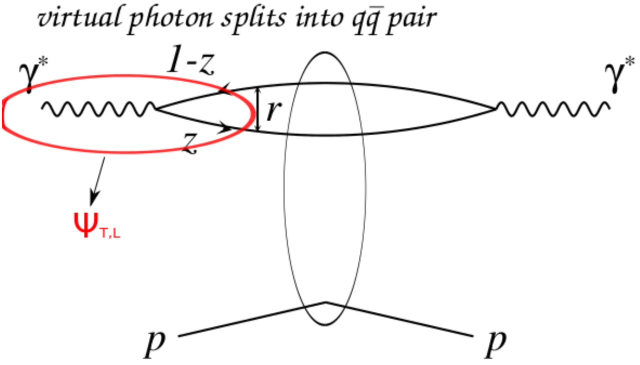


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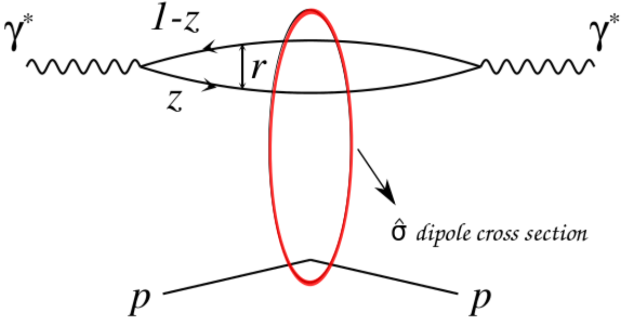


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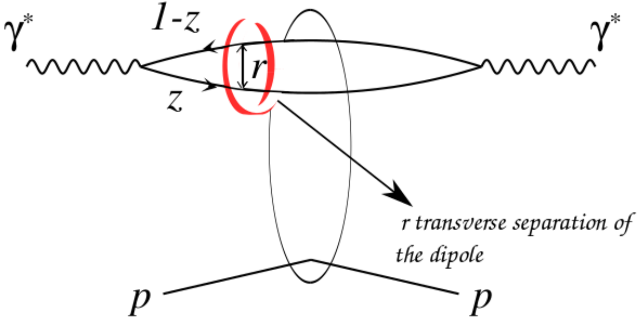


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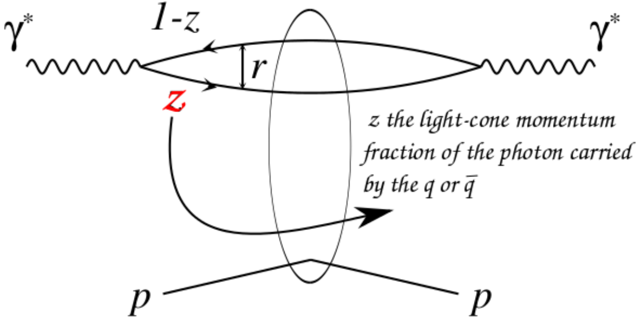


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# The total $\gamma^* p$ cross section

$$\sigma_{T,L}(x, Q^2) = \int d^2r \int_0^1 dz |\Psi_{T,L}(r, z, Q^2)|^2 \hat{\sigma}(r, x) \quad (2)$$

$$|\Psi_T|^2 = \frac{3\alpha_{em}}{2\pi^2} \sum_f e_f^2 \{[z^2 + (1-z)^2] \bar{Q}_f^2 K_1^2(\bar{Q}_f r) + m_f^2 K_0^2(\bar{Q}_f r)\} \quad (3)$$

$$|\Psi_L|^2 = \frac{3\alpha_{em}}{2\pi^2} \sum_f e_f^2 \{[4Q^2 z^2 (1-z^2) K_0^2(\bar{Q}_f r)\}, \bar{Q}_f^2 = z(1-z)Q^2 + m_f^2 \quad (4)$$

- $K_{0,1}$ are the Bessel-Mc Donald functions

# Assumption of the saturation model

- The adoption of the $x$–dependent radius = saturation radius $R_0(x)$
- Scaling of the $q\bar{q}$ pair separation $r$ in the dipole cross section:

$$\hat{\sigma}(x,r) = \sigma_0 \; g(\bar{r}), \;\; \bar{r} = \left(\frac{r}{R_0(x)}\right) \tag{5}$$

- The function $g$:
  - small $\bar{r} \to$ rise (in sat. model is the quadratic rise) $\Longrightarrow$ scaling behaviour $\sigma^{\gamma^* p} \sim 1/Q^2$
  - large $\bar{r} \to$ flattering off $\Longrightarrow$ saturation $\sigma^{\gamma^* p} = \sigma_T + \sigma_L \sim$ *const* for small $Q^2$
- $\Longrightarrow$ chosen the Ansatz: $g(\bar{r}) = 1 - exp(-\bar{r}/4)$

The main assumption of the saturation model is the saturation property of the dipole cross section.

# Geometric scaling

- $R_0(x)$ has the dimension of length
- Decreases with decreasing $x$
- Normalization $\sigma_0$ is $x$-independent $\implies$ dipole cross section, eq. (5), is limited by the energy independent $\sigma_0$ = unitarity bound
- The small $x$ increase of DIS structure functions generated by DGLAP or linear BFKL evolution are tamed by unitarization effects

The $\hat{\sigma}(x, r)$ only depends on the dimensionless ratio $r/R_0(x)$. Its energy dependence is entirely driven by the saturation radius $R_0(x)$ = "geometric scaling ".

# Qualitative Analysis

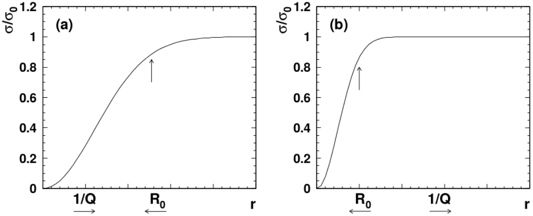
- Neglect the $m_f$ in:
  $|\Psi_T|^2 = \frac{3\alpha_{em}}{2\pi^2}\sum_f e_f^2\{[z^2+(1-z)^2]\bar{Q}_f^2 K_1^2(\bar{Q}_f r) + \cancel{m_f^2 K_0^2(\bar{Q}_f r)}\}$
  and
  $|\Psi_L|^2 = \frac{3\alpha_{em}}{2\pi^2}\sum_f e_f^2\{[4Q^2 z^2(1-z^2)K_0^2(\bar{Q}_f r)\}$,
  $\bar{Q}_f^2 = z(1-z)Q^2 + \cancel{m_f^2}$
- $\sigma_T$ dominates over $\sigma_L$
- The behaviour of $K_1(\bar{Q}_f^2 r)$
  - small values of $\bar{Q}_f^2 r$ is $K_1(\bar{Q}_f^2 r) \sim \frac{1}{\bar{Q}_f^2 r}$
  - large values of $\bar{Q}_f^2 r$, the $K_1(\bar{Q}_f^2 r)$ is exponentially suppressed
- Dominant contribution $\rightarrow$ the integration of (1) for $\bar{Q}_f^2 r < 1$
- Rescaling the dipole size $r \rightarrow r/R_0(x)$
- Photon virtuality introduces the scale $1/Q$ for the transverse dimension of the $q\bar{q}$ pair

# Qualitative Analysis

- Small dipole: $r < 1/Q$, large dipole: $r > 1/Q$
- Analysis of small dipole with condition $\bar{Q}_f^2 r = z(1-z)Q^2 r < 1$ for $\forall\ z$ in the case of $1/Q < R_0$
- The small $q\bar{q}$ pair is smaller than the $R_0$, Fig. (a) $\Longrightarrow$ integration of eq. (1) $\rightarrow \sigma_{\gamma^* p}(x, Q^2) = \sigma_{\gamma^* p}(\tau),\ \ \tau = Q^2 R_0^2$;

$$\sigma_{\gamma^* p} \sim \sigma_0 \qquad \rightarrow \qquad \sigma_{\gamma^* p} \sim \sigma_0/\tau \tag{6}$$

- For large values of $\tau$ - modulo logarithmic modification in $\tau$



# The aim of the article

- Demonstration that the DIS data exhibit approximately the geometric scaling $\sigma_{\gamma^* p}(x, Q^2) = \sigma_{\gamma^* p}(\tau)$ with the property $\sigma_{\gamma^* p} \sim \sigma_0 \to \sigma_{\gamma^* p} \sim \sigma_0/\tau$
- Parametrization of the saturation radius:

$$R_0(x) = \frac{1}{Q_0}\left(\frac{x}{x_0}\right)^{\frac{\lambda}{2}}, \tag{7}$$

  $Q_0 = 1$ GeV, $x_0$, $\lambda > 0$ and $\sigma_0$ fitted to all inclusive DIS data with $x < 0.01$
- Very good description of data down to $Q^2 < 1$ GeV$^2$
- Saturation model able to describe transition from the region of DIS to low values of $Q^2$

# Different approach

# Extension of the saturation model

- $R_0(x)$ can be determined in a less model-dependent way
- Extension of the saturation model to the low $Q^2$ region, including $Q^2 = 0$ photoproduction limit
- Right threshold behaviour $\to$ modification of the Bjorken $x$:

$$\bar{x} = x\left(1 + \frac{4m_f^2}{Q^2}\right) \tag{8}$$

- Relation between $\bar{x}$ and the total energy $W$ is:

$$\bar{x} = \frac{Q^2 + 4m_f^2}{W^2} \tag{9}$$

- Keeping $m_f \neq 0$, sat. model extended to the region $Q^2 = 0 \to \bar{Q}_f^2 = m_f^2$ and $\bar{x} = 4m_f^2/W^2$

# Extension of the saturation model

- In the eq. (1) the dominant contribution is from the integration region $1/m_f^2 > r^2 > R_0^2(x)$
- Few steps $\rightarrow$ the relation between saturation radius and the photoproduction cross section given by:

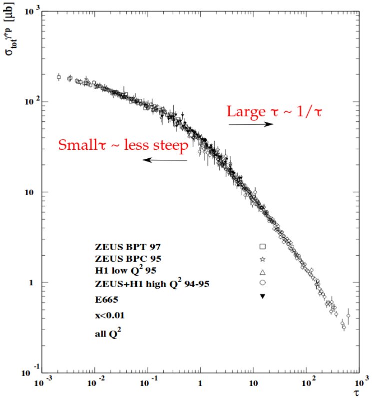
$$R_0^2(\bar{x}) = \frac{1}{m_f^2} exp\left( - \frac{\sigma_{\gamma p}}{\bar{\sigma}_0} \right), \tag{10}$$

where $\bar{\sigma}_0 = (2\alpha_{em}/3\pi)\sigma_0$.

- Use the Donnachie-Landshoff parametrization: $\sigma_{\gamma p} = a\bar{x}^{-0.08}$ and $m_f = 140$ MeV
- Using results from the fit $\rightarrow$ $a = 68\mu b \left(4m_f^2/1\mathrm{GeV}^2\right)^{0.08}$ with $\bar{\sigma}_0 = 23\ \mu b$ $\rightarrow$ good description of data

# Confrontation of geometric scaling on experimental DIS data

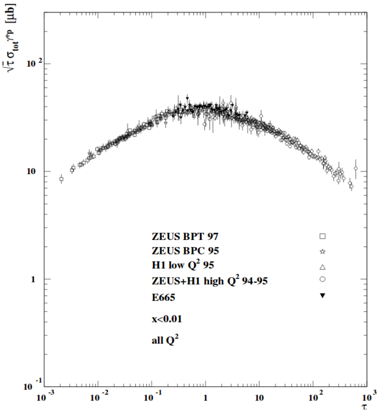
# Dependence on $\tau$




- $\sim 1/\tau$ reflects that $\sigma_{\gamma^* p}$ scales as $1/Q^2$ and energy dependence is $\sim 1/R_0^2(x)$
- Small values of $\tau$: $\sigma_{\gamma^* p}$ grows weaker - saturation of the dipole cross section

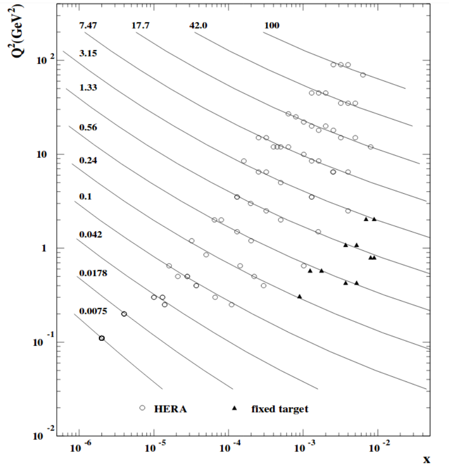
- The total cross section $\sigma_{\gamma^* p}$ vs. $\tau = Q^2 R_0^2(x)$
- $R_0(x)$ obtained from eq. (10), data for $x < 0.01$ and $Q^2$ from $(0.045, 450)$ GeV$^2$
- Asymptotic relations $\sigma_{\gamma^* p} \sim \sigma_0 \to \sigma_{\gamma^* p} \sim \sigma_0/\tau$ ✓

# Symmetry



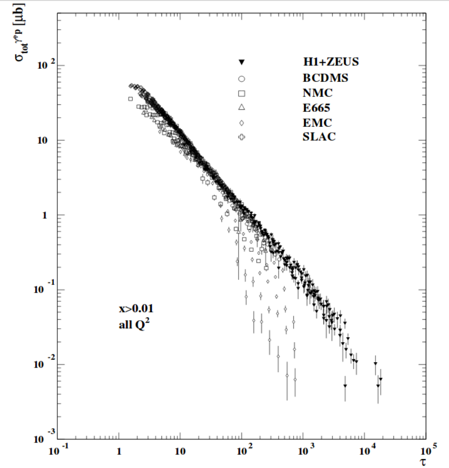

- Indicates the symmetry of $\sqrt{\tau}\sigma_{\gamma^* p}$ with respect to transformation $\tau \longleftrightarrow 1/\tau$
- For the whole region of $\tau$
- Again manifestation of asymptotic relations: $\sigma_{\gamma^* p} \sim \sigma_0 \to \sigma_{\gamma^* p} \sim \sigma_0/\tau$ ✓

# Exhibition of geometric scaling




- Contours for different values of $\tau$ in $(x, Q^2)$ plane
- Geometric scaling = $\sigma_{\gamma^* p}$ is *const* along each line
- For each $\tau$: several experimental points
- Variation of $x$ as much as 2 orders of magnitude, $Q^2$ changes by a factor of 4
- Exp. points in narrow lines $\rightarrow$ geometric scaling

# Geometric scaling ONLY in low region of $x$




- Violation of the geometric scaling for $x > 0.01$
- It is confined ONLY to the small $x$ region

# Take-home-message

The scaling is a manifestation of the presence of an internal scale - saturation scale - characterizing dense partonic systems, $Q_s(x) \sim 1/R_0(x)$.

# Summary

- The experimental data on DIS *ep* scattering at low $x$ exhibit geometric scaling
- That means: the total cross section $\sigma_{\gamma^* p}$ is the function of only one dimensionless variable $\tau = Q^2 R_0^2(x)$
- Demonstrated for a very broad region of $Q^2$: (0.045, 450) GeV$^2$