Going through the paper:

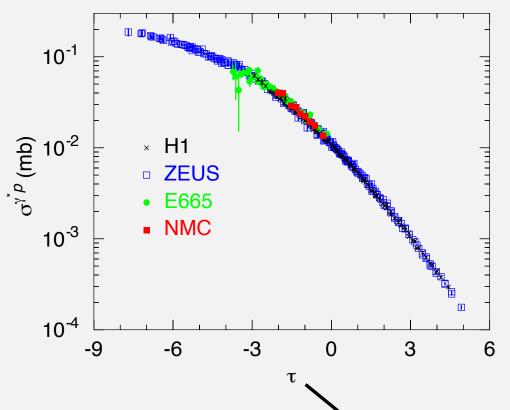
Exclusive vector meson production at HERA from QCD with saturation C. Marquet, R. Peschanski, G. Soyez arXiv:hep-ph/0702171v2 27 Jun 2007

GEOMETRIC SCALING IN VECTOR MESON PRODUCTION

Marek Matas
UPC group meeting at Decin 2.5.-3.5.2018

WHAT IS GEOMETRIC SCALING?

GEOMETRIC SCALING AT HERA



Geometric scaling was observed at HERA.

Defining a new variable τ , we can describe the DIS data.

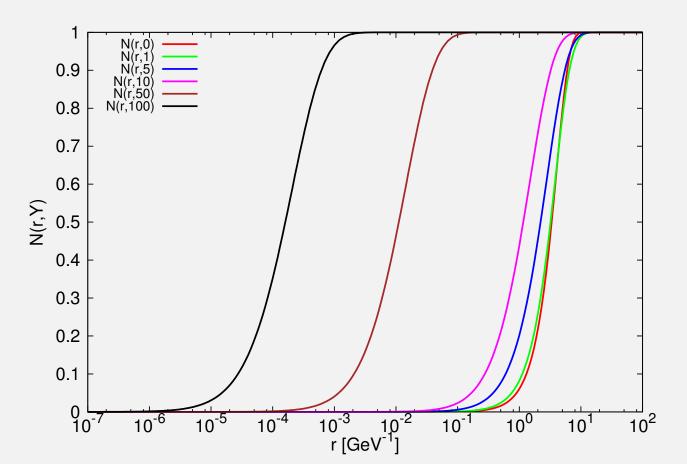
With it, we do not need a separate Y and Q^2 dependence.

$$\sigma^{\gamma^* p}(Y, Q) = \sigma^{\gamma^* p} \left(\frac{Q^2}{Q_s^2(Y)} \right)$$

$$Q_s^2 \propto e^{\lambda Y} \qquad \lambda \sim 0.3$$

GEOMETRIC SCALING AT BK

Geometric scaling of the BK solution means, that its shape does not change with evolution. It only shifts towards higher values of r.



GEOMETRIC SCALING AT BK

Geometric scaling of the BK solution means, that its shape does not change with evolution. It only shifts towards higher values of r.

Mathematically, this relates to the fact, that the b-independent scattering amplitude N now depends only on one variable $rQ_s(Y)$ instead of two separate r and Y.

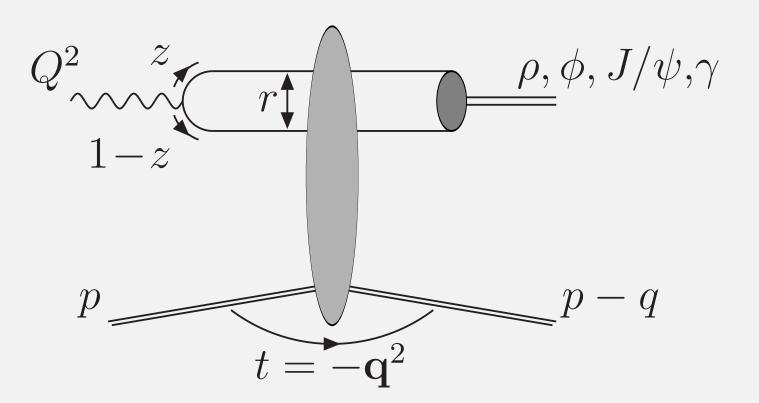
$$N(r,Y) \sim N(rQ_s(Y))$$

Following the convention of the paper discussed in this talk, we will define variable T(r,b,Y), which is the scattering amplitude weighted with the protons area.

Then we can write $\tilde{T}(\mathbf{r},\mathbf{q}=0;Y)=2\pi R_p^2 \, N(r^2Q_s^2(Y))$ for the case of zero momentum transfer.

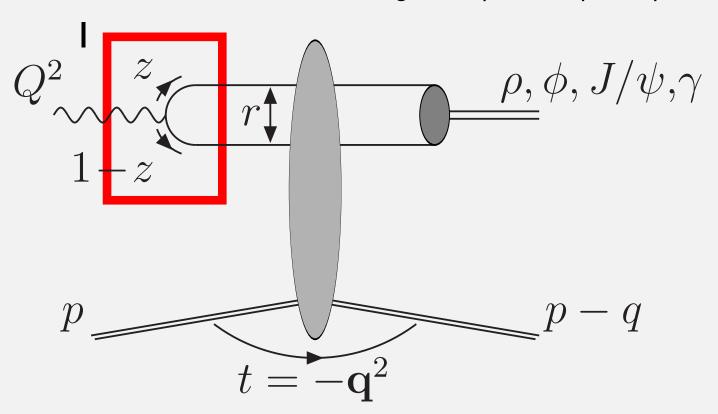
HOW TO COMPUTE VECTOR MESON CROSS PRODUCTION WITH BK?

Vector meson production in the dipole model approach looks as



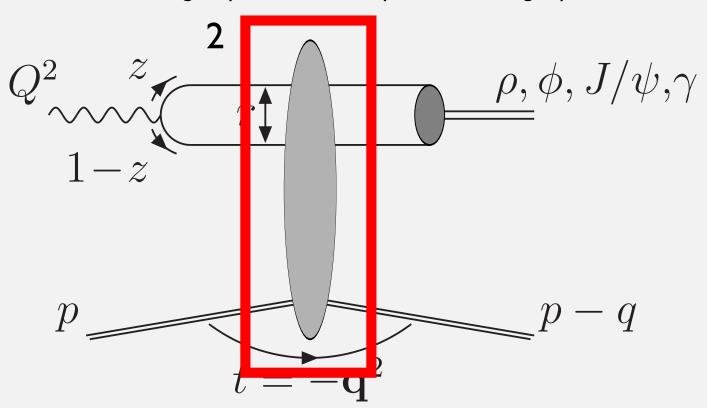
For the computation of vector meson production, we need three main ingredients.

Photon wave functions of it fluctuating into a quark-antiquark dipole.



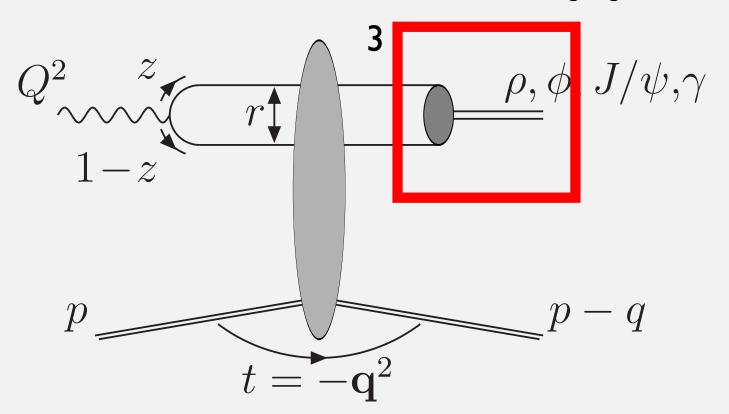
For the computation of vector meson production, we need three main ingredients.

Scattering amplitude of the dipole of the target proton.



For the computation of vector meson production, we need three main ingredients.

Vector meson wave function to account for the outgoing state.



The vector meson and photon wave functions have already been computed in other papers. $\Phi^{\gamma^*\gamma^*}(x, \mathbf{r}, O^2) = \sum_{c} \frac{\alpha_c N_c}{4O^2 x^2(1-x)^2 K^2(\mathbf{r}, \bar{O})}$

$$\Phi_L^{\gamma^*\gamma^*}(z,\mathbf{r},Q^2) = \sum_f e_f^2 \frac{\alpha_e N_c}{2\pi^2} 4Q^2 z^2 (1-z)^2 K_0^2(r\bar{Q}_f),$$

$$\Phi_T^{\gamma^*\gamma^*}(z,\mathbf{r},Q^2) = \sum_f e_f^2 \frac{\alpha_e N_c}{2\pi^2} \left\{ [z^2 + (1-z)^2] \bar{Q}_f^2 K_1^2(r\bar{Q}_f) + m_f^2 K_0^2(r\bar{Q}_f) \right\},$$

$$\Phi_T^{\gamma^*\gamma}(z,\mathbf{r},Q^2) = \sum_f e_f^2 \frac{\alpha_e N_c}{2\pi^2} \left\{ [z^2 + (1-z)^2] \bar{Q}_f K_1(r\bar{Q}_f) m_f K_1(rm_f) + m_f^2 K_0(r\bar{Q}_f) K_0(rm_f) \right\}$$

$$\mathbf{3} \quad \Phi_L^{\gamma^* V}(z, \mathbf{r}, Q^2) = \hat{e}_f \sqrt{\frac{\alpha_e}{4\pi}} N_c \, 2Q K_0(r\bar{Q}_f) \left[M_V z (1-z) \phi_L(r, z) + \delta \frac{m_f^2 - \nabla_r^2}{M_V} \phi_L(r, z) \right],$$

$$\Phi_T^{\gamma^* V}(z, \mathbf{r}, Q^2) = \hat{e}_f \sqrt{\frac{\alpha_e}{4\pi}} N_c \frac{\alpha_e N_c}{2\pi^2} \left\{ m_f^2 K_0(r\bar{Q}_f) \phi_T(r, z) - [z^2 + (1-z)^2] \bar{Q}_f K_1(r\bar{Q}_f) \partial_r \phi_T(r, z) \right\},$$

	common parameters			BG parameters			LCG parameters			
Vector-meson	M_V (GeV)	$m_f \text{ (GeV)}$	\hat{e}_f	$R^2 (\text{GeV}^{-2})$	N_L	N_T	$R_L^2 ext{ (GeV}^{-2})$	$R_T^2 (\text{GeV}^{-2})$	N_L	N_T
ρ	0.776	0.14	$1/\sqrt{2}$	12.9	0.853	0.911	10.4	21.0	1.79	4.47
ϕ	1.019	0.14	1/3	11.2	0.825	0.919	9.7	16.0	1.41	4.75
J/Ψ	3.097	1.4	2/3	2.3	0.575	0.578	3.0	6.5	0.83	1.23

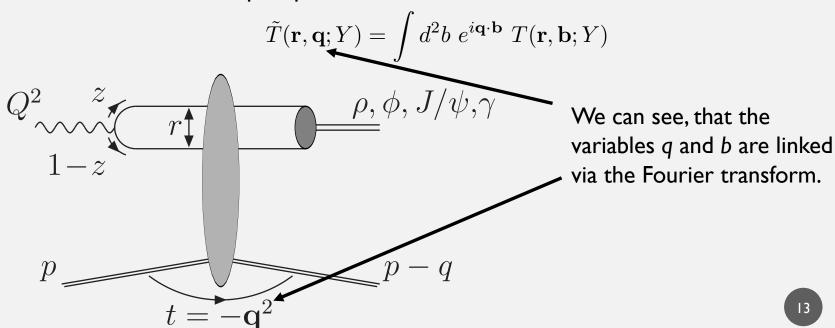
BK equation in impact parameter independent frame can give us the scattering amplitude N(r,Y).

How to put it all together?

How do we compute the scattering amplitude for a finite momentum transfer? (e.q. b-dependent)

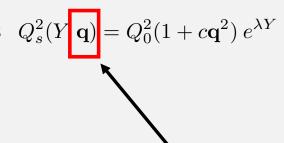
BK equation in impact parameter independent frame can give us the scattering amplitude N(r,Y).

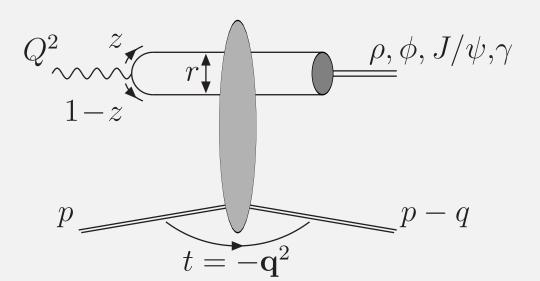
The connection between impact parameter and momentum transfer is:



If we would like to obtain the scattering amplitude for non-zero momentum transfer, we can take advantage of geometric scaling.

Parametrizing saturation scale value phenomenologically as $Q_s^2(Y|\mathbf{q}) = Q_0^2(1+c\mathbf{q}^2) e^{\lambda Y}$





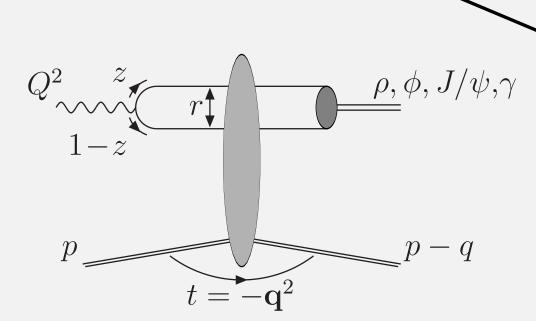
Now we make it dependent on transferred momentum.

If we would like to obtain the scattering amplitude for non-zero momentum transfer, we can take advantage of geometric scaling.

Parametrizing saturation scale value phenomenologically as $Q_s^2(Y,\mathbf{q}) = Q_0^2(1+c\mathbf{q}^2) \ e^{\lambda Y}$

We can then write

$$\tilde{T}(\mathbf{r}, \mathbf{q}; Y) = 2\pi R_p^2 f(\mathbf{q}) N(\mathbf{r}^2 Q_s^2(Y, \mathbf{q}))$$

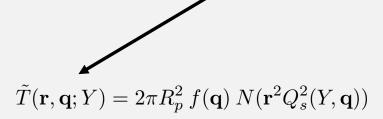


This factor incorporates the non-perturbative behavior of the impact parameter dependent scattering amplitude.

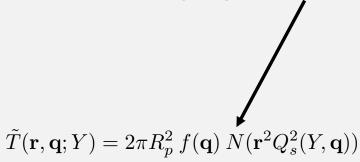
$$f(\mathbf{q}) = \exp(-B\mathbf{q}^2)$$

Has been used.

So in order to get the desired complete scattering amplitude and compute the σ ,



we need a prescription for the scattering amplitude N.



$$\tilde{T}(\mathbf{r}, \mathbf{q}; Y) = 2\pi R_p^2 f(\mathbf{q}) N(\mathbf{r}^2 Q_s^2(Y, \mathbf{q}))$$

Lets go through all the ingredients for the scattering amplitude.

$$\tilde{T}(\mathbf{r}, \mathbf{q}; Y) = 2\pi R_p^2 f(\mathbf{q}) N(\mathbf{r}^2 Q_s^2(Y, \mathbf{q}))$$

$$R_p = 3.34 \text{ GeV}^{-1}$$

$$\tilde{T}(\mathbf{r}, \mathbf{q}; Y) = 2\pi R_p^2 f(\mathbf{q}) N(\mathbf{r}^2 Q_s^2(Y, \mathbf{q}))$$

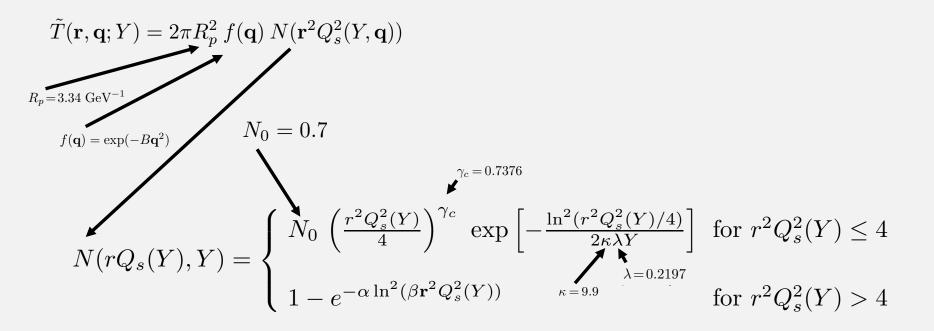
$$R_p = 3.34 \text{ GeV}^{-1}$$

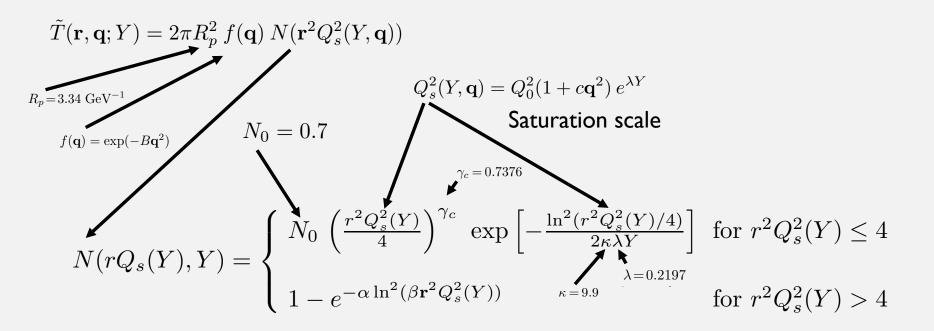
$$f(\mathbf{q}) = \exp(-B\mathbf{q}^2)$$

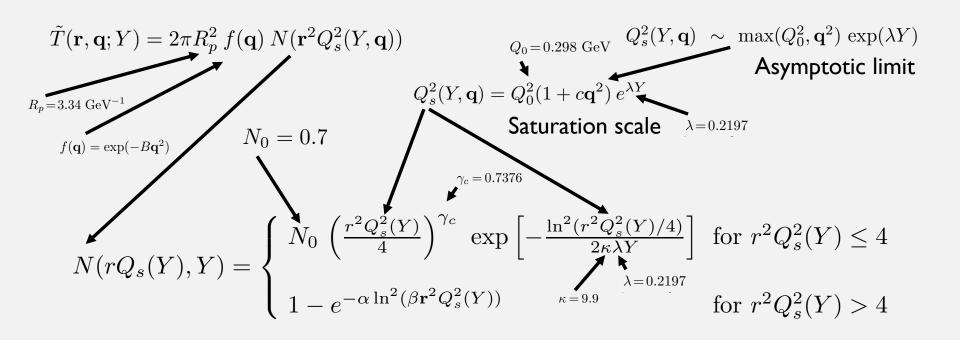
$$\tilde{T}(\mathbf{r}, \mathbf{q}; Y) = 2\pi R_p^2 f(\mathbf{q}) N(\mathbf{r}^2 Q_s^2(Y, \mathbf{q}))$$

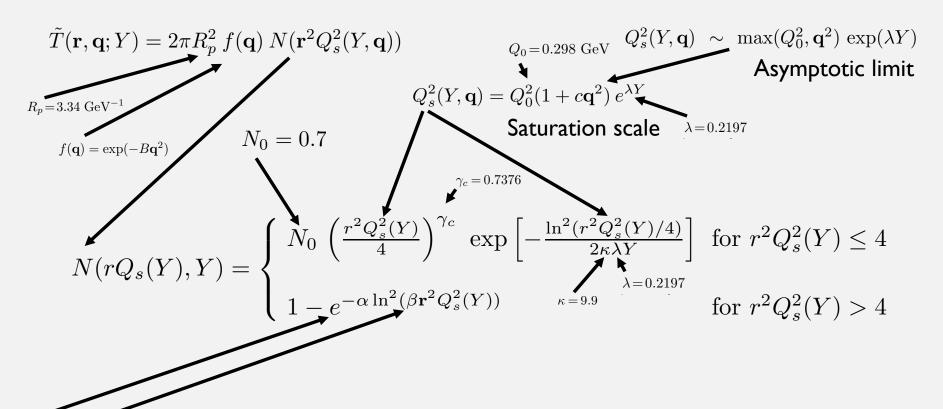
$$f(\mathbf{q}) = \exp(-B\mathbf{q}^2)$$

$$N(rQ_s(Y), Y) = \begin{cases} N_0 \left(\frac{r^2 Q_s^2(Y)}{4}\right)^{\gamma_c} \exp\left[-\frac{\ln^2(r^2 Q_s^2(Y)/4)}{2\kappa\lambda Y}\right] & \text{for } r^2 Q_s^2(Y) \le 4 \\ 1 - e^{-\alpha \ln^2(\beta \mathbf{r}^2 Q_s^2(Y))} & \text{for } r^2 Q_s^2(Y) > 4 \end{cases}$$

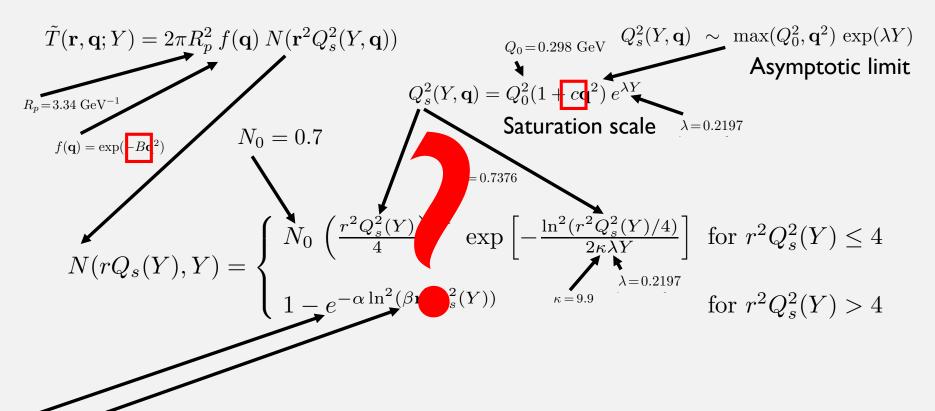








 α and β are obtained from the condition of continuous N and its derivation at $r^2Q_s^2(Y) = 4$



 α and β are obtained from the condition of continuous N and its derivation at $r^2Q_s^2(Y) = 4$

These are the different values for the parameter c and B for four different parametrizations of the wave functions.

They turn out to be similar to each other.

	t -dependent Q_s								
Parameter	BG	LCG	BLL	BLB					
	4.077 ± 0.310								
$B (\text{GeV}^{-2})$	3.754 ± 0.095	3.724 ± 0.093	3.708 ± 0.097	3.713 ± 0.096					

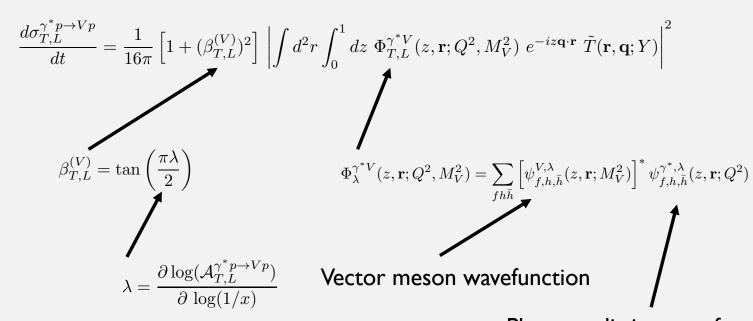
Now we have everything we need!

Now we have scattering amplitude, how about the cross section?

$$\frac{d\sigma_{T,L}^{\gamma^* p \to V p}}{dt} = \frac{1}{16\pi} \left[1 + (\beta_{T,L}^{(V)})^2 \right] \left| \int d^2 r \int_0^1 dz \; \Phi_{T,L}^{\gamma^* V}(z, \mathbf{r}; Q^2, M_V^2) \; e^{-iz\mathbf{q}\cdot\mathbf{r}} \; \tilde{T}(\mathbf{r}, \mathbf{q}; Y) \right|^2$$

$$\frac{d\sigma_{T,L}^{\gamma^*p\to Vp}}{dt} = \frac{1}{16\pi} \left[1 + (\beta_{T,L}^{(V)})^2 \right] \left| \int d^2r \int_0^1 dz \; \Phi_{T,L}^{\gamma^*V}(z,\mathbf{r};Q^2,M_V^2) \; e^{-iz\mathbf{q}\cdot\mathbf{r}} \; \tilde{T}(\mathbf{r},\mathbf{q};Y) \right|^2$$

$$\beta_{T,L}^{(V)} = \tan\left(\frac{\pi\lambda}{2}\right) \qquad \Phi_{\lambda}^{\gamma^*V}(z,\mathbf{r};Q^2,M_V^2) = \sum_{fh\bar{h}} \left[\psi_{f,h,\bar{h}}^{V,\lambda}(z,\mathbf{r};M_V^2) \right]^* \psi_{f,h,\bar{h}}^{\gamma^*,\lambda}(z,\mathbf{r};Q^2)$$



Photon splitting wavefunction

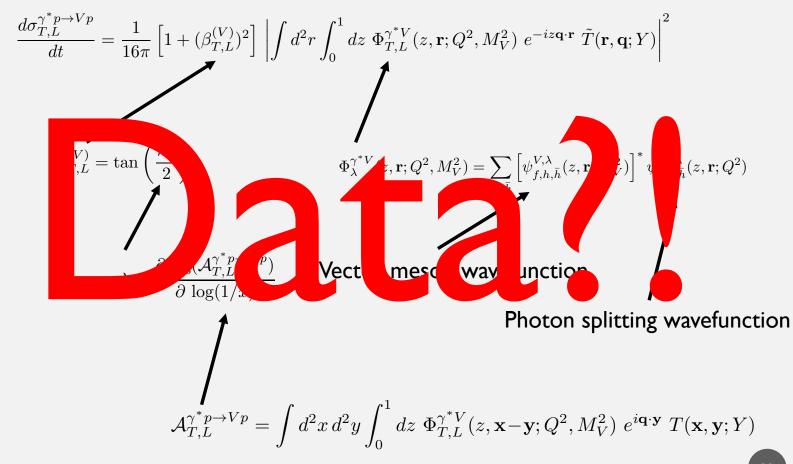
$$\frac{d\sigma_{T,L}^{\gamma^*p\to Vp}}{dt} = \frac{1}{16\pi} \left[1 + (\beta_{T,L}^{(V)})^2 \right] \left| \int d^2r \int_0^1 dz \; \Phi_{T,L}^{\gamma^*V}(z,\mathbf{r};Q^2,M_V^2) \; e^{-iz\mathbf{q}\cdot\mathbf{r}} \; \tilde{T}(\mathbf{r},\mathbf{q};Y) \right|^2$$

$$\beta_{T,L}^{(V)} = \tan\left(\frac{\pi\lambda}{2}\right) \qquad \Phi_{\lambda}^{\gamma^*V}(z,\mathbf{r};Q^2,M_V^2) = \sum_{fh\bar{h}} \left[\psi_{f,h,\bar{h}}^{V,\lambda}(z,\mathbf{r};M_V^2) \right]^* \psi_{f,h,\bar{h}}^{\gamma^*,\lambda}(z,\mathbf{r};Q^2)$$

$$\lambda = \frac{\partial \log(\mathcal{A}_{T,L}^{\gamma^*p\to Vp})}{\partial \log(1/x)} \qquad \text{Vector meson wavefunction}$$

$$Photon splitting wavefunction$$

$$\mathcal{A}_{T,L}^{\gamma^*p\to Vp} = \int d^2x \, d^2y \int_0^1 dz \; \Phi_{T,L}^{\gamma^*V}(z,\mathbf{x}-\mathbf{y};Q^2,M_V^2) \; e^{i\mathbf{q}\cdot\mathbf{y}} \; T(\mathbf{x},\mathbf{y};Y)$$



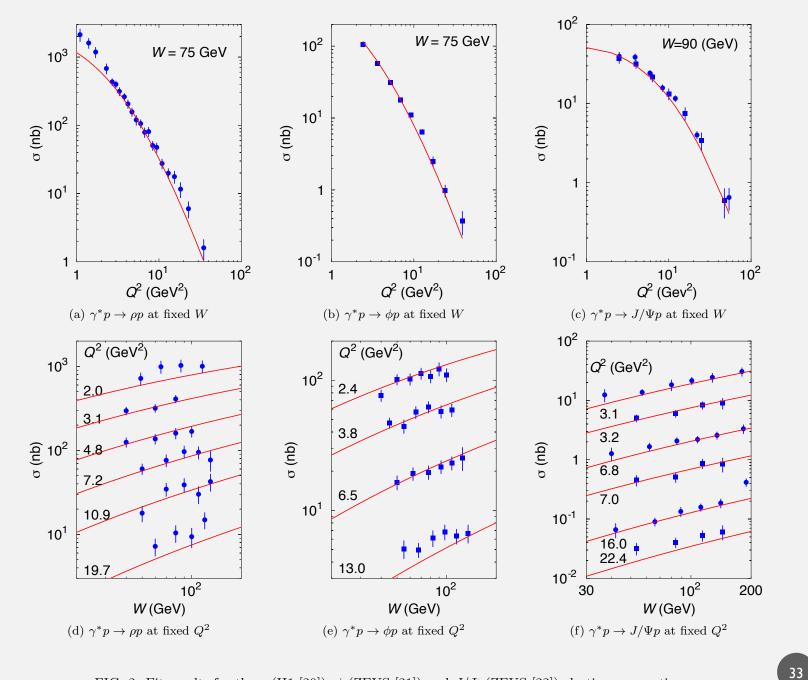


FIG. 2: Fit results for the ρ (H1 [20]), ϕ (ZEUS [21]) and J/Ψ (ZEUS [22]) elastic cross-sections.

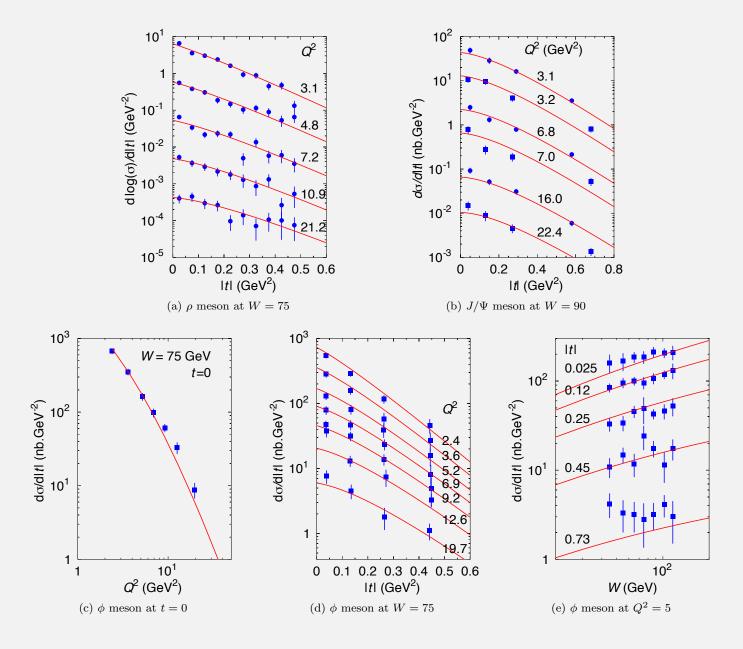


FIG. 3: Fit results for the ρ (H1 [20]), ϕ (ZEUS [21]) and J/Ψ (ZEUS [22], H1 [23]) differential cross-section.

CONCLUSIONS

- In this paper, the usual prescription for GS in vector meson production was extended for processes with non-zero momentum transfer.
- This was done with the use of the BK GS properties with the use of the color dipole model.
- Two new parameters have been produced with this generalisation and they were fit to data.
- Data was then described well for both total and differential cross sections.

THANK YOU FOR YOUR ATTENTION

No matter what, don't lose hope. We are all bombastic.

- Dan Nekonečný