IMPACT PARAMETER DEPENDENT BALITSKY-KOVCHEGOV EQUATION

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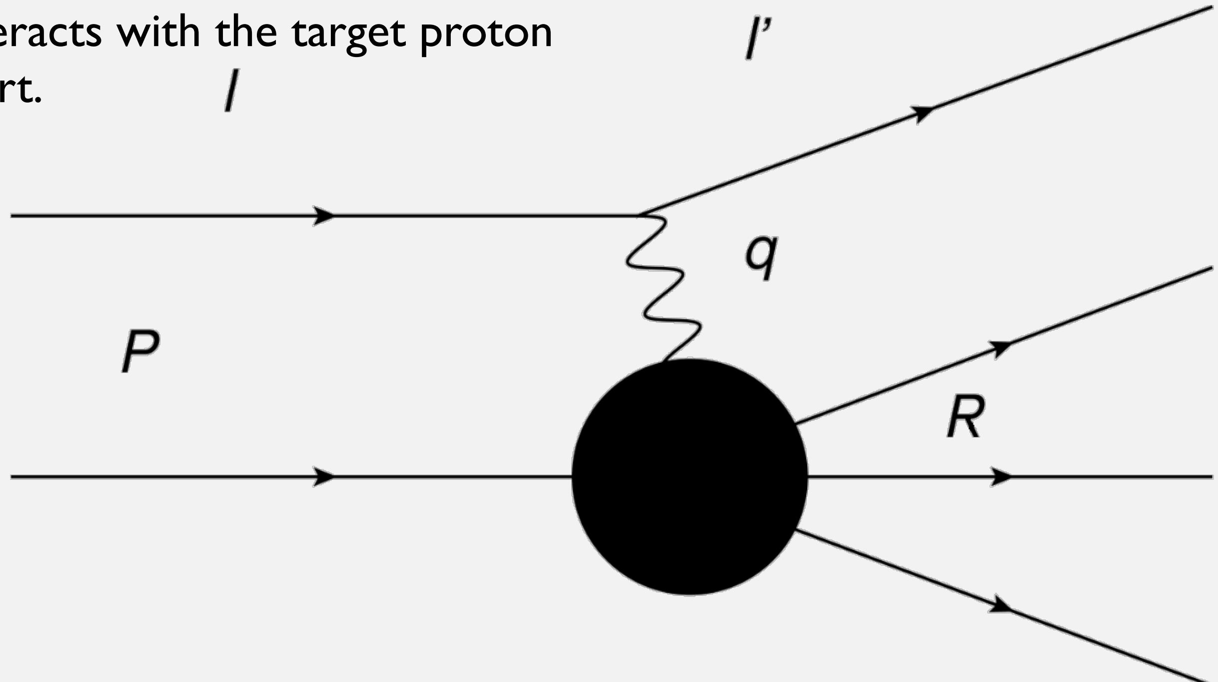
UPC group meeting at Decin 2.5.-3.5.2018

WHAT IS BK EQUATION?

DEEP INELASTIC SCATTERING

The electron-proton collisions are considered to happen as:

1. The incoming electron emits a virtual photon.
2. The virtual photon interacts with the target proton
3. The proton breaks apart.

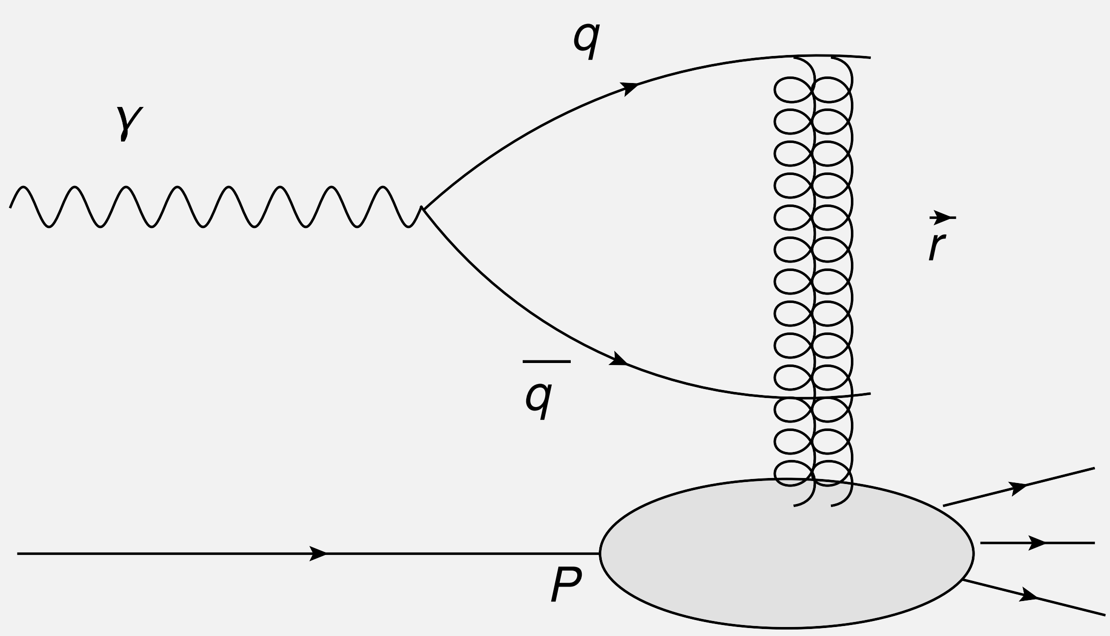


HOW DOES A PHOTON INTERACT WITH A PROTON?

DIPOLE MODEL

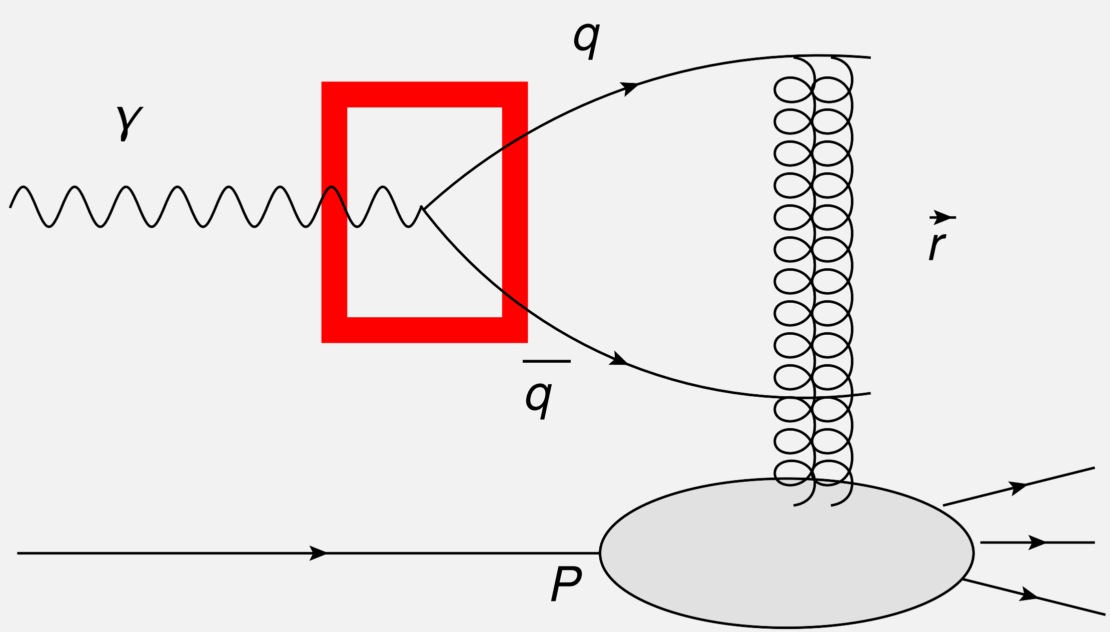
The photon must interact strongly with the target proton, how is that possible?

1. The virtual photon first fluctuates into a quark-antiquark pair
2. Then it exchanges an object with vacuum quantum numbers with the proton



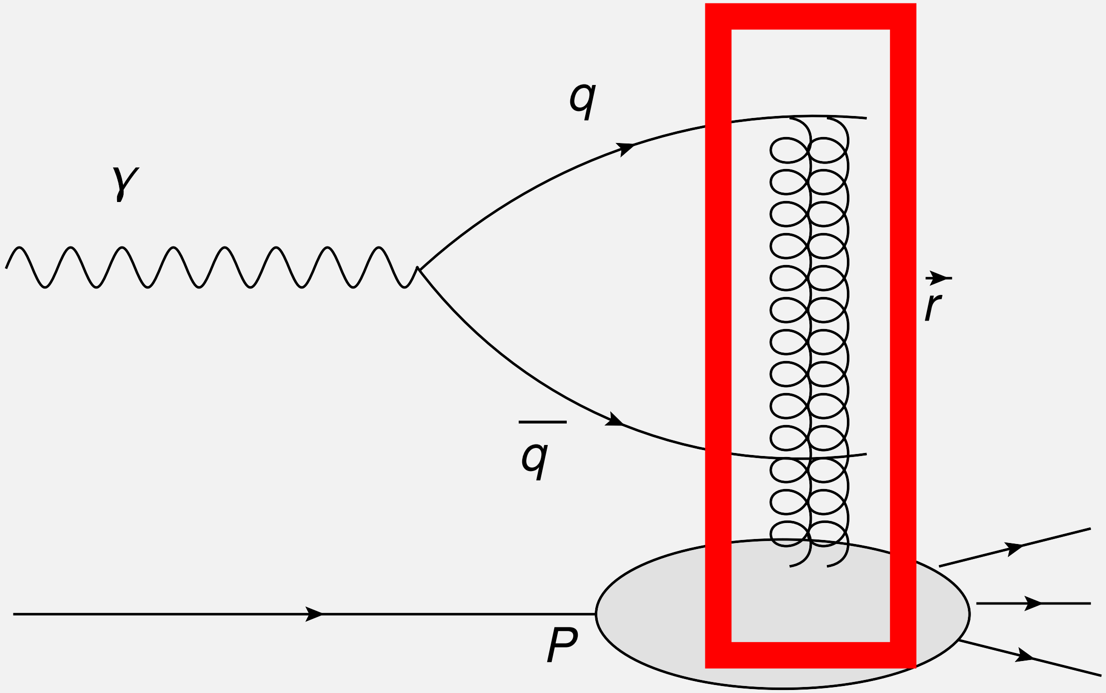
DIPOLE MODEL

The probability of a photon splitting to a quark-antiquark pair is computed from QFT.



DIPOLE MODEL

To compute the cross section of the interaction, we are missing the $\sigma_{dipole\text{-}proton}$



HOW DO WE OBTAIN THE DIPOLE-PROTON CROSS SECTION?

BK EQUATION

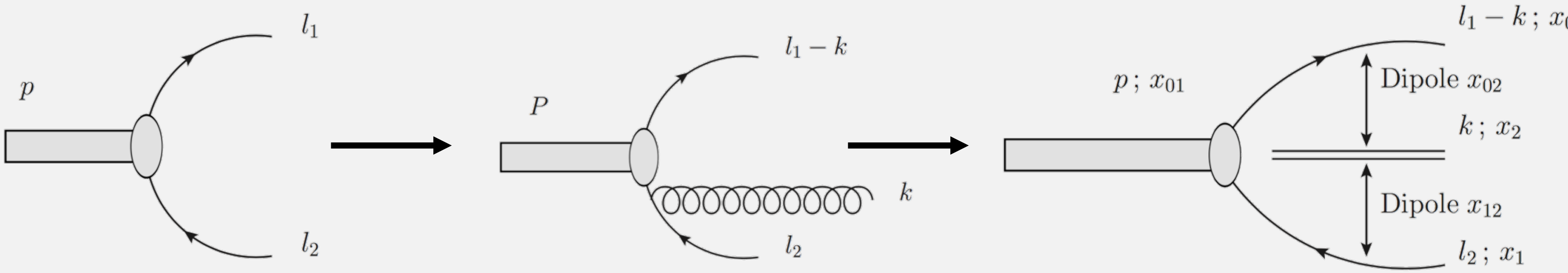
The BK equation governs the $\sigma_{dipole\text{-}proton}$, also called the scattering amplitude.
(Thanks to the optical theorem)

The main idea of the computation of the scattering amplitude is as follows:

1. Boost into a frame, where the dipole is at rest. Here the dipole is bare.
2. Then boost a bit, so that we add a bit energy into the system.
3. One of the quarks emits a gluon.
4. In the limit of high number of colors, this gluon fluctuates into another qq pair.
5. Two daughter dipoles are created. These contribute independently to the scattering amplitude

BK EQUATION

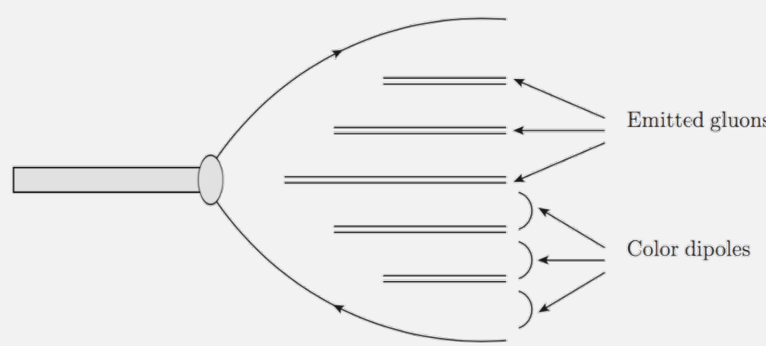
Schematically this means:



Add a bit of energy

High N_c limit

After some time, the initial dipole becomes dressed.



BK EQUATION

Mathematically, this realates to:

$$\frac{\partial N(r,Y)}{\partial \ln Y} = \int d\vec{r}_1 K(\vec{r},\vec{r}_1,\vec{r}_2)(N(\vec{r}_1,Y) + N(\vec{r}_2,Y) - N(\vec{r},Y) - N(\vec{r}_1,Y)N(\vec{r}_2,Y))$$

$$K(\vec{r},\vec{r}_1,\vec{r}_2) = \frac{\alpha_s(r^2)N_c}{2\pi}\left(\frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_1^2}\left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1\right) + \frac{1}{r_2^2}\left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1\right)\right)$$

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This is the change of the scattering amplitude, when we add a bit of energy into the system.

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Kernel is computed from QCD to reflect the probability of the gluon emission.

BK EQUATION

Okay, we know how to evolve to higher energy, but how about the initial condition?

Initial shape of the scattering amplitude is obtained under some approximations for example with the use of the MV model.

$$N^{MV}(r) = 1 - \exp\left(\frac{-(r^2 Q_{s0}^2)^{\gamma}}{4} \ln\left(\frac{1}{r^2 \Lambda_{QCD}^2} + e\right)\right)$$

Where Λ_{QCD}, γ and Q_{s0}^2 are constants.

WHAT DOES IMPACT PARAMETER HAVE TO DO WITH ALL THIS?

IMPACT PARAMETER

Allowing the dependence of the scattering amplitude not only on transverse size and energy, but also on impact parameter makes things much more complicated.

Option a) Factorizing impact parameter dependence.

$$N(\vec{r}, \vec{b}, x) \cong T(\vec{b}) N(\vec{r}, x)$$

If we factorize impact parameter dependence, we can integrate over it and replace it with a multiplicative factor.

$$\sigma^{q\bar{q}}(\vec{r}, x) = \int d\vec{b} N(\vec{r}, \vec{b}, x) = \sigma_0 N(x, \vec{r})$$

This factor then stays the same for all energies and dipole sizes and is usually fit to data.

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Option b) Solving the equation with the full impact parameter dependence.

$$\frac{\partial N(\vec{r},\vec{b},Y)}{\partial Y} = \int d\vec{r_1} K^{run}(r,r_1,r_2)(N(\vec{r_1},\vec{b_1},Y) + N(\vec{r_2},\vec{b_2},Y) -$$

$$- N(\vec{r},\vec{b},Y) - N(\vec{r_1},\vec{b_1},Y)N(\vec{r_2},\vec{b_2},Y))$$

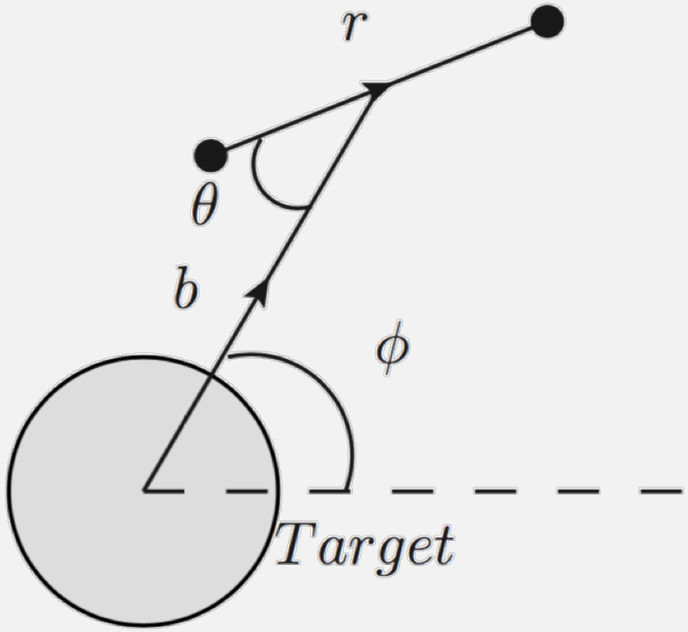
This adds two additional dimension to the computation. The usual grid size in these two dimensions is 225x20. This then means, that the CPU time gets increased with a factor of 4500.

IMPACT PARAMETER

Furthermore, we have to generalize our initial condition!

The initial condition now depends in the size of *b* as well as on the orientation of the angle θ.

We can still safely assume the rotational symmetry of the proton and neglect the dependence on ϕ.



IS THE BK EQUATION READY FOR ALL THIS?

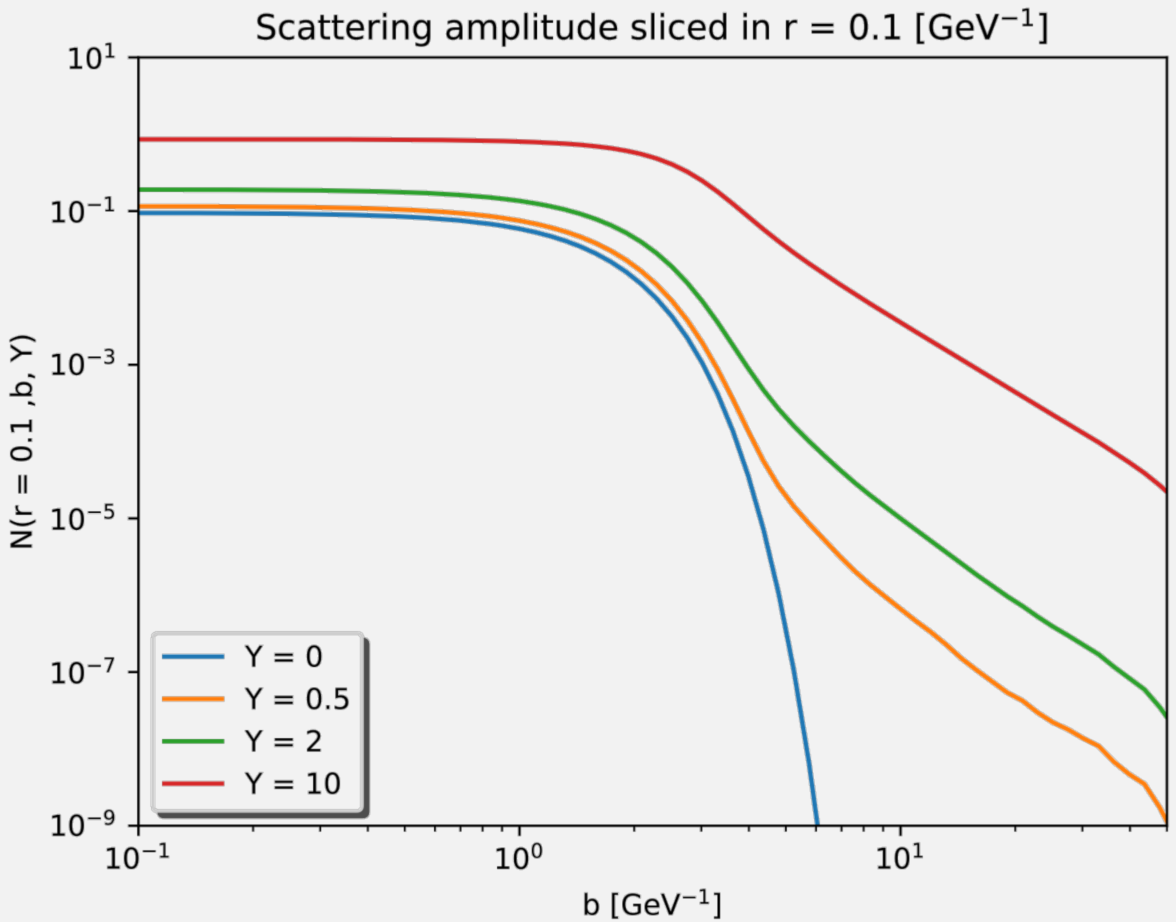
BK CONSTRAINTS

The BK equation was derived in purely perturbative way (remember the kernel).

Solving it at large values of impact parameter (distances) means, that the running coupling gets close to one and we get to non-perturbative region!

Is it a real problem? What if we try to run the equation as it is?

BK CONSTRAINTS



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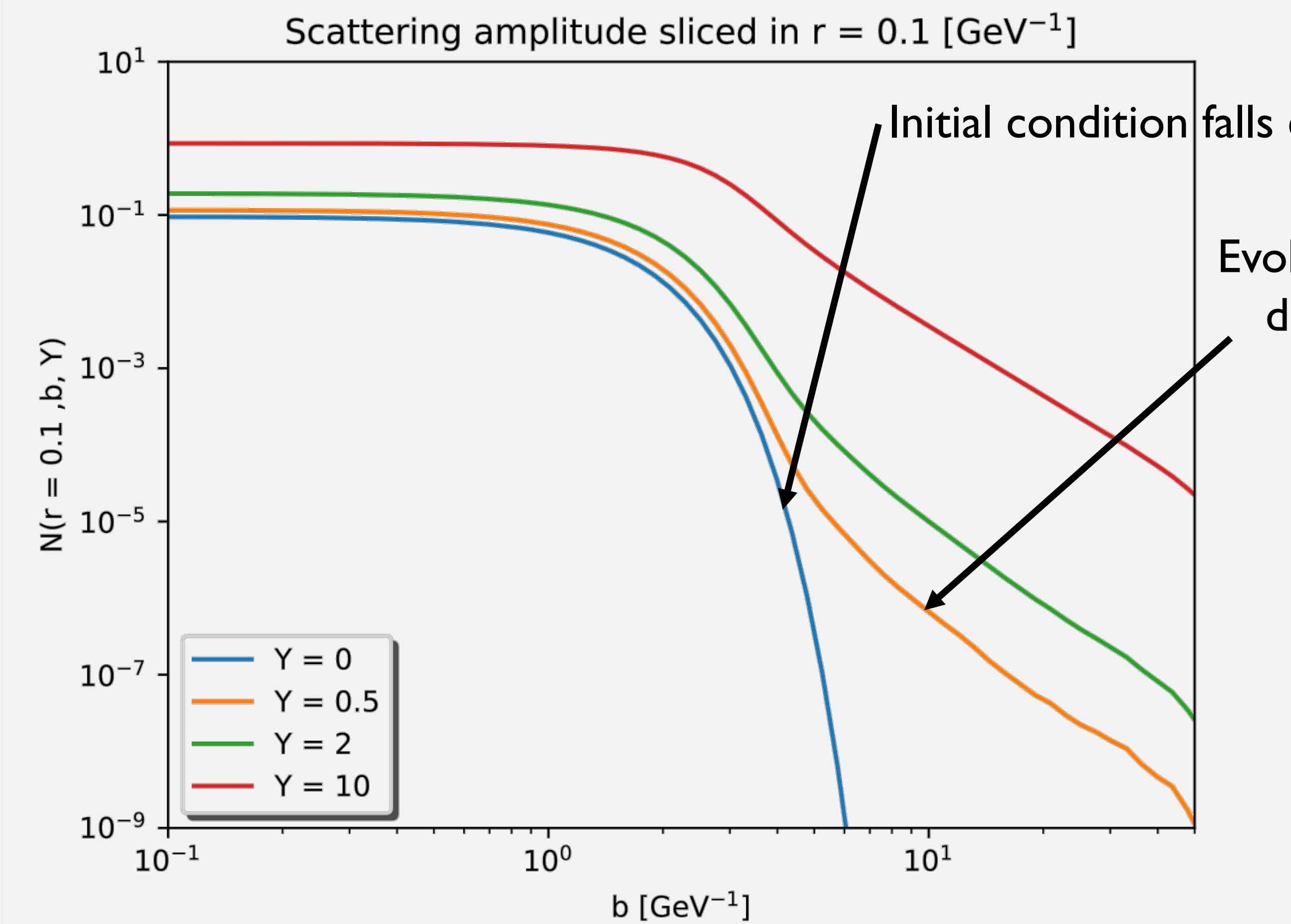
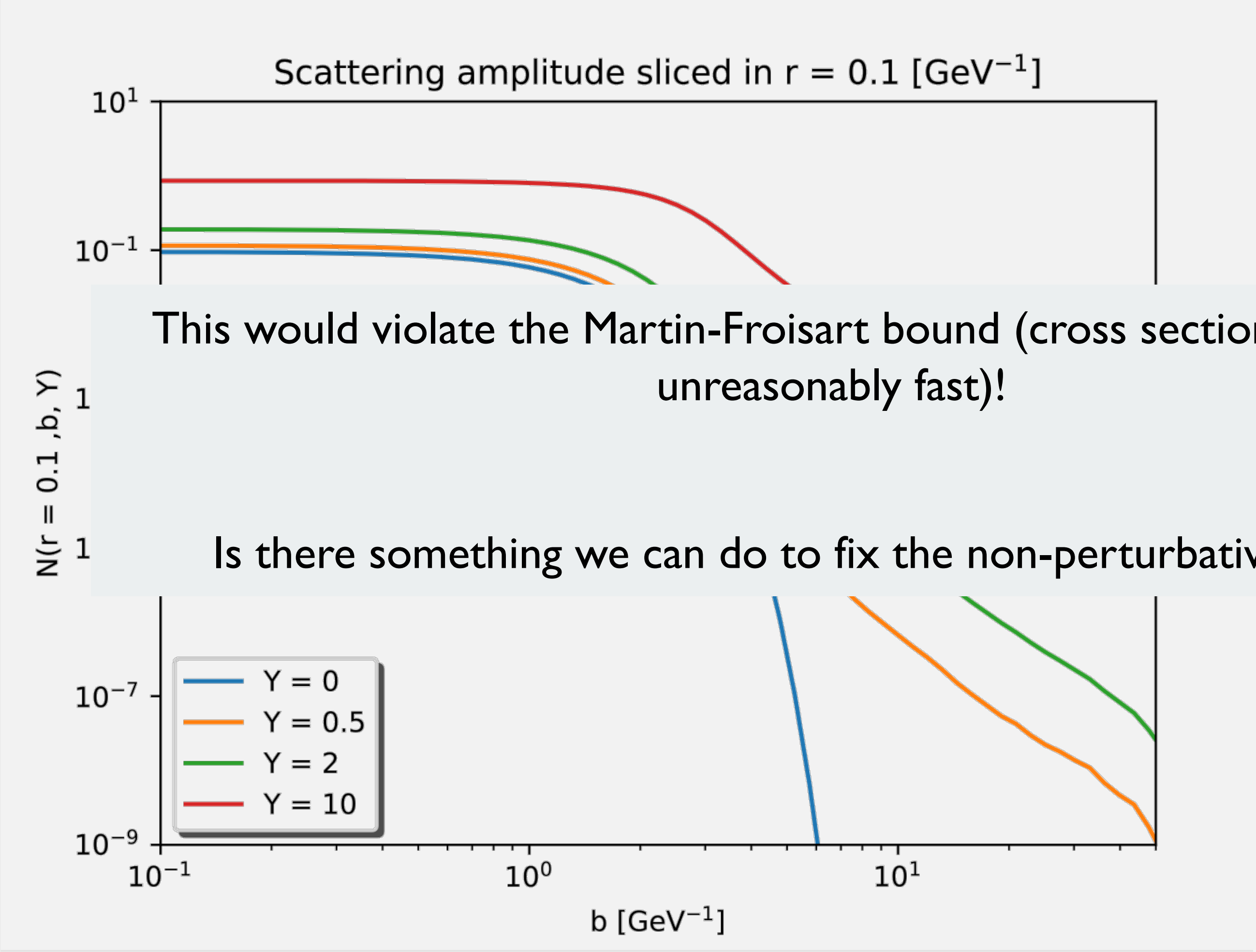


Initial condition falls exponentially

Evolution increases the larger dipoles into a power-like growth.

BK CONSTRAINTS



This would violate the Martin-Froisart bound (cross section would grow unreasonably fast)!

Is there something we can do to fix the non-perturbative regions?

KERNEL CUTOFF

The simple solution to this problem is that we can cut the kernel, so that dipoles, that are too big would not contribute to the evolution.

$$\frac{\partial N(r,\vec{b},Y)}{\partial Y} = \int d\vec{r_1} K^{run}(r,r_1,r_2)\Theta\left(\frac{1}{m^2}-r_1^2\right)\Theta\left(\frac{1}{m^2}-r_2^2\right)$$
$$\left(N(r_1,\vec{b_1},Y)+N(r_2,\vec{b_2},Y)-N(r,\vec{b},Y)-N(r_1,\vec{b_1},Y)N(r_2,\vec{b_2},Y)\right)$$

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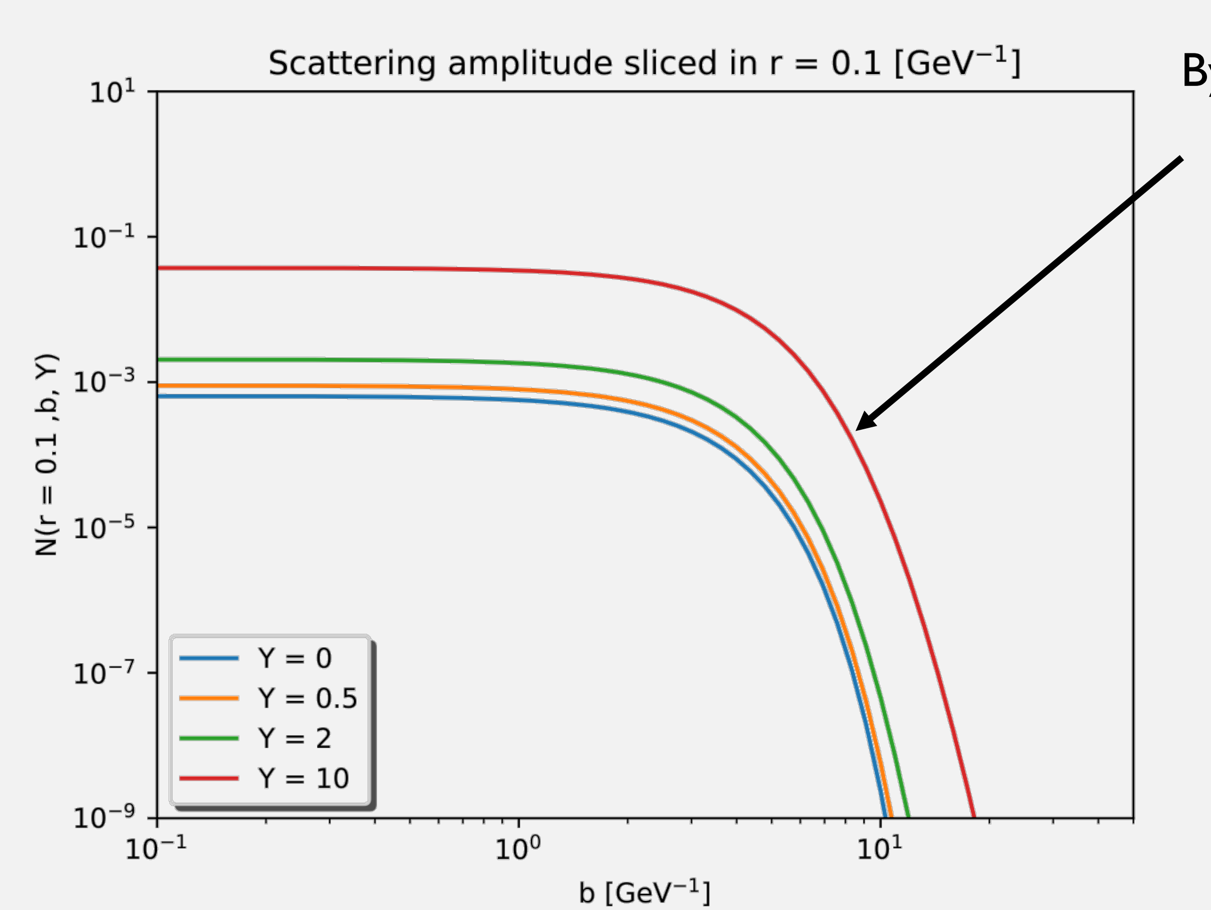
$$\left(N(r_1,\vec{b_1},Y)+N(r_2,\vec{b_2},Y)-N(r,\vec{b},Y)-N(r_1,\vec{b_1},Y)N(r_2,\vec{b_2},Y)\right)$$

Mass of the emitted gluon is a free parameter, that is fitted to data.

Does this remedy solve the problem?

KERNEL CUTOFF



By imposing the cutoff of the kernel, we maintain the exponential falloff of the scattering amplitude.

WHAT DO THE DATA SAY TO THIS?

STRUCTURE FUNCTION

$$F_2 = F_2^{Bal} + F_2^{soft}$$

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$$\sigma_0 = 75.98 \text{ GeV}^{-2}$$

$$F_2^{Bal}(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}} \int \sum_i d\vec{r} d\vec{b} dz \mid \Psi^i_{T,L}(z, \vec{r}) \mid^2 \sigma^{q\bar{q}}(\vec{r}, \vec{b}, \tilde{x})$$

$$F_2^{soft} = \frac{Q^2}{2\pi\alpha_{em}} \sigma_0 \int_{\frac{1}{m}} r dr \int_0^1 dz (|\Psi_L|^2 + |\Psi_T|^2)$$

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$$\sigma^{q\bar{q}}(\vec{r},\vec{b},x) = 2N(\vec{r},\vec{b},x)$$

$$\mid \Psi^i_{T,L}(z,\vec{r}) \mid^2 = \mid \Psi^i_T(z,\vec{r}) \mid^2 + \mid \Psi^i_L(z,\vec{r}) \mid^2$$

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$$\mid \Psi^i_T(z,\vec{r},Q^2) \mid^2 = \frac{3\alpha_{em}}{2\pi^2} e^2_{q_i}((z^2+(1-z)^2)\epsilon^2 K_1^2(\epsilon r) + m^2_{q_i} K_0^2(\epsilon r))$$

$$\mid \Psi^i_L(z,\vec{r},Q^2) \mid^2 = \frac{3\alpha_{em}}{2\pi^2} e^2_{q_i}(4Q^2 z^2(1-z)^2 K_0^2(\epsilon r))$$

EW coupling

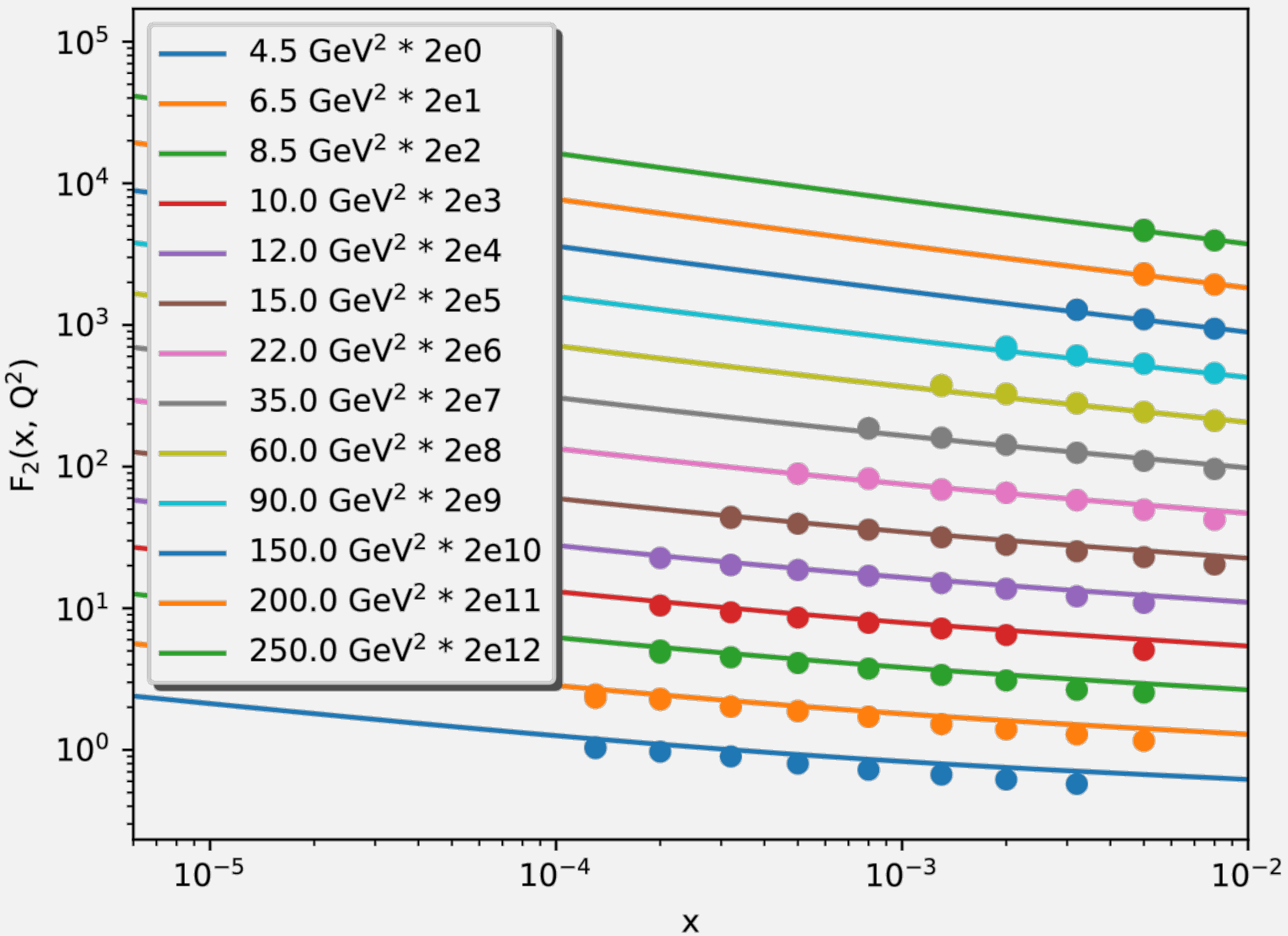
quark charge

$$\epsilon^2 = z(1-z)Q^2 + m^2_{q_i}$$

MacDonald functions

quark mass

STRUCTURE FUNCTION



CONCLUSIONS

- The BK evolution equation is one of the ways to solve the photon-hadron interactions.

- Incorporating impact parameter brings many difficulties, such as tremendous CPU time and necessity for a massive kernel-cutoff.

- The strong dependence of the results on the value of the cutoff parameter *m* is a problem.

- We are currently working on solving the Kernel and initial condition cutoff by imposing geometrical constraints in the impact parameter plane.

THANK YOU FOR YOUR ATTENTION

No matter what, don’t lose hope. We are all bombastic.

- Dan Nekonečný

REFERENCES

- Numerical solution of the nonlinear evolution equation at small x with impact parameter and beyond the LL approximation: Jeffrey Berger and Anna M. Sta´sto. 1010.0671

- Small x nonlinear evolution with impact parameter and the structure function data: Jeffrey Berger and Anna M. Sta´sto. 1106.5740v3

- On solutions of the Balitsky-Kovchegov equation with impact parameter: K. Golec–Biernat and A. M. Sta´sto. 0306279