A novel approach to detection of interaction range

Zuzana Szabová

Czech technical univeristy in Prague
Faculty of nuclear sciences and physical engineering

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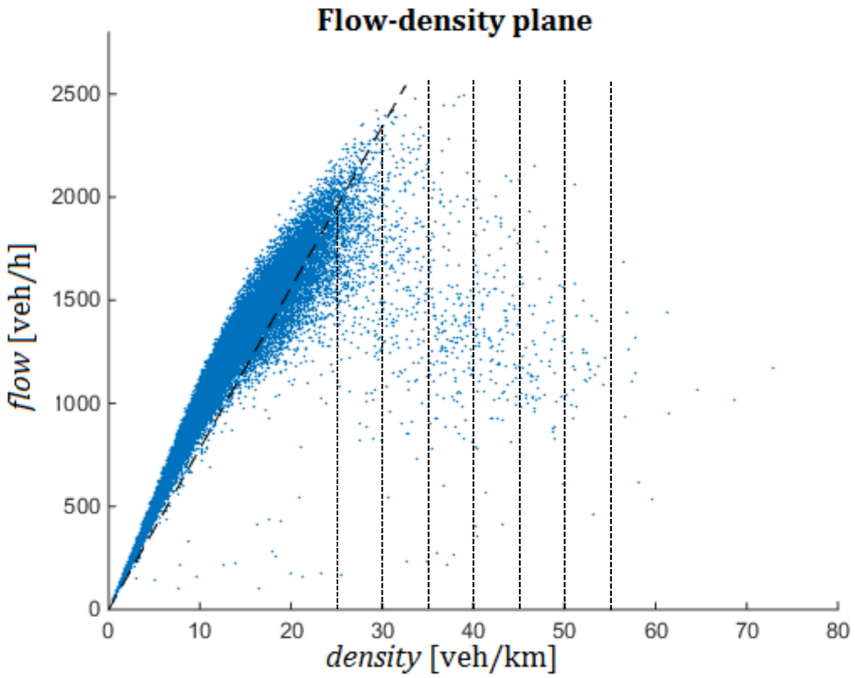
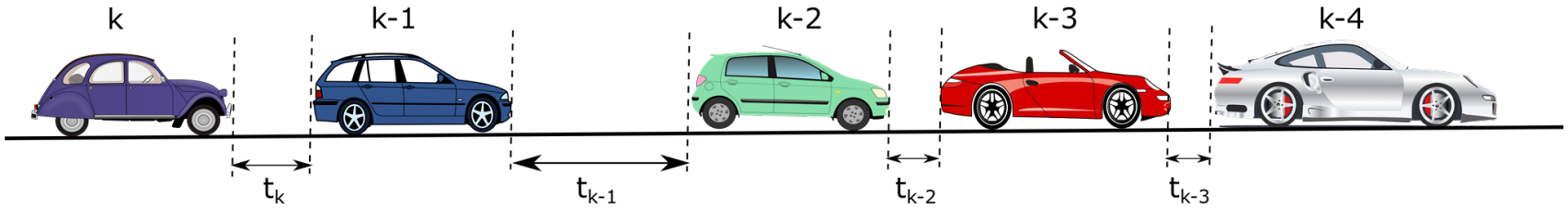
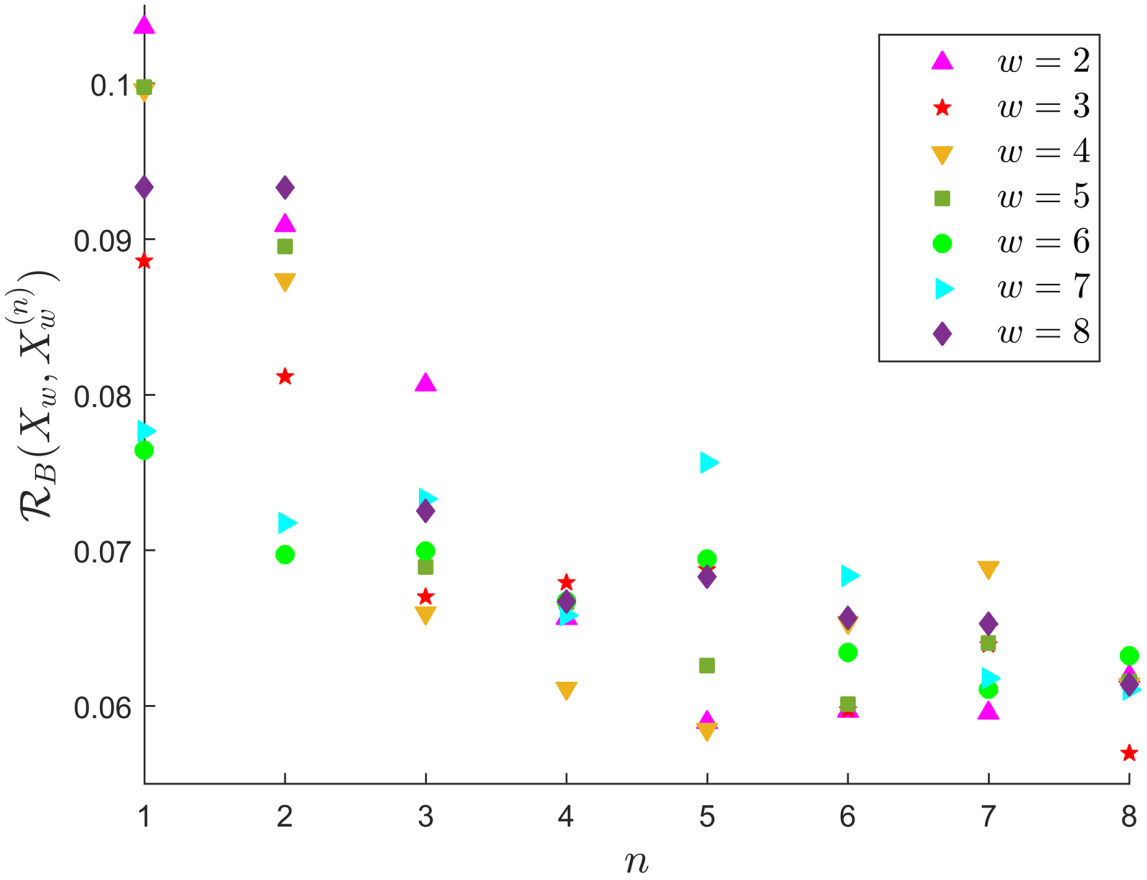
Fundamental diagram and density regions

Traffic phases
- Free flow
- Congested traffic
  - Synchronized flow [S]
  - Wide mowing jam [J]



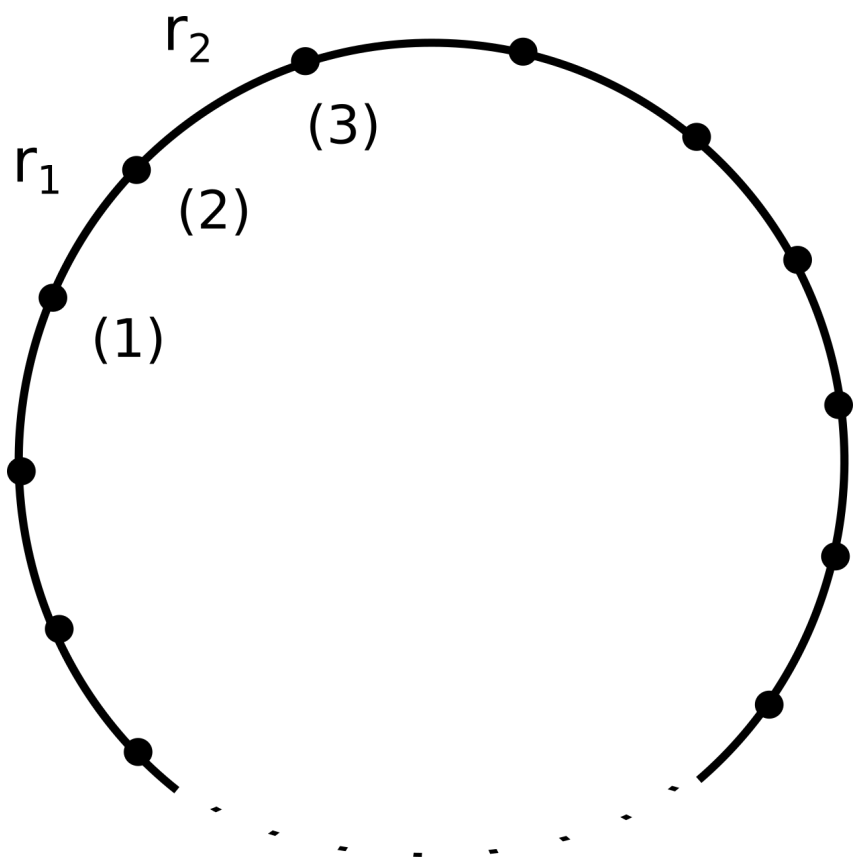
Brownian distance-correlation of clearances.





Thermodynamic traffic gas

- Types of used repulsive potential:
  1. logarithmic: $\varphi(r) = \ln(r)$
  2. hyperbolic: $\varphi(r) = \frac{1}{r}$
- Considered types of repulsive potential with respect to interaction range:
  1. short-ranged potential
  2. middle-ranged potential
- Model is characterised by parameter $\beta = \frac{1}{k_B T}$ - resistivity



Multiclearance distribution

Distribution of multiclearances is given by convolution formula:

$$\rho(x \mid \mu) = \rho(x) \star \rho(x \mid \mu - 1) \equiv \int_{\mathbb{R}} \rho(s)\rho(x - s \mid \mu - 1)\mathrm{d}s.$$

We introduce two parametric family of densities

$$p(x) = \Theta(x) A x^{\alpha} \mathrm{e}^{-\frac{\beta}{x}} \mathrm{e}^{-Dx},$$

where

$$D = \frac{\alpha}{\mu} + \frac{\beta}{\mu^2} + \frac{3 - \mathrm{e}^{-\sqrt{\frac{\beta}{\mu}}}}{2\mu}, \quad A = \left(2\sqrt{\frac{\beta}{D}} \mathscr{K}_{\alpha+1}(2\sqrt{\beta D})\right)^{-1}.$$

Perturbation function

Perturbation function is given by formula:

$$\psi(x \mid \mu) = \int_0^x (\hat{\rho}(y \mid \mu) - \hat{\rho}(y \mid 1) \star \hat{\rho}(y \mid \mu - 1))\mathrm{d}y$$

Kolmogorov distance:

$$G(\mu) = \sup_{x \in \mathbb{R}} \mid \psi(x \mid \mu) \mid$$

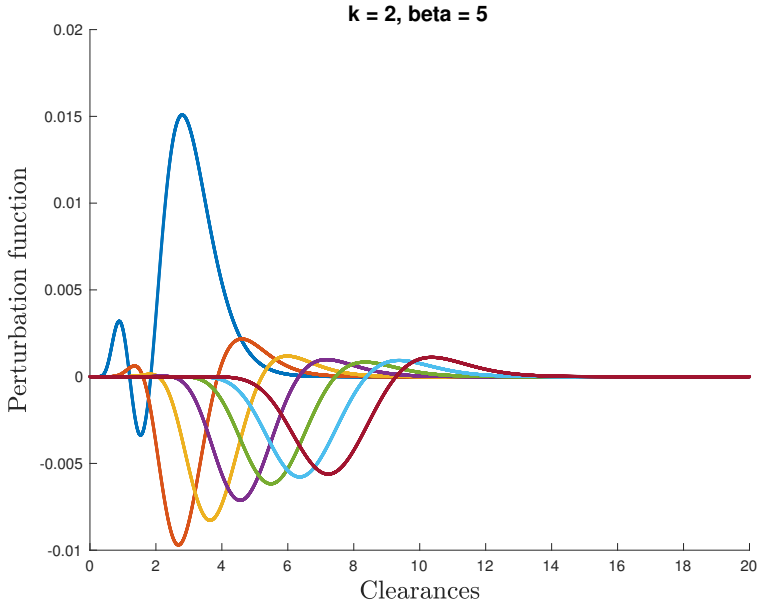


Figure: Courses of perturbation functions for specific value of stochastic resistivity β, interaction range and degree of multi-clearance.

Monte Carlo methods and correlation analysis

- Metropolis-Hastings algorithm: generates distribution of particles for a specific repulsive potential after the thermodynamic balance has been reached

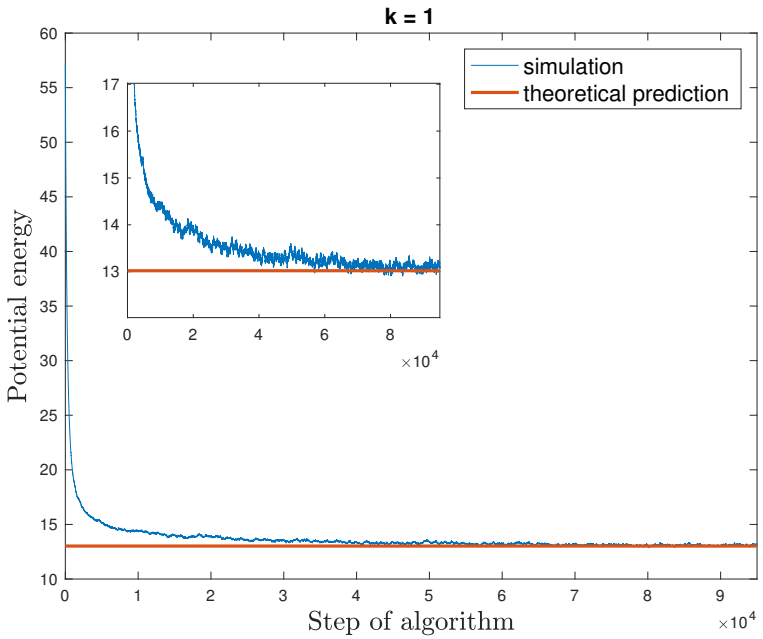
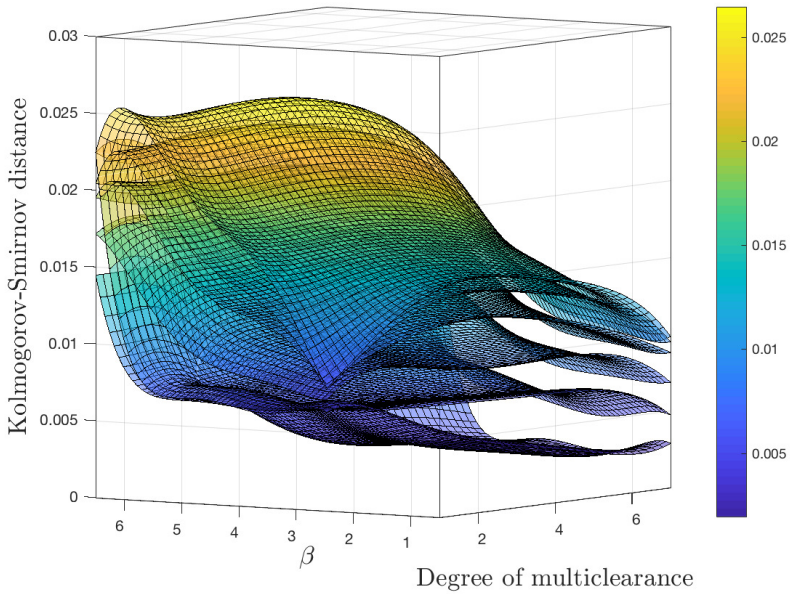


Figure: Course of potential energy with respect to number of steps of algorithm.

Kolmogorov distance of perturbation function



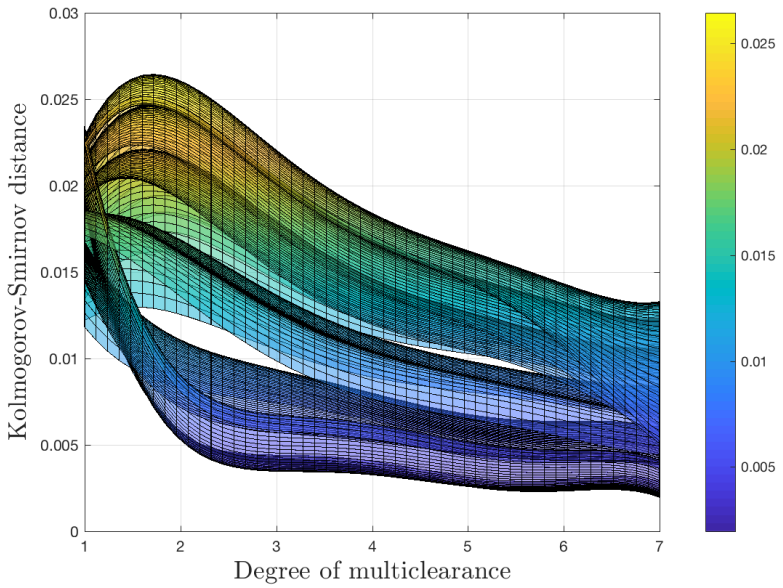


Figure: Courses of Kolmogorov distance of perturbation function with respect to stochastic resistivity β, interaction range and degree of multi-clearance.

Precision of simulation method

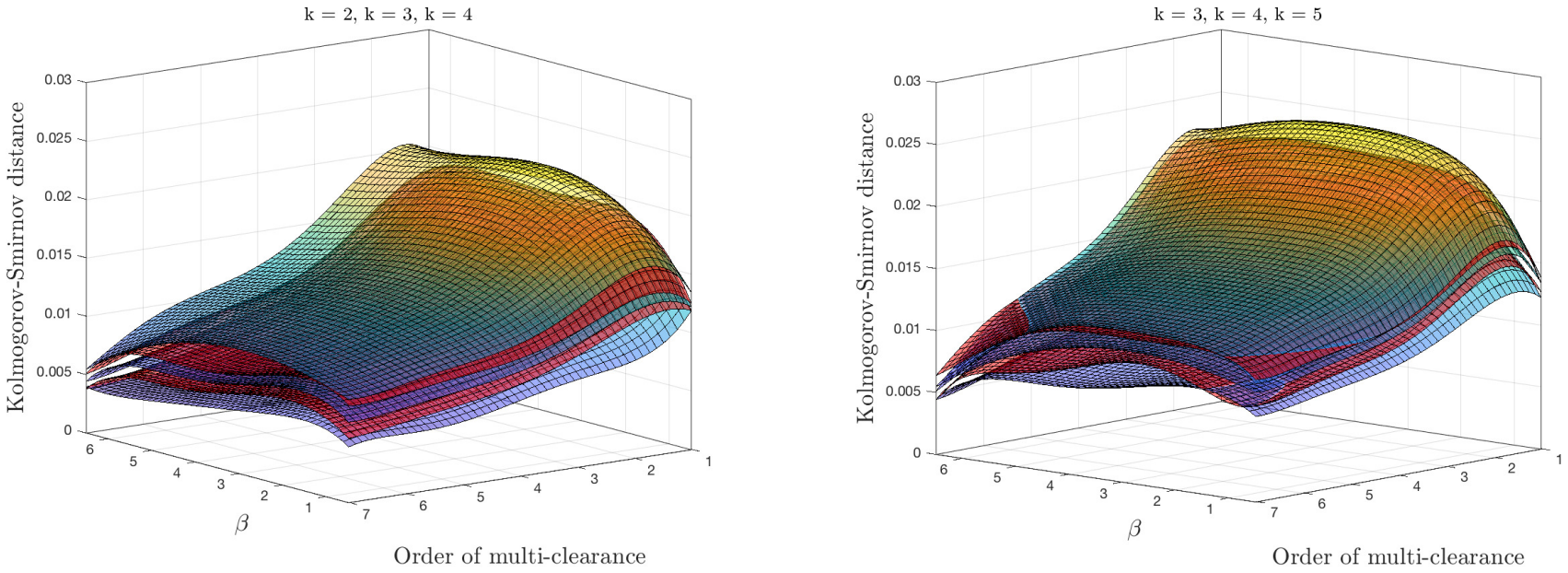


Figure: Kolmogorov Smirnov distance of perturbation function with error bars (red planes) with respect to stochastic resistivity and order of multi-clearance for different interaction ranges.

Detection of interactions by means of perturbation function

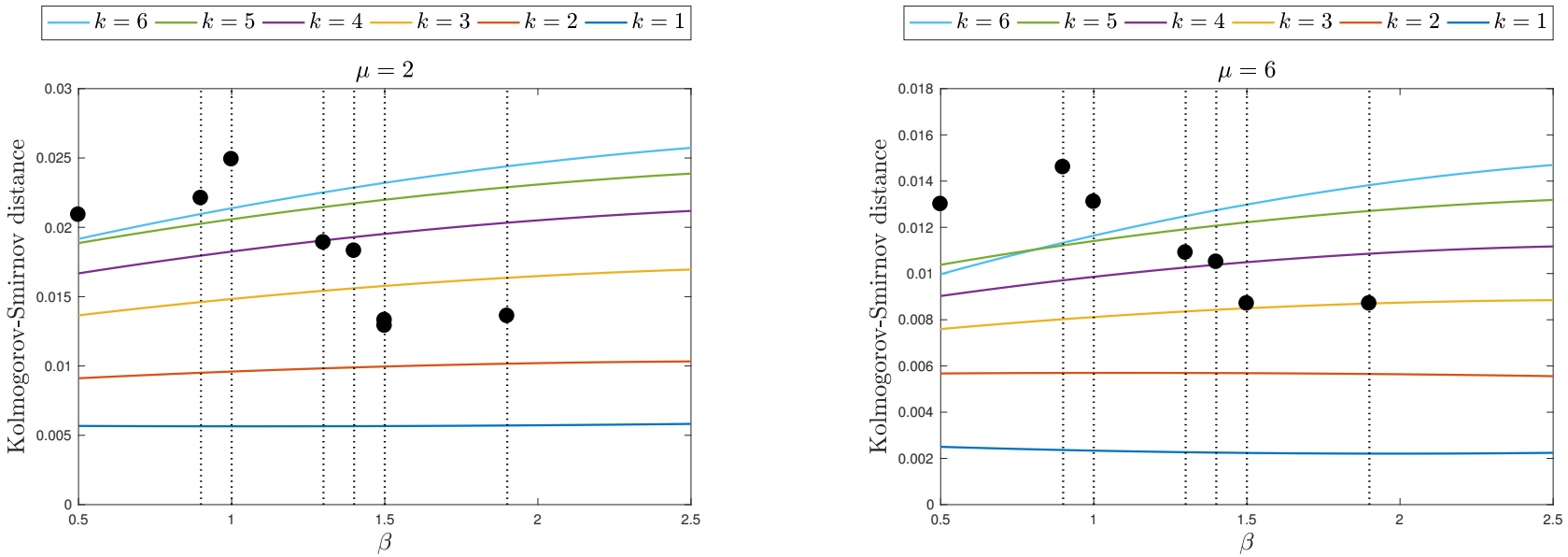
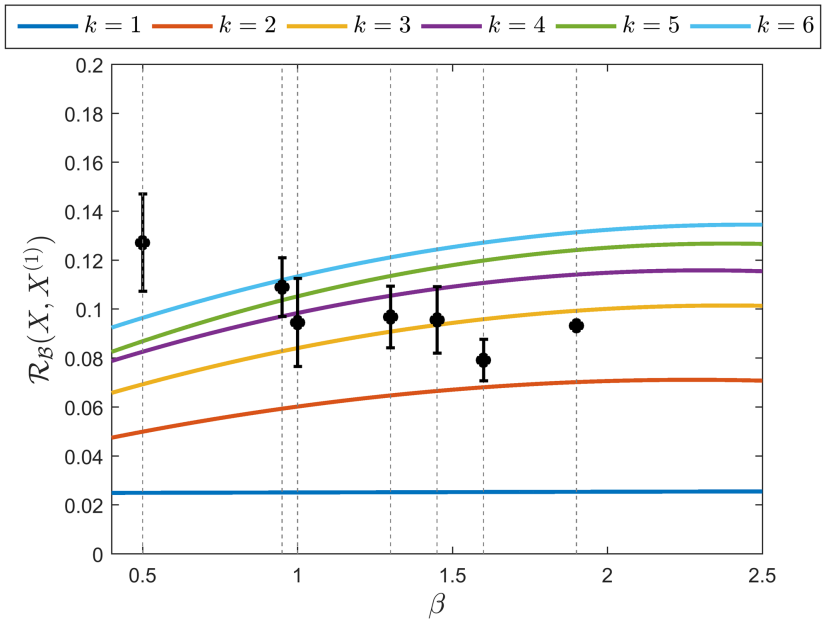
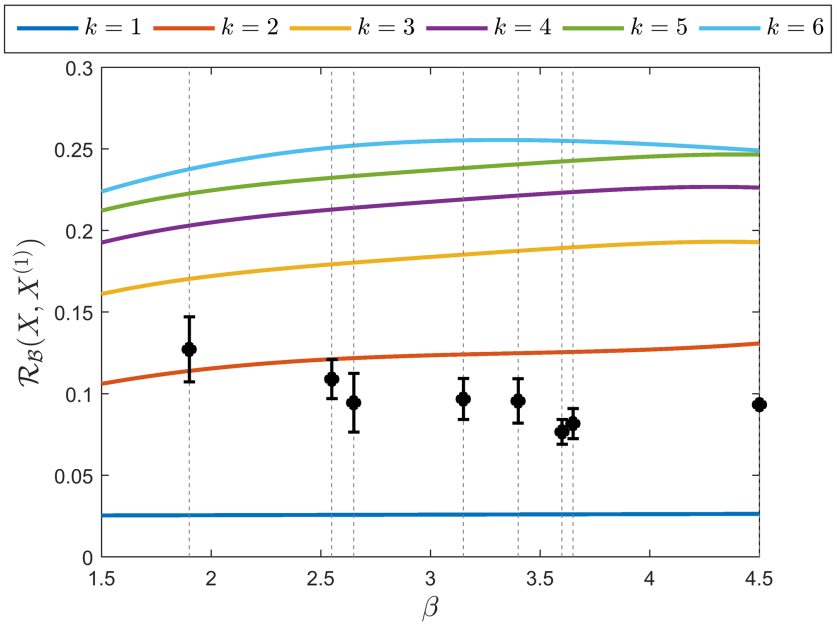


Figure: Curves represent polynomial interpolations of values of Kolmogorov distance of simulations of perturbation function $G(\mu)$. Black bullets represent empirical values of Kolmogorov distance of perturbation function obtained by MDE for hyperbolic potential.

Detection of interaction range in density regions



(a) Hyperbolic potential



(b) Logarithmic potential

Balanced density

Let there be a Lebesgue space $\{M_\lambda, \mathbb{R}, \lambda(x)\}$ with one-dimensional Lebesgue measure. λ-measurable function $f(x) : \mathbb{R} \to \mathbb{R}$ is called balanced density (function) if

1. $\mathrm{Dom}(f) = \mathbb{R}$
2. $\mathrm{Ran}(f) \subset [0, +\infty)$
3. $\mathrm{supp}(f) \subset [0, +\infty)$
4. $f(x) \in PC(\mathbb{R})$ (piecewise continuous)
5. $f(x) \in \mathscr{L}(\mathbb{R})$
6. $\exists\, \omega > 0$ *such that*

$$\forall \alpha > \omega : \lim_{x \to +\infty} f(x) \mathrm{e}^{\alpha x} = +\infty,$$
$$\forall \alpha < \omega : \lim_{x \to +\infty} f(x) \mathrm{e}^{\alpha x} = 0.$$

Interesting results

Theorem

Let $f(x), g(x) \in \mathscr{B}$ *and* $\mathrm{inb}(f) = \omega, \mathrm{inb}(g) = s$. *Then* $(f \star g)(x) \in \mathscr{B}$ *and* $\mathrm{inb}(f \star g) = \min\{\omega, s\}$.

Theorem

Let $f(x), g(x) \in \mathscr{B}$ *such that* $\mu_k^f = \mu_k^g, \forall k \in \mathbb{N}_0$, *where* $(\mu_k^f)_{k=0}^{\infty}, (\mu_k^g)_{k=0}^{\infty}$ *are moment codes of densities* $f(x), g(x)$. *Then*

$$f(x) \sim g(x).$$

Thank you for your attention.