

Traffic modelling
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Simulation
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Hypothesis
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Varification process
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Conclusion
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Statistical resistivity of non-equilibrium states in transport gases

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Outline

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Traffic modelling

- Microscopic model
 - each car is an individual agent with individual properties
- Traffic gas
 - statistical resistivity β - a parameter describing the mental state of the driver
 - short-range interaction potential
- Simulating headway-distributions φ_i
 - N particles on a circle
 - using Metropolis-Hastings Algorithm (MCMC method)

Short-range traffic gas

- Short-range - only the closest agents interact
- Using logarithmic potential energy
 - it has been proven that φ_i has Gamma distribution, with parameter β when in equilibrium state
- Gamma distribution:

$$f(x; \beta) = \theta(x) \frac{(\beta + 1)^{\beta+1}}{\Gamma(\beta + 1)} x^{\beta} e^{-(\beta+1)x}$$

Distribution of φ_i

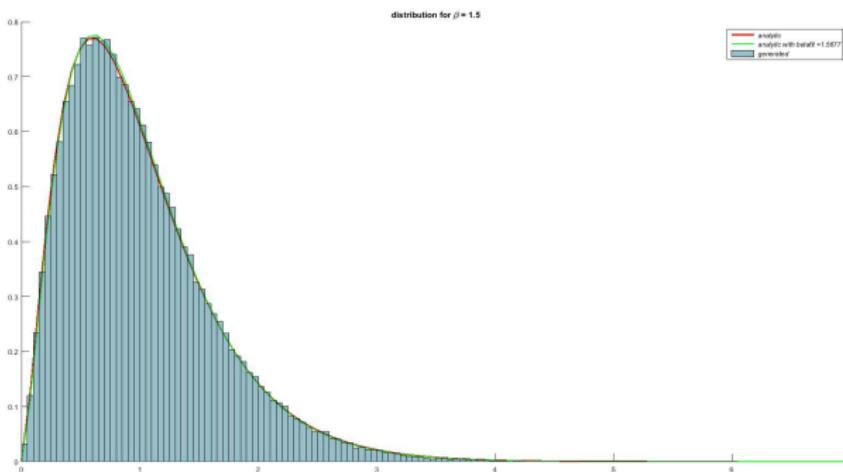


Figure: A histogram of φ_i in equilibrium state for $\beta = 1, 5$.

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Potential energy

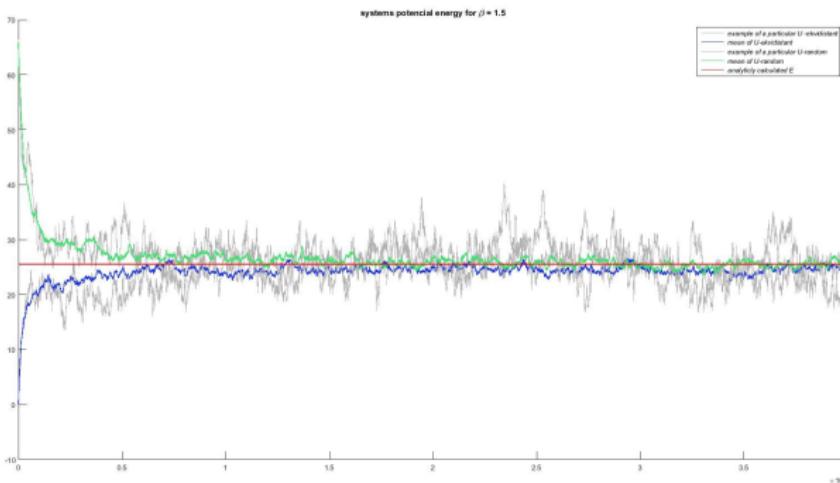


Figure: Convergence to the equilibrium state with $E[U]$

Equidistant vs. random initial distributions

Equidistant

- Potencial goes $0 \rightarrow E[U]$
- Converges to equilibrium state faster
- Initial distribution has $\beta \rightarrow \infty$
- $f(x; \beta \rightarrow \infty) = \delta(x - 1)$

Random

- Potencial goes $\infty \rightarrow E[U]$
- Converges to equilibrium state slower
- Initial distribution has $\beta = 0$
- $f(x; 0) = \theta(x)e^{-x}$

Hypothesis

- Does the non-equilibrium state have the same distribution as the equilibrium state, only having a different parameter β_{non} ?

$$f(x; \beta_{non}) = \theta(x) \frac{(\beta_{non} + 1)^{\beta_{non} + 1}}{\Gamma(\beta_{non} + 1)} x^{\beta_{non}} e^{-(\beta_{non} + 1)x}$$

- Does a time dependence of the parameter β_{non} exist?

Convergence towards the equilibrium state

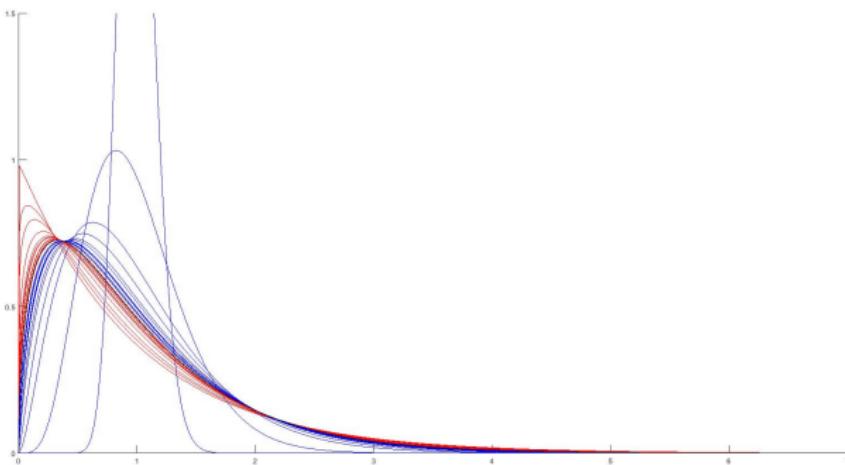
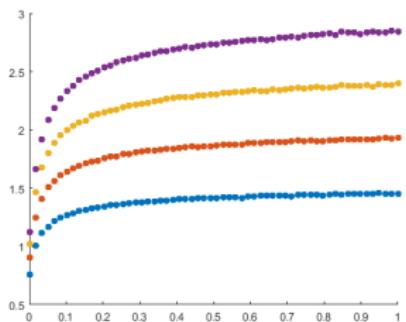


Figure: Convergence of non-equilibrium state Gamma distributions towards the equilibrium state

Fitting of β_{non} in non-equilibrium states



Fitting function $g(x)$ must fulfil these requirements:

$$g(0) = 0$$

$$\lim_{x \rightarrow \infty} g(x) = \beta$$

Figure: Convergence of parameter β_{non} towards β for $\beta = 1.5, 2, 2.5, 3$.

Time dependance of the parameter β

$$\beta_{non}(x) = p_1 \left(\frac{x}{x + p_2} \right)^{p_3}$$

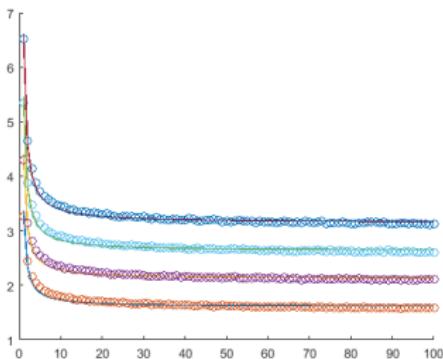
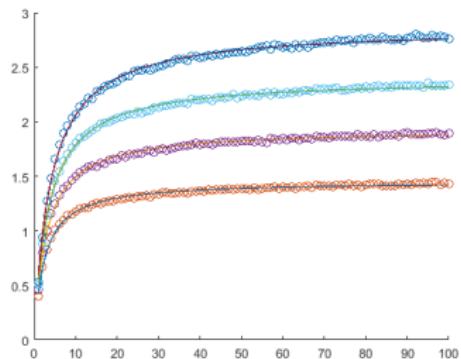


Figure: Functions $\beta_{non}(x)$ for distributions of φ with random and equidistant initial distributions.

Bootstrap method

1. Generate data for time τ ,
2. randomly choose a sample set of the data,
3. calculate the parameter β for time τ and the appropriate Gamma distribution,
4. test the empirical distribution of the sample data, and the calculated Gamma distribution (K-S test), and
5. for time τ repeat steps 2 - 4.

Expected significance level is $\alpha \approx 0.05$.

Results

time	3 parameter fit	2 parameter fit
1000	0,0706666666666667	0,0643333333333333
2000	0,0583333333333333	0,0576666666666667
3000	0,0583333333333333	0,0513333333333333
4000	0,0486666666666667	0,0620000000000000
5000	0,0596666666666667	0,0596666666666667
6000	0,0633333333333333	0,0616666666666667
7000	0,0556666666666667	0,0480000000000000
8000	0,0510000000000000	0,0520000000000000
9000	0,0546666666666667	0,0593333333333333
10000	0,0486666666666667	0,0406666666666667
11000	0,0536666666666667	0,0536666666666667
12000	0,0473333333333333	0,0513333333333333
13000	0,0570000000000000	0,0556666666666667
14000	0,0416666666666667	0,0343333333333333
15000	0,0346666666666667	0,0560000000000000

Conclusion

- The system's initial distribution influences the rate of convergence.
- The analytic formula for β_{non} is a time dependent, rational function.
- The results for non-equilibrium states have $\alpha \approx 0.05$.

References

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